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On Throughput-Reliability Tradeoff Analysis of MIMO Channels under Generalistic Fading Scenario

S. Rajarajan[a]* and Sanjay K. Chaturvedi[a]

[a]*Subir Chowdhury School of Quality and Reliability, IIT Kharagpur, Kharagpur- 721302, India*

Abstract

MIMO wireless communication systems have extensively been employed due to their ability to transfer data efficiently even at higher data rates and are often being used to increase either the multiplexing gain and/or the diversity gain with a tradeoff. The Diversity-Multiplexing Tradeoff (DMT) had been proven to be a powerful evaluation and comparison tool for the existing and new technologies. However, Throughput-Reliability Tradeoff (TRT) has the notable advantage of revealing the interplay among transmission rate, signal-to-noise ratio and outage/error probability parameters.

This paper proposes TRT analysis of MIMO channels under broad class of fading distributions that includes, different identical fading distributions, correlation between channels, non-identical fading distributions, and non-zero channel means. The analysis is carried out by characterizing the joint pdf of the eigenvalues of Gram matrix in high SNR regime. This work also investigates the relation between DMT and TRT of MIMO in broad class of fading distributions. Further, our study sheds light on different channel parameters and their usefulness in TRT analysis, such as, error/outage probability vs. Signal-to-Noise Ratio (SNR) curves, amount of SNR increase to increase transmission rate and/or decrease error/outage probability, etc.

Keywords: Throughput-reliability tradeoff; Diversity-multiplexing tradeoff; Fading channels

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1. Introduction

MIMO wireless system has extensively been in action due to their ability to transfer data efficiently at higher transfer rates. To evaluate their performance, a framework termed as Diversity-Multiplexing Trade-off has been proposed [1, 2]. It has been proved in the seminal work of [1] that a MIMO system can be used to increase two gains simultaneously in the high Signal-to-Noise Ratio (SNR) regime. It has also been stated that the diversity (reliability) and/or throughput (transfer rate) of wireless MIMO systems can be increased by increasing SNR [1, 3, 4]. The definition of DMT for a system is that the diversity gain will not exceed the diversity gain, $(d(r))$, for the multiplexing gain r. Many popular transmission schemes like D-BLAST, V-BLAST and Alamouti have been evaluated using DMT [1]. The DMT is considered to be the benchmark for comparing and evaluating the performance of existing and newly proposed schemes [3, 4]. Despite being proven to be a powerful tool of performance evaluation of MIMO systems, the DMT fails to provide answer to the question [5]- how much reliability (or transmission rate) is increased in a system by increasing SNR by 3-dB? Instead, it states that by doubling the operating SNR level, we get both $2^{-d(r)}$ increase in reliability and r additional bits per channel use. The reason of this is that the DMT allows the rate of codebook increases linearly with $\log \rho$, limiting answer to the question raised about the

* Corresponding author. Tel.: +919434502388.
E-mail address: s.rajarajan@iitkgp.ac.in.

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performance of the system in [5]. This leads to the formulation of Throughput-Reliability Trade-off (TRT) at high SNR regime ρ - an another form of DMT, which is capable of answering the question by considering $(\mathrm{R}, \rho, \mathrm{P_{otg}})$. The TRT has been used to evaluate performance of popular schemes such as Alamouti and V-BLAST [5] to show its versatility. It is due to the fact that in the TRT framework, the codebook rate evolves potentially by considering $\limsup_{\rho\to\infty} \mathrm{R}/\log\rho \neq \liminf_{\rho\to\infty} \mathrm{R}/\log\rho$ [†].

The functional significance of the TRT lies in its predicting capability of local slopes and horizontal displacement between the error waterfall curves drawn at various levels of SNR. Earlier work in TRT made an independent and identically distributed Gaussian fading channel assumption [5-7], however, is violated in some scenarios. For example, most outdoor and indoor mobile fading channels are modeled by Nakagami-m distribution [8], the line-of-sight micro-cellular fading channels by Rician distribution [8], and many other versatile digital communication channels by using Weibull distribution [9-11]. Further, due to the size limitation of transceiver systems, the placements of transmit and/or receiver antennas results in a correlation between signals at the receiver [12-14].

The DMT analysis of general fading channel, including, Rayleigh, Rician, Nakagami-m, Weibull, Nakagami-q fading distributions has been established by characterizing the joint pdf of the eigenvalues of the Gram matrix in high SNR regime [16]. It has also covered the effects of channel correlation, nonzero channel means and non-identical distributions. Our contributions in this paper are to provide TRT analysis of general fading MIMO channels to overcome the drawback of DMT, besides, showing that a relationship between DMT and TRT. In addition, we derive their corresponding TRT expressions, specifically for different fading channel distributions, for any optimal coding/decoding schemes, effect of correlation among channels and non-zero channel means. Besides, we compare the waterfall curves predicted by TRT with the theoretic curves by using simulation. The numerical results obtained reveal the interplay between the triplets $\mathrm{R}, \log\rho, \mathrm{P_{otg}}(\mathrm{R},\rho)$.

