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A Sub-Modeling Technique to Balance Force System Boundary Condition

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Abstract

Revolute joints and sliding joints are essential mechanical elements connecting moving parts of machineries. When implementing the whole machine/global modeling, the connection joints and their local detailed geometric features are usually simplified in order to improve the computational efficiency and avoid convergence difficulties. These kinds of simplification strategies may lead to the problem that the accurate stress in the vicinity of the simplified local regions could not be obtained from the analysis of the global model. In such cases, subsequent sub-modeling analyses are usually employed to obtain the accurate stress results in these regions. Simplification of the local features in the model may sometimes result in significant changes in stiffness of these local regions. When traditional interpolated displacement type of boundary conditions are used to indirectly apply loading to the sub-model, the actual loading added to the sub-model boundary could be much different to the loading derived from the global model due to the stiffness error of the global model. In contrast to applying the displacement boundary condition, a method of directly implementing forces to the sub-model cut-boundaries has been proposed in this paper. These forces applied to the cut-boundaries could be obtained from the analysis results of the global model. Numerical analysis results have shown that the newly proposed sub-modeling technique can load the model more accurately than the traditional sub-model method and the analysis accuracy is not sensitive to the degree of simplification of the global model. The predicted stress results for a telescopic boom of a truck-mounted crane have also been validated by experimental results.

Keywords: Finite element analysis, Sub-model, Force boundary condition, Displacement boundary condition, Experimental verification.

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1. Introduction

The contact between the adjacent parts of construction machinery is a commonly existing problem. In order to increase the computational efficiency and avoid the difficulties of convergence, when implementing whole machine/global modeling, the connection joints and their local detailed geometric features are usually simplified. This may lead to the problem that the accurate stress in the vicinity of the simplified local zones could not be obtained from the analysis of the global model. The rationally and the most effective way to solve this problem is the sub-model method [1-3].

Sub-model method has been widely used in analysis of mechanical structure. Some scholars utilized the sub-model method to analyze the mechanical structural strength [4-7]. Wu and Wang used sub-model method to improve and optimize mechanical structure [8]. Li made an analysis on the stress of steel box beam of super-span suspension bridge [9]. Li and Giglio utilized sub-model method to make fatigue analysis on mechanical structure [10-11]. The application of sub-model technique greatly increased the computational efficiency of model.

Traditional sub-model analysis method takes the displacements of nodes at the cut-boundary of global model and boundary condition of the sub-model. If there is no large difference between the structural rigidity of sub-model and that corresponding to the sub-model in the simplified global model, the sub-model boundary stress obtained after sub-model analysis should be an equivalent force system highly approximating to the stress distribution of cut-boundary of global model. Though the displacement boundary conditions are used to apply loading to the sub-model, the equivalent force system has been is actually added to the sub-model. Therefore, we can get a correct analysis result, and the displacement boundary

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condition applied to the sub-model is feasible. Because the displacement boundary condition is easily carried out in finite element software, the traditional displacement sub-model method can be widely used. However, when finite element analysis is made on large-scale machine, in order to increase the speed of calculation, local detailed geometric features are usually simplified. This will result in a large difference between the structural rigidity of detailed sub-model used in local reanalysis and the structural rigidity of corresponding sub-model of the simplified global model. In such case, if, according to traditional sub-model solving method, the displacement obtained from the global model is applied to the detailed sub-model boundary, the boundary stress obtained after sub-model solution will be significantly different from the boundary stress of the global model. This will lead to a remarkable load error in sub-model analysis. In contrast to applying the displacement boundary condition, a method of directly implementing forces to the sub-model has been proposed in this paper, ensuring the boundary condition applied to the sub-model is correct.

2. Finite element principle of traditional sub-model method

The core concept of traditional sub-model method (specific boundary displacement sub-model method) is to take the displacement values of the nodes at the cut-boundary of the global model as the displacement boundary condition of the sub-model. According to basic theory of finite element, in sum-model calculating, the stress and displacement should satisfy the relation below:

$$\boldsymbol{K}\vec{U}=\vec{F} \tag{1}$$

In above equation, $\boldsymbol{K}$ refers to total rigidity matrix of sub-model, $\vec{F}$ refers to external load vector of sub-model, and $\vec{U}$ refers to unsolved displacement vector of sub-model.