The paper is organized as follows. In section 2, presents system model and commonly used channel distribution. Section 3, contains prerequisite on random matrices, outage formulation and TRT analysis of general fading channels. A relation between DMT and TRT of general fading channel is provided in section 5. In section 6, various extensions of TRT analysis of general fading channels, like, channels with optimal coding scheme, effect of spatial correlation and, effect of non-zero channel mean are given. Finally, section 7 covers the conclusion of this work.

We have used bold capital letters to represent matrices and bold small letters to represent single valued vectors. Plain letters represent channel realization or constants. Frobenius norm is represented by $\|\cdot\|_{\mathrm{F}}$. Unless otherwise mentioned $\mathrm{g_{ij}}$ or $[\mathrm{G}]_{\mathrm{ij}}$ is the (i,j) th entry of the matrix G. $\mathrm{I_N}$ and 0_{N} represents $\mathrm{N}\times\mathrm{N}$ identity matrix and $\mathrm{N}\times\mathrm{N}$ zero matrix, respectively. $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ represents conjugate, transpose and conjugate-transpose of matrices, respectively.

2. System Model

Consider a wireless MIMO system of M transmitting antennas with N receiving antennas and that has been depicted in Fig. 1. The complex channel fading coefficient from ith transmit antenna to jth receive antenna is given by g_{ij}. These channel coefficients are independent random variables and every transmit-receive antenna path gain is collectively represented by $\mathbf{G} \in \mathbb{C}^{N\times M}$. $\mathbf{G}$ is assumed to be time invariant for at least T symbol intervals. We also assume a coherent transmitter-receiver model which implies channel state information available only at the receiver. We do not assume any particular distribution for the channel matrix, however, elements of channel matrix behave polynomial near zero, fast decaying near infinity and are upper bounded by a constant K. This holds true for fading path distributions like Rayleigh, Rician, Nakagami and Weibull, also applicable for paths that may contain non-zero mean, non-identical distributions or channel correlation as stated earlier in Section 1. With these assumptions, a block of received data can be represented as,

$$\boldsymbol{Y} = \sqrt{\frac{\rho}{\mu}}\boldsymbol{G}\boldsymbol{C} + \boldsymbol{W} \tag{1}$$

where $\mathbf{Y} \in \mathbb{C}^{N\times T}$ is the received vector, $\mathbf{C} \in \mathbb{C}^{M\times T}$ is the codeword transmitted from M transmit antennas over T symbol intervals, $\mathbf{W} \in \mathbb{C}^{N\times T}$ is the Additive White Gaussian Noise (AWGN) matrix whose entries are i.i.d. random variables with the distribution $\mathcal{CN}(0,1)$, ρ denotes the average signal-to-noise ratio per receive antenna and μ is a normalization factor that ensures the average energy transmitted from all antennas per time slot is unity. Since $\mathbf{G}$ has iid elements, the pdf of $\mathbf{G}$ can be written as [16]

[†] Until otherwise stated, all logarithms in this paper are assumed to be on base 2.

$$p_{\mathbf{G}}(G) = \prod_{i=1}^{N} \prod_{j=1}^{M} p_{\mathbf{g}_{ij}}([G]_{ij}). \tag{2}$$

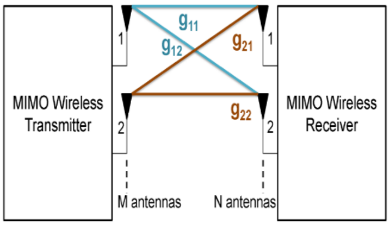

Fig. 1: MIMO Communication Systems

A codebook C of rate R bits per channel use (bpcu) has $|\mathsf{C}| = \lfloor 2^{Rl} \rfloor$ codewords $\{\mathbf{C}(1), \cdots, \mathbf{C}(\mathsf{C})\}$. We further assume a power constraint on the codebook C:

$$\frac{1}{|\mathsf{C}|} \sum_{i=1}^{|\mathsf{C}|} \|\mathbf{C}(i)\|_F^{\,2} \leq \mu l \tag{3}$$

This shows that ρ is the average transmit power irrespective of the value of μ.

3. Outage Formulation

3.1. Outage Probability

Any wireless link in (1) can be evaluated by outage, which can be termed as the event that the mutual information of channel does not endure the intended data rate R. The outage probability can be written as given in [1, 5] as,

$$P_{otg} \geq \Pr\left\{\log\left(\prod_{i=1}^{q}(1+\rho\lambda_i)\right) < R\right\} \tag{4}$$

$$P_{otg} \leq \Pr\left\{\log\left(\prod_{i=1}^{q}\left(1+\frac{\rho}{M}\lambda_i\right)\right) < R\right\} \tag{5}$$

where q is $\min(M, N)$. The above equation signifies that the eigenvalues of $\mathbf{GG}^H$ is responsible for the outage probability of a system.

3.2. Distribution of Eigen Values

To obtain the distribution of eigenvalues of $\mathbf{GG}^H$, generally matrix decompositions are employed due to its simplicity [16-19]. The distribution of eigen values can be written as,

$$p_{\mathbf{\Lambda}}(\Lambda) = \frac{1}{(4\pi)^q}\left(\prod_{i=1}^{q}\lambda_i^{|M-N|}\right)\prod_{i<j}(\lambda_i-\lambda_j)^2$$
$$\int_{V_{M,M}}\int_{V_{M,N}} p_{\mathbf{G}}\left(\mathbf{U}\mathbf{\Lambda}^{1/2}\mathbf{Q}\right)d\mathbf{Q}d\mathbf{U} \tag{6}$$

where $V_{M,M}$ and $V_{M,N}$ are complex Stiefel manifolds and $1/(2\pi)^q$ is from the fact that we assumed the first row of $\mathbf{U}$ is non-negative and integrate $\mathbf{U}$ over the whole manifold $V_{M,M}$. For further details on Steifel manifolds and Jacobians, readers can refer [16-18].

Now, in the following Theorem 1 we derive TRT of general fading channels.

Theorem 1: *For MIMO system with independent and regular fading paths described in equation (3) has*

$$\lim_{\substack{\rho\to\infty \\ (R,\rho)\in\mathcal{R}(k)}} \frac{log\, P_{otg}(R,\rho) - c(k)R}{log\, \rho} = -g(k) \tag{7}$$

where $P_{otg}(R,\rho)$ denotes the outage probability at rate R, SNR ρ, and $\mathcal{R}(k)$ denotes the k^{th} operating region defined by

$$\mathcal{R}(k) \triangleq \left\{(R,\rho)|k+1 > \frac{R}{log\,\rho} > k\right\} \tag{8}$$

for $k \in \mathbb{Z}$, $min\{M,N\} > k \geq 0$. In equation (19), $c(k)$ and $g(k)$ are accordingly defined as

$$\{c(k), g(k)\} \triangleq \begin{cases} \left\{M+N-1+\frac{\Sigma\Sigma t_{i,j}}{2}, NM+\frac{\Sigma\Sigma t_{i,j}}{2}\right\}, & k=0 \\ \{M+N-(2k+1), MN-k(k+1)\}, & 0 < k \leq q \end{cases}. \tag{9}$$

The reliability gain coefficient $g(k)$ describes the slope of the outage probability curve. A smaller $g(k)$ value implies a less steep curve and thus less realizable. The throughput gain coefficient $t(k) \triangleq g(k)/c(k)$ implies that the variations of the rate and the SNR must satisfy $\Delta R \approx \frac{g(k)}{c(k)}\Delta\, log\,\rho$ in the operating region $\mathcal{R}(k)$ by keeping P_{otg} constant.

Proof: The proof of Theorem 1 is provided in Appendix A.

The following two propositions given here are as excerpts from Theorem 1, which are used as tools to prove results in Section 6.

Proposition 1: For a channel matrix $\mathbf{G} = \mathbf{U}_1\widetilde{\mathbf{G}}\mathbf{U}_2$, where $\mathbf{U}_1$ and $\mathbf{U}_2$ are square matrices then the eigenvalue distribution of $\mathbf{G}\mathbf{G}^H$ and $\widetilde{\mathbf{G}}\widetilde{\mathbf{G}}^H$ are same [19].

Proposition 2: For a channel matrix $\mathbf{G} = \mathbf{\Sigma}_1\widetilde{\mathbf{G}}\mathbf{\Sigma}_2$ such that both $\mathbf{\Sigma}_1$ and $\mathbf{\Sigma}_2$ are diagonal matrices with positive elements on the diagonal. It is easy to prove that the distributions of elements of $\mathbf{G}$ are same as that of the elements of $\widetilde{\mathbf{G}}$ except whose variances need scaling [14].

4. Simulation Results and Discussion

The TRT formulation predict the outage curves of MIMO general fading channels by

$$\log P_{otg}\,(R,\rho) \approx c(k)R - g(k)\log\rho \tag{10}$$

which clearly reveals the interplay between SNR, rate and outage probability. From (7) it can be observed that $-g(k)$ is the slope of outage waterfall curve for a fixed rate and the SNR difference between two waterfall curves to provide a transmission rate difference of ΔR is given by $3\Delta R/t(k)$ for a fixed outage probability. Thus, the larger $g(k)$ implies larger the slope of the outage curve and the larger $t(k)$ implies smaller the SNR gain needs to be to support the increased rate. Hence, $g(k)$ is the reliability gain coefficient and $t(k)$ is the throughput gain coefficient.

In Fig. 2 to Fig. 4 we provide simulated results based on Theorem 1 to show, the correctness of our TRT analysis, the ability of TRT equations to simulate various scenarios and the significance of operating region, respectively.

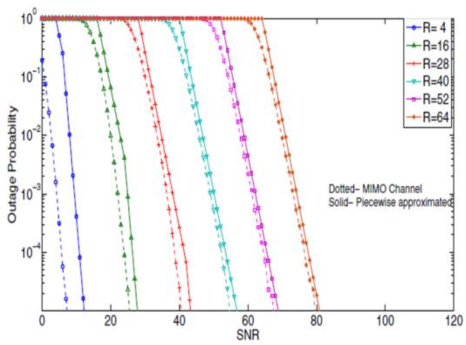

Fig. 2: Theoretical outage curves (dotted) corresponding to $\boldsymbol{M} = \mathbf{3}$ and $\boldsymbol{N} = \mathbf{3}$ over Rayleigh fading channel, along with the approximated curve (solid) suggested by TRT.