Divide $\vec{U}$ into two parts: the first part is that the sub-model shares displacement coordinating relation with other substructure or unit, which belongs to boundary node displacement, denoted by $\overrightarrow{U_B}$, i.e. known displacement vector (which can be obtained/interpolation from the global coarse model). The second part involves those having no displacement coordination relation with other substructure or unit, denoted by $\overrightarrow{U_S}$, i.e. unsolved displacement vector. Therefore, Equation (1) can be decomposed as:

$$\begin{bmatrix}\boldsymbol{K}_{BB} & \boldsymbol{K}_{BS}\\ \boldsymbol{K}_{SB} & \boldsymbol{K}_{SS}\end{bmatrix}\begin{bmatrix}\overrightarrow{U_B}\\ \overrightarrow{U_S}\end{bmatrix}=\begin{bmatrix}\overrightarrow{F_B}\\ \overrightarrow{F_S}\end{bmatrix} \tag{2}$$

in which, $\overrightarrow{U_B}$ refers to the displacement vector of the boundary nodes of the sub-model, $\overrightarrow{U_S}$ refers to the displacement vector of the internal nodes of the sub-model; $\boldsymbol{K}_{BB}$ is the rigidity matrix of the boundary nodes of the sub-model; $\boldsymbol{K}_{SS}$ is the rigidity matrix of the internal nodes of the sub-model; $\overrightarrow{F_B}$ is the load vector of the boundary nodes of the sub-model; $\overrightarrow{F_S}$ is the load vector of the internal nodes of the sub-model.

Expand Equation (2) and obtain:

$$[\boldsymbol{K}_{BB}][\overrightarrow{U_B}]+[\boldsymbol{K}_{BS}][\overrightarrow{U_S}]=[\overrightarrow{F_B}] \tag{3}$$

$$[\boldsymbol{K}_{SB}][\overrightarrow{U_B}]+[\boldsymbol{K}_{SS}][\overrightarrow{U_S}]=[\overrightarrow{F_S}] \tag{4}$$

By Equation (4), we can determine the displacement of the internal nodes as follows

$$[\overrightarrow{U_S}]=[\mathbf{K}_{SS}]^{-1}[\overrightarrow{F_S}]-[\mathbf{K}_{SS}]^{-1}[\mathbf{K}_{SB}][\overrightarrow{U_B}] \tag{5}$$

Expand Equation (2) and obtain:

It can be found in Equation (5) that, for the unsolved $\overrightarrow{U_S}$, the given displacement vector $\overrightarrow{U_B}$ has become one part of load vector. In other words, for a structure, the displacement boundary condition may produce the load effect. In this way, though the sub-model has applied displacement boundary condition, the actual loading added to the sub-model is an equivalent force system load.

From this, it is observed that the key that affects the precision of sub-model solution lies in the similarity degree between the applied equivalent force system load and the actual stress distribution of cut-boundary of global model. When loading by displacement boundary condition, the similarity degree depends on the level of similarity between sub-model rigidity $\boldsymbol{K}$ and the structural rigidity ($\boldsymbol{K}_{coarse}$) of corresponding sub-model of the simplified global model.

3. Finite element principle of new sub-model method

The balance force system boundary condition sub-model method proposed in this article (hereinafter abbreviated as new sub-model method) is a new sub-model analysis method. It was proposed for the implementation of accurate stress analysis of the contact structures of hinge or sliding local joint position between two or more components that commonly exists in machinery. Sub-model generally owns multiple local cut-boundaries (see Fig. 1.). Assume that the deformation pattern (shape) of each sub-model cut-boundary surface after loading is the same with that in global model, but relative to its cut-boundary surface of global model, each local cut-boundary have six additional rigid body generalized displacement degrees of freedom, i.e. three translational degrees of freedom and three rotational degrees of freedom. Randomly select one node $n_{0i}(x_{0i}, y_{0i}, z_{0i})$ in the ith cut-boundary of sub-model as the datum node, and its degree of freedom is $\vec{\delta_i} = \left(u_{0xi}, u_{0yi}, u_{0zi}, \theta_{0xi}, \theta_{0yi}, \theta_{0zi}\right)^{\mathrm{T}}$. The degree of freedom of the datum node can be defined as the additional rigid body degree of freedom of this cut-boundary. Then, for each node $n(x_j, y_j, z_j)$ (j=1,2…N_{total}) among all N_{total} nodes in the ith cut-boundary of sub-model, its deformation after loading can be determined by the equation below:

$$\overrightarrow{U_j} = \overrightarrow{U_{0i}} + \boldsymbol{R}\vec{L} \tag{6}$$

in which,

$$\overrightarrow{U_j} = \left[u_{xj}, u_{yj}, u_{zj}\right]^T, \quad j=1,2\ldots N_{total} \tag{7}$$

is the translational degree of freedom column vector of the cut-boundary nodes,

$$\overrightarrow{U_{0i}} = \left[u_{0xi}, u_{0yi}, u_{0zi}\right]^T \tag{8}$$

is the translational degree of freedom column vector of datum node,

$$\boldsymbol{R} = \begin{bmatrix} 0 & -\theta_{0zi} & \theta_{0yi} \\ \theta_{0zi} & 0 & -\theta_{0xi} \\ -\theta_{0yi} & \theta_{0xi} & 0 \end{bmatrix} \tag{9}$$

is the rotation matrix of additional rigid body in cut-boundary,

$$\vec{L} = \begin{bmatrix} (x_j + u'_{xj}) - (x_{0i} + u'_{0xi}) \\ (y_j + u'_{yj}) - (y_{0i} + u'_{0yi}) \\ (z_j + u'_{zj}) - (z_{0i} + u'_{0zi}) \end{bmatrix} \tag{10}$$

is the relative displacement column vector of the position of node n in sub-model cut-boundary after loading and deformation corresponding to the position of datum node n_{0i}.

u'_{0xi}, u'_{0yi} and u'_{0zi} are the translation displacement component of the position in global model cut-boundary corresponding to node n0i in sub-model cut-boundary, which is obtained from the global model analysis. u'_{xj}, u'_{yj} and u'_{zj} are the displacement component of the position corresponding to node n, which also can be obtained from the global model analysis. Substituting Equation (7)-(10) into Equation (6), we can obtain the node deformation field of the ith boundary in explicit formulation form:

$$\begin{bmatrix} u_{xj} \\ u_{yj} \\ u_{zj} \end{bmatrix} = \begin{bmatrix} u_{0xi} \\ u_{0yi} \\ u_{0zi} \end{bmatrix} + \begin{bmatrix} 0 & -\theta_{0zi} & \theta_{0yi} \\ \theta_{0zi} & 0 & -\theta_{0xi} \\ -\theta_{0yi} & \theta_{0xi} & 0 \end{bmatrix} \begin{bmatrix} (x_j + u'_{xj}) - (x_{0i} + u'_{0xi}) \\ (y_j + u'_{yj}) - (y_{0i} + u'_{0yi}) \\ (z_j + u'_{zj}) - (z_{0i} + u'_{0zi}) \end{bmatrix} \tag{11}$$

in which, j=1,2…N_{total}

When implementing the New sub-model method, we often adopt solid element. Each node of the element has only three translational degrees of freedom, without rotational degree of freedom. Three additional rotational generalized degree of freedom need to be introduced in the required position of datum node. It can be found from Equation (11) that the degree of freedom of the nodes in each cut-boundary of sub-model shall satisfy this equation, and they are merely the linear equation of six generalized degree of freedom in this boundary, which can be satisfied through constraint equation in the process of solving system equation. Therefore, in the solution process of new the sub-model method, only six generalized degree of freedom of datum node in each cut-boundary are unknown. The component of resultant force of six generalized force corresponding to the six generalized degree of freedom of each local cut-boundary can be obtained from the results of global model analysis. These six generalized force components are three resultant force components f_{xi}, f_{yi} and f_{zi} and three resultant force moment components at datum point M_{xi}, M_{yi} and M_{zi}. In sub-model solution, these generalized force components can be directly applied into the generalized displacement of its corresponding datum node. The generalized force applied to all cut-boundary of sub-model actually forms a whole balanced force system, because they are obtained from the balanced force system of global model. Thus, any subdivision of it will definitely be balanced force system.

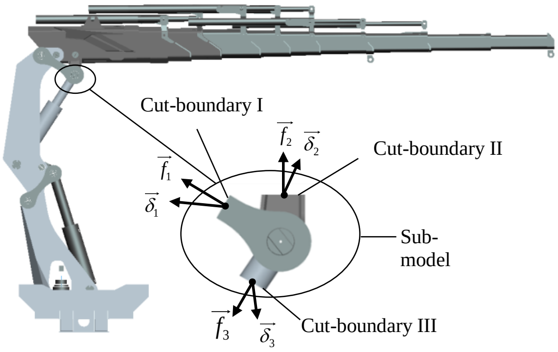


Fig. 1: Schematic diagram of a sub-model with multi-cut boundaries

4. Numerical example

Fig. 2 shows a typical telescopic boom truck-mounted crane. The telescopic boom system is composed of one basic boom and nine telescopic booms nesting together one after another. The inner booms can slide within the outer neighboring booms. The loads transferred between the neighboring booms are realized through the sliding contacts between the sliding blocks and the sliding surfaces of the booms. The upper sliding blocks are fixed on top of the out surface at the end of the inner boom, and the lower sliding blocks are fixed on the lower inner surface of the out boom. The telescopic boos consist of thin-walled hollow segments with octagonal cross-section, as shown in Fig. 3, and the geometric data of the cross-section parameters of the telescopic boom are given in Table 1.

Telescopic booms are key elements for payload lifting in construction machineries. The mechanical properties of the telescopic booms directly determine the lifting capacity of these machines. The overlapping contact regions between the adjacent booms are critical locations where high stresses occur, thus the stress level in these regions is the key factor representing the strength of the telescopic boom structures. In order to precisely analyze the stress distribution in contact regions between adjacent booms, it is required to carry out an accurate contact analysis between sliding blocks and booms.

In order to increase computational efficiency and avoid convergence difficulties in nonlinear contact calculation of overlapped contact regions between the adjacent booms, sub-model method is often applied. The global model is carried out first, and the node freedom coupling method is used to simulate the contact relation between adjacent booms, which is efficient but inaccurate. Then detailed contact calculation in sub-model analysis is conducted.

Fig. 2: A typical knuckle boom truck-mounted crane.

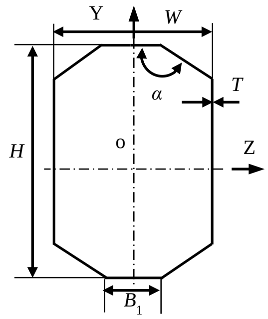


Fig. 3: Octagonal cross-section

Table 1: Cross-section dimensions of the telescopic boom

cross section parameter	base boom	first boom	second boom	third boom	fourth boom	fifth boom	sixth boom	seventh boom	eighth boom	ninth boom
H/mm	666	620	574	528	482	436	390	344	298	252
W/mm	450	420	390	360	330	300	270	240	210	180
B_1/mm	210	196	181	167	152	138	123	109	94	80
T/mm	8	8	8	8	8	8	8	8	8	8
α/ °	145	145	145	145	145	145	145	145	145	145

4.1. Sub model analysis

Before using sub-model method to analyze the contact region between first boom and second boom of telescopic boom structure, it is necessary to make a global coarse analysis first. To avoid the modeling error caused by the structural geometric simplification, the whole upper carriage structure was included in the finite element models. To increase the analysis accuracy, solid brick elements were used throughout. The working condition of calculation is that the telescopic booms fully extended and was kept at horizontal position with operation range of 21.3 meters and raising load of 4000Kg. The model is built according to real structural size. The node freedom coupling method is used to simulate the sliding connection between booms and sliding blocks. The bottom of slewing platform is fully restrained. The payload is added to the head of the ninth telescopic boom. Then, we can build a local structural model for contact area between first and second boom, and contact elements were used to simulate the realistic contact behavior at the overlapping regions between neighboring booms. The global FE model and sub-model are shown in Fig. 4.