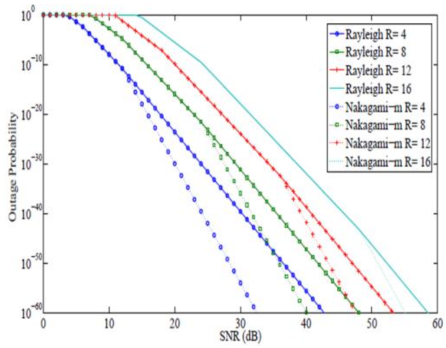

Fig. 3: Outage curves corresponding to $\boldsymbol{M} = \mathbf{4}$ and $\boldsymbol{N} = \mathbf{4}$ over Rayleigh and Nakagami- $\boldsymbol{m}$ ($\boldsymbol{m} = \mathbf{1.5}$) fading channels.

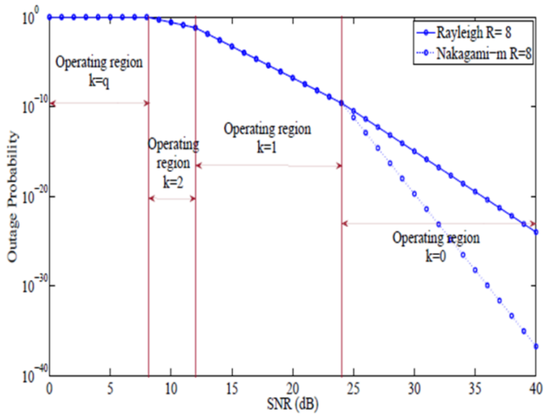

Fig. 4: Outage curves corresponding to $\boldsymbol{M} = \boldsymbol{3}$ and $\boldsymbol{N} = \boldsymbol{3}$ over Rayleigh and Nakagami-m ($\boldsymbol{m} = \boldsymbol{2}$) fading channels.

It can be observed from Fig. 2 that the TRT curves converge to theoretical curves at infinite SNR proves the correctness of our TRT analysis of channels that may follow any given fading distributions.

Fig. 3 depicts the outage curves for Rayleigh and Nakagami-m $(\mathrm{m} = 1.5)$ channels for $\mathrm{M} = 4$ and $\mathrm{N} = 4$ with different transmission rates. In Fig. 2, at operating region $\mathcal{R}(0)$ of any given transmission rate R, the Nakagami-m channel with $\mathrm{g(k)} = 24$ and $\mathrm{t(k)} = 8/5$ whereas, the Rayleigh channel with $\mathrm{g(k)} = 16$ and $\mathrm{t(k)} = 16/7$ precisely needs $3\Delta\mathrm{R}/\mathrm{t(k)}$ SNR gain for ΔR increase in transmission rate.

Finally, Fig. 4 represents the outage curves of Rayleigh and Nakagami-m $(\mathrm{m} = 2)$ channels. The results clearly indicate that the systems in Nakagami-m fading channel perform better than Rayleigh fading channel at lower operating regions $\left(\mathcal{R}(0)\right)$ whereas, the systems perform identical in any of the channels at higher operating regions of TRT analysis.

5. Relationship between TRT and DMT of General Fading Channels

Note that the relationships between TRT and DMT for MIMO and co-operative systems under Gaussian fading conditions are given by [1, 6, 7],

$$g(k) = d(k) - kd'(k^{+}), \qquad \text{and} \qquad c(k) = -d'(k^{+}). \tag{11}$$

The DMT expression for general fading MIMO channels is given by,

$$d(k) = \begin{cases} (N-k)(M-k) & if\ k = 1, \cdots, 1 \\ \sum_{i=1}^{N}\sum_{j=1}^{M}\left(1 + \frac{t_{i,j}}{2}\right) & if\ k = 0 \end{cases}. \tag{12}$$

The DMT curves for general fading channels have inflexions points. Each of these inflexions split the DMT curve into segments, which corresponds to the operating region of TRT for general fading channels. Comparing the DMT of general fading channel in (12) and TRT of general fading channel equations in Theorem 1, we find that relation between DMT and TRT of MIMO with general fading channel conditions also satisfy (11), like MIMO and co-operative systems under Gaussian fading conditions [3, 6, 7]. Thus, any MIMO system under any fading scenario may have same relationship between DMT and TRT.

6. Extensions

After establishing the TRT analysis for MIMO systems with generalistic channel fading distributions, we now extend the results to error probability analysis for optimal space-time codes, effect of spatial correlation and effect of non-zero channel mean.

6.1. Optimal Tradeoff Curve

Theorem 2: *For any optimal coding/decoding scheme in conjunction with MIMO channel represented in (1), the probability of error satisfies*

$$\lim_{\rho\to\infty} \frac{P_e - c(k)R}{\log \rho} = -g(k) \tag{13}$$

where $\mathcal{R}(k)$, $c(k)$ and $g(k)$ are given in (19). Moreover, there exists a coding scheme that achieves for $l \geq M + N - 1 + \frac{\Sigma\Sigma t_{i,j}}{2}$.