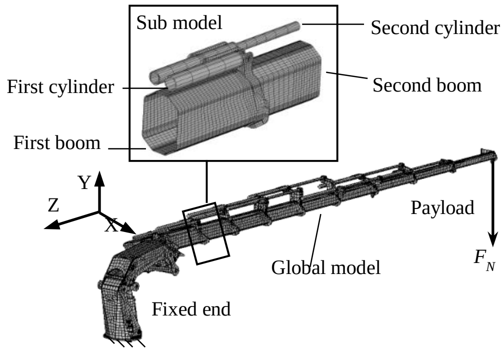
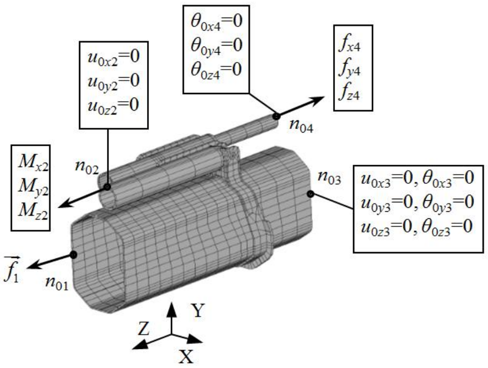


Fig. 4: Finite element meshes for global model and local sub-model of the crane



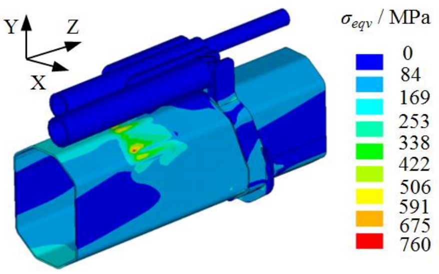


Fig. 5: Boundary condition for the new sub-model **Fig. 6:** Equivalent stress in the contact zone of first boom and second boom

When implementing traditional sub-model analysis, interpolated displacement conditions are used to apply to both the cut-boundaries of first and second booms. When implementing new sub-model analysis, in order to eliminate the model's rigidity displacement, we set the generalized displacement of cut-boundary datum node n_{03} in second boom to be 0, and restrain three translation degrees of freedom of cut-boundary datum node n_{02} of first cylinder, and restrain three rotation degrees of freedom of cut-boundary datum node n_{04} of second cylinder. Then, we apply resultant force and resultant moment component to cut-boundary datum node n_{01} of first boom. Three bending moment components are applied to cut-boundary datum node n_{02} of first cylinder, and three resultant force components are applied to cut-boundary datum node n_{04} of second cylinder, as shown in Fig. 5.

Through new sub-model analysis, the equivalent stress distribution at the contact zone between the first and second boom can be obtained as shown in Fig. 6. From this figure, we can see an obvious high stress region in contact zone between neighboring booms.

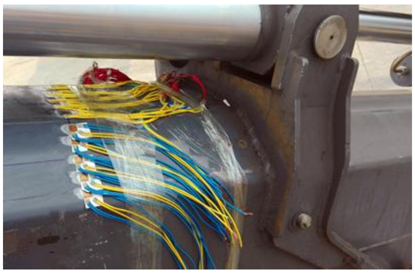
Table 2 compares the resultant force f_y and resultant moment component M_x on cut-boundary surface of first boom obtained from the analysis results of three finite element models. From Table 3, we can clearly see that the cut-boundary resultant force and resultant moment of new sub-model is identical with that of global model; in comparison, traditional sub-model using interpolated displacement condition indirectly adds force and moment to sub-model, which are much smaller than that should be applied.

Table 2: Comparison of predicted resultant force and moment at the cut-boundary of the first boom from different models

	Global model	New sub-model		Traditional sub-model	
	Calculation Result	Calculation Result	Relative Error	Calculation Result	Relative Error
f_y/N	137827	137827	0%	114857	16.6%
M_x/KNm	994	994	0%	873	12.1%

4.2. Experimental test

To further verify the accuracy of the sub-model method proposed in this article, experiments on a real telescopic boom structure have been carried out. The test area is the contact region between first boom and the sliding blocks at second boom end. The pictures showing the strain gauges on the surface of the tested boom are given in Fig.7. The location distribution of the measurement points is illustrated in Fig.8.