The proof of this theorem has been provided in Appendix B.

6.2. Effect of Correlation

Theorem 3: For MIMO system, say, the channel matrix $\mathbf{G} = \mathbf{\Sigma}_1 \widetilde{\mathbf{G}} \mathbf{\Sigma}_2$, where $\widetilde{\mathbf{G}}$ has independent elements and, $\mathbf{\Sigma}_1$ and $\mathbf{\Sigma}_2$ are covariance matrices with arbitrarily full rank at receiver and transmitter, respectively. The TRT of this channel is same as given in (7). The optimal TRT equation of this channel is given in (13).

One way to prove Theorem 3 is to prove eigenvalue distribution of $\mathbf{G}\mathbf{G}^H$ is same as $\widetilde{\mathbf{G}}\widetilde{\mathbf{G}}^H$. By eigenvalue decomposition of the covariance matrices, we can write

$$\mathbf{\Sigma}_1 \widetilde{\mathbf{G}} \mathbf{\Sigma}_2 = \mathbf{U}_1 \mathbf{\Lambda}_1 \mathbf{U}_1^H \widetilde{\mathbf{G}} \mathbf{U}_2 \mathbf{\Lambda}_2 \mathbf{U}_2^H \tag{14}$$

where $\mathbf{U}_1$ and $\mathbf{U}_2$ are square matrix and $\mathbf{\Lambda}_1$ and $\mathbf{\Lambda}_2$ by definition are diagonal matrices with positive elements.

From Propositions 1 and 2, this theorem can easily be proved.

6.3. Effect of Channel Mean

From Table 1 for Rayleigh and Rician channel, the parameter $\Sigma\Sigma t_{i,j}$ is zero and the only channel distribution with non-zero channel mean is Rician distribution. Therefore, the channels under Gaussian distributions with non-zero mean will not affect the TRT results.

For other channels distributions, the channel with non-zero mean can be written as $\mathbf{G} = \widetilde{\mathbf{G}} + \bar{G}$ where $\widetilde{\mathbf{G}}$ is the channel matrix with independent regular entries and $\bar{G}_{i,j}$ is the mean added to every channel path gain. The pdf of channel can be rewritten as

$$p_{\mathbf{G}_{ij}}(g) = p_{\widetilde{\mathbf{G}}_{ij}}\left(g - [\bar{G}]_{ij}\right) \tag{15}$$

when $|g| \to \infty$, the pdf $p_{\mathbf{G}_{ij}}(g) = O\left(e^{-b_{ij}|g|^{\beta_{ij}}}\right)$ ‡ and bounded by constant K. For this scenario, the pdfs $p_{\mathbf{G}_{ij}}(g)$ and $p_{\widetilde{\mathbf{G}}_{ij}}(\tilde{g})$ are same. For $|g| \to 0$, is near $-[G]_{ij}$ for $p_{\widetilde{\mathbf{G}}_{ij}}$, which the TRT result of channel **G** depends upon.

Consider a scalar channel $(p(\cdot), \bar{g})$, where the channel mean $\bar{g}$ and the pdf of the channel $p(\cdot)$ subtracting the channel mean. We claim a channel is regular only if

1. elements of channel matrix behave polynomial near zero, fast decaying near infinity and are upper bounded by constant K.
2. if $\bar{g} \neq 0$, when $|g + \bar{g}| \to 0$, $p(g) = \Theta(|g|^{t_{ij}})$

when $\bar{g} \neq 0$, it can be noted that a regular channel is equivalent to Gaussian channel in TRT's perspective meaning that nonzero mean is non-beneficial for throughput gain and multiplexing gain. This can be summarized in the following corollary.

Corollary 1: Consider that the channel fading matrix **G** is strictly regular. Let $\bar{G} = E\{\mathbf{G}\}$ and $\widetilde{\mathbf{G}} = \mathbf{G} - \bar{G}$. Then TRT of channel **G** is same as those of MIMO systems with channel matrix **B** with independent elements such as:

- $\forall i, j$ with $\bar{G}_{ij} \neq 0$, $\mathbf{B}_{ij}$ has complex Gaussian distribution with zero mean;
- $\forall i, j$ with $\bar{G}_{ij} = 0$, $\mathbf{B}_{ij}$ has the same distribution as $\bar{\mathbf{G}}_{ij}$.

The optimal TRT for channel **G** is same as that of channel **B,** which can be realized through Theorem 1.

We remark that for nonzero channel mean in other scenarios is beneficial for throughput gain or multiplexing gain or both.

6.4. Effect of Combination of Channel Mean and Channel Correlation

Using the results obtained from Section 6.2 and 6.3 we can write:

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‡ The usage $f(x) = \Theta(g(x))$ denotes that there exists positive c_1 and c_2 such as $c_1(g(x)) \leq f(x) \leq c_2(g(x))$ valid, and $f(x) = O(g(x))$ denotes that there exists a positive c_2 such that $f(x) \leq c_2 g(x)$.

Corollary 2: For MIMO system, say, that the channel matrix **G** has the form of $G = \Sigma_1 \widehat{\mathbf{G}} \Sigma_2$, where Σ_1 and Σ_2 are the covariance matrices with full rank at the receiver and the transmitter, respectively; $\widehat{\mathbf{G}}$ has independent and strictly regular entries. Then TRT of channel **G** are similar to those of MIMO system with channel matrix $\widehat{\mathbf{G}}$, which can be realized through Corollary 1.

7. Conclusion

The notable advantage of Throughput-Reliability Trade-off (TRT) over Diversity- Multiplexing Trade-off (DMT) is that it reveals the interplay among transmission rate, signal-to-noise ratio and outage/error probability parameters. In this paper, we have provided TRT analysis of general fading channels, which includes different fading distributions, correlation between channels, non-identical fading distributions, and non-zero channel means. The analysis has been carried-out by considering a generalized probability distribution function by considering different channel fading scenarios. We have also proved that a relationship do exists between DMT and TRT for general fading channels. The simulated and TRT predicted results provided have validated the veracity of our derivation of TRT expression for MIMO system in a generalized fading scenarios. We are working on the TRT analysis of MIMO systems in generalized fading scenarios under finite SNR region and would be our extension of this work.

Appendix

A. Proof of Theorem 1

Lower Bound: Lower bound for outage probability is given in equation (4). Let us define the following variable related to the channel coefficients and the transmission rate as

$$\beta_i = \frac{\log(1 + p\lambda_i)}{R}. \tag{16}$$

By equations (6) and (16), the $p(\lambda)$ can be written in terms of $p(\beta)$. Defining $\mathcal{B} \triangleq \{\beta | \beta_q \geq \cdots \geq \beta_1 \geq 0, 1 - \sum_i \beta_i > 0\}$ and multiplying both sides with $2^{-c(k)R}$ we get,

$$P_{otg} 2^{-c(k)R} \geq K 2^{-c(k)R} R^q \left(\frac{1}{\rho}\right)^{MN} \int_{\mathcal{B}} \prod_{i=1}^{q} \left(2^{\beta_i R} - 1\right)^{|M-N|}$$
$$\prod_{i<j} \left(2^{\beta_j R} - 2^{\beta_i R}\right)^2 \prod_{i=1}^{q} 2^{\beta_i R} \tag{17}$$
$$\int_{V_{N,N}} \int_{V_{M,M}} \prod_{i=1}^{M} \prod_{j=1}^{N} \left| [\mathbf{UDQ}]_{ij} \right| d\mathbf{Q} d\mathbf{U} d\beta$$

where $K \triangleq \left(\frac{\ln(2)}{4\pi}\right)^q$. We now look up for a subset region $\mathcal{OB}_{\varepsilon_1} \subset \mathcal{B}$ such that

$$\mathcal{OB}_{\varepsilon_1} \triangleq \left\{\beta \in \mathcal{B} | \mathcal{E}_1 > 0, s.t. \frac{\log \rho}{R} - \varepsilon_1 \geq \beta_q, \left|\beta_j - \beta_i\right| > \varepsilon_1 \quad \forall i \neq j\right\}. \tag{18}$$

Then we have

$$P_{otg} 2^{-c(k)R} \geq K 2^{-c(k)R} R^q \left(\frac{1}{\rho}\right)^{MN} \int_{\mathcal{OB}_{\varepsilon_1}} \prod_{i=1}^{q} (1 - 2^{-\varepsilon_1 R})^{|M-N|}$$
$$2^{\beta_i R |M-N|} \prod_{i<j} (1 - 2^{-\varepsilon_1 R})^2 2^{2\beta_j R} \prod_{i=1}^{q} 2^{\beta_i R} \tag{19}$$
$$\int_{\mathcal{U}_{N,N}(\varepsilon)} \int_{Q_{M,M}(\varepsilon)} X(U, Q)\, d\mathbf{Q} d\mathbf{U} d\beta$$

where $X(U,Q)=\left[\left(\frac{1}{\rho}\right)^{1/2} 2^{\beta_q R/2}(1-2^{-\varepsilon_1 R})^{1/2}\left[\varepsilon^2+(q-1)2^{-\varepsilon_1 R/2}\right]\right]^{\Sigma t_{i,j}}$. The derivation of $X(U,Q)$ has not been provided due to space constraint. Using $X(U,Q)$, (19) is simplified as,

$$P_{otg}2^{-c(k)R} \geq 2^{-c(k)R}KR^q\left(\frac{1}{\rho}\right)^{MN+\frac{\Sigma t_{i,j}}{2}}(1-2^{-\varepsilon_1 R})^{MN+\frac{\Sigma t_{i,j}}{2}}$$
$$\left(\varepsilon^2+(q-1)2^{-\varepsilon_1 R/2}\right)^{\Sigma t_{i,j}}\int_{\mathcal{OB}_{\varepsilon_1}} 2^{f(\beta)R}\,d\beta \tag{20}$$

where

$$f(\beta) \triangleq \sum_{i=1}^{q}(|M-N|+2i-1)\beta_i + \frac{\beta_q}{2}\sum_{i,j} t_{i,j}.$$