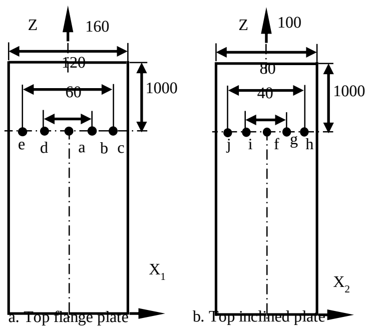
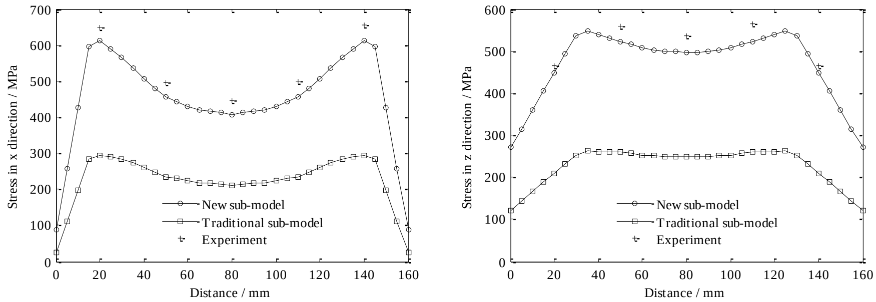
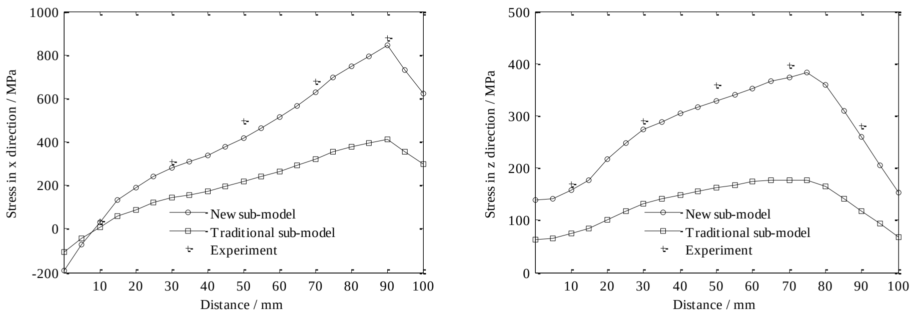


Fig. 7: Measuring points for testing the first boom **Fig. 8:** Measuring points for testing the first boom



a. Stress in X_1 direction b. Stress in Z direction

Fig. 9: Comparison of stress component on the top flange of the first boom



a. Stress in X_2 direction b. Stress in Z direction

Fig. 10: Comparison of stress component on the top inclined plate of the first boom

Fig. 9 shows stress distribution of test points on the top flange plate of the first boom, and Fig. 10 shows stress distribution of test points on the top inclined plate of the first boom.

From these comparison figures, it can be seen that the results from the new sub-model correlate excellent with those from the experimental data. The maximum relative differences of stress components along directions X_1 and Z on the top flange plate between the new sub-model and the experimental results is 4.2% and 3.2% respectively, and the stress components along directions X_2 and Z on the top inclined plate has a relative error of 5.5% and 5.8% respectively. However the traditional sub-model analysis shows significant discrepancies.

5. Conclusions

The balance force system boundary condition sub-model analysis method has been proposed in this article. It is aimed at the local stress analysis problems in hinge or sliding connecting structure of multiple parts existing in engineering machinery. This method directly applies the balanced force system of local model obtained from analysis results of global model to detailed sub-model boundary for accurately solving local stress, which prevents the inevitable load error in solving through traditional displacement boundary sub-model method due to the discrepancy between the structural rigidity of precise sub-model and the structural rigidity of the local structure correspond to sub-model in simplified global model, and ensures the load accuracy of sub-model cut-boundary. Both numerical analysis results and experimental verification have shown that the newly proposed sub-modeling technique can load the model more accurately than the traditional sub-model method and the analysis accuracy is not sensitive to the degree of simplification of the global model. Therefore, the implementation of new sub-model analysis strategy avails to increase the efficiency of modeling and calculation of global model, meanwhile ensure the accuracy of stress analysis of local structure.

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