We can now defined β^* as

$$\beta^* \triangleq \arg\sup_{\beta\in\mathcal{OB}_{\varepsilon_1}} f(\beta). \tag{21}$$

Because of the continuity of function f for some $\varepsilon_2 > 0$, there exists a neighborhood $I_{\varepsilon_2} \subset \mathcal{B}$ around $\mathcal{OB}_{\varepsilon_1}$ in which $f(\beta) \geq f(\beta_{\varepsilon_1}) - \varepsilon_2$. So in the intersection we have $\mathcal{OB}_{\varepsilon_1} \cap I_{\varepsilon_2}$. The equation (20) is then written as,

$$P_{otg}2^{-c(k)R} \geq KR^q(1-2^{-\varepsilon_1 R})^{MN+\frac{\Sigma t_{i,j}}{2}}$$
$$\left(\varepsilon^2+(q-1)2^{-\varepsilon_1 R/2}\right)^{\Sigma t_{i,j}}$$
$$2^{-c(k)R}\left(\frac{1}{\rho}\right)^{MN+\frac{\Sigma t_{i,j}}{2}} 2^{(f(\beta_{\varepsilon_1})-c(k)-\varepsilon_2)R}\mathrm{Vol}\{\mathcal{OB}_{\varepsilon_1}\cap I_{\varepsilon_2}\} \tag{22}$$

where $\mathrm{Vol}\{\mathcal{OB}_{\varepsilon_1}\cap I_{\varepsilon_2}\} = \int_{\mathcal{OB}_{\varepsilon_1}\cap I_{\varepsilon_2}} d\beta$. Then we have

$$\frac{\log P_{otg} - c(k)R}{\log\rho} \geq \frac{\mathrm{Vol}\{\mathcal{OB}_{\varepsilon_1}\cap I_{\varepsilon_2}\}}{\log\rho} - \left(MN+\frac{\Sigma t_{i,j}}{2}\right) + \left(f(\beta_{\varepsilon_1})-c(k)\right)\frac{R}{\log\rho} +$$
$$\frac{\log\left(KR^q(1-2^{-\varepsilon_1 R})^{MN+\frac{\Sigma\Sigma t_{i,j}}{2}}\left(\varepsilon^2+(q-1)2^{-\varepsilon_1 R/2}\right)^{\Sigma\Sigma t_{i,j}/2}\right)}{\log\rho}. \tag{23}$$

To obtain $f(\beta^*)$ under the constraint $\mathcal{OB}_{\varepsilon_1}$, we partition the operating region according to the value of $\frac{\log\rho}{R}$. The first case is when $(R,\rho) \in \mathcal{R}_\delta(k)$ in which case

$$f = M+N-1+\sum_{i,j}\frac{t_{i,j}}{2}. \tag{24}$$

The supremum is achieved at $\beta^* = (0,0,\cdots,0,1)$ in this case. When $(R,\rho) \in \mathcal{R}_\delta(k)\ \ \forall k \in \mathbb{Z}$ and $q > k > 0$ the second case happens, where

$$f = \left(M+N-(2k+1)\right) + \sum_{i,j}\frac{t_{i,j}}{2}\left(\frac{\log\rho}{R}+\varepsilon\right) + k(k+1)\left(\frac{\log\rho}{R}+\varepsilon\right). \tag{25}$$

whose supremum occurs at

$$\beta^* = \left(0,\cdots,1-k\left(\frac{\log\rho}{R}+\varepsilon\right),\frac{\log\rho}{R}+\varepsilon,\cdots,\frac{\log\rho}{R}+\varepsilon\right). \tag{26}$$

Considering the values of the function $f(\beta^*)$ in different operating regions given in (8), we have

$$\frac{\log P_{otg} - c(k)R}{\log \rho} \geq \frac{log\ \left(KR^q(1-2^{-\varepsilon_1 R})^{MN+\frac{\Sigma\Sigma t_{i,j}}{2}}\left(\varepsilon^2+(q-1)2^{-\varepsilon_1 R/2}\right)^{\Sigma\Sigma t_{i,j}/2}\right)}{log\ \rho} - \left(NM+\sum\sum\frac{t_{i,j}}{2}\right)+\left[M+N-1+\sum\sum\frac{t_{i,j}}{2}-c(k)\right]\frac{R}{\log\rho}. \tag{27}$$

Comparing equations (7), (9) and (27) we can write

$$\lim_{\rho\to\infty}\inf\frac{\log P_{otg}-c(k)R}{\log\rho} \geq -g(k)+\left(\tilde{c}(k)-c(k)\right)\times\lim_{\rho\to\infty}\inf\frac{R}{\log\rho} \tag{28}$$

where

$$\tilde{c}(k) \triangleq \begin{cases} M+N-1+\dfrac{\Sigma\Sigma t_{i,j}}{2}, & k=0 \\ M+N-(2k+1), & 0<k\leq q \end{cases}. \tag{29}$$

Upper Bound: Upper bound for outage probability given in equation (5). Let us define the following variable related to the channel coefficients and the transmission rate as

$$\alpha_i = \frac{\log\left(1+\frac{\rho}{M}\lambda_i\right)}{R}. \tag{30}$$

By equations (6) and (30), $p(\lambda)$ can be written in terms of $p(\alpha)$. Defining $\mathcal{A} \triangleq \left\{\alpha|\alpha_q \geq \cdots \geq \alpha_1 \geq 0, 1-\sum_i \alpha_i > 0\right\}$ and multiplying both sides with $2^{-c(k)R}$ the outage probability is written as,

$$P_{otg}2^{-c(k)R} \leq K_1 2^{-c(k)R}R^q\left(\frac{1}{\rho}\right)^{MN}\int_{\mathcal{A}}\prod_{i=1}^{q}(2^{\alpha_i R}-1)^{|M-N|} \prod_{i<j}\left(2^{\alpha_j R}-2^{\alpha_i R}\right)^2\prod_{i=1}^{q}2^{\alpha_i R} \int_{V_{N,N}}\int_{V_{M,M}}\prod_{i=1}^{M}\prod_{j=1}^{N}\left|[\mathbf{UDQ}]_{ij}\right|d\mathbf{Q}d\mathbf{U}d\alpha \tag{31}$$

where $K_1 = \left(\frac{\ln(2)}{R}\right)^q M^{MN}$. Since $\mathbf{U}$ and $\mathbf{Q}$ are unitary matrices, their elements are bounded by 1. Note that $[\mathbf{UDQ}]_{ij}$ is a polynomial with the highest order of $\left(\frac{2^{\alpha_q R}-1}{\rho}\right)^{t_{i,j}/2}$, therefore there exists a positive constant $N_1^{t_{i,j}}$ such that $\left|[\mathbf{UDQ}]_{ij}\right| \leq N_1^{t_{i,j}}\left(\frac{2^{\alpha_q R}-1}{\rho}\right)^{t_{i,j}/2}$. Thus, equation (31) is modified as

$$P_{otg}2^{-c(k)R} \leq A_1 + A_2$$

where

$$A_1 = 2^{-c(k)R}\int_{\mathcal{A}_1}p(\alpha)d\alpha \qquad A_2 = 2^{-c(k)R}\int_{\mathcal{A}_2}p(\alpha)d\alpha$$

and

$$\mathcal{A}_1 \triangleq \left\{\alpha \in \mathcal{A}|\alpha_q > \frac{\log\rho}{R}+\varepsilon\right\}$$
$$\mathcal{A}_2 \triangleq \left\{\alpha \in \mathcal{A}|\alpha_q \leq \frac{\log\rho}{R}+\varepsilon\right\}. \tag{32}$$

Using A_1 and A_2, we can write

$$\lim_{\rho\to\infty}\sup\frac{\log P_{otg}-c(k)R}{\log\rho}\le\lim_{\rho\to\infty}\sup\frac{\log(1+A_1/A_2)}{\log\rho}+\lim_{\rho\to\infty}\sup\frac{\log(1+A_2)}{\log\rho}. \tag{33}$$

Now by considering the first term at the right-hand side of (33), we write that

$$A_1\le N_1^{\Sigma\Sigma t_{i,j}}K_1R^q\left(\frac{1}{\rho}\right)^{MN+\frac{\Sigma\Sigma t_{i,j}}{2}}2^{-c(k)R}\int_{\mathcal{A}_1}2^{f(\alpha)R}d\alpha \tag{34}$$

where

$$f(\alpha)\triangleq\sum_{i=1}^{q}(|M-N|+2i-1)\alpha_i+\alpha_q\sum_{i,j}t_{i,j/2}.$$

Realizing that since $\mathcal{A}_1\subseteq\mathcal{A}\subseteq\{\alpha|1\ge\alpha_i\ge 0,\forall i\}.\mathrm{Vol}\{\mathcal{A}_1\}\le 1$, we conclude

$$A_1\le N_1^{\Sigma\Sigma t_{i,j}}K_1R^q\left(\frac{1}{\rho}\right)^{MN+\frac{\Sigma\Sigma t_{i,j}}{2}}(2^{\varepsilon R})^{\frac{f_1c(k)}{\varepsilon}} \tag{35}$$

where

$$f_1=\sup_{\mathcal{A}_1}f(\alpha).$$

On the other hand, we can write

$$\begin{gathered}A_2\ge K_1R^q(1-2^{-\varepsilon_3R})^{MN+\frac{\Sigma\Sigma t_{i,j}}{2}}2^{-c(k)R}\\ \left(\frac{1}{\rho}\right)^{MN+\frac{\Sigma\Sigma t_{i,j}}{2}}\int_{\mathcal{A}_{\varepsilon_3}}2^{f(\alpha)}\,d\alpha.\end{gathered} \tag{36}$$

where

$$\mathcal{A}_{\varepsilon_3}\triangleq\left\{\alpha\in\mathcal{A}_2|\,\frac{\log\rho}{R}-\alpha_q>\varepsilon_3,\alpha_1>\varepsilon_3,\left|\alpha_j-\alpha_i\right|>\varepsilon_3,\forall i\ne j\right\}.$$

Let us define α^* as

$$\alpha^*\triangleq\arg\sup_{\alpha\in\mathcal{A}_{\varepsilon_3}}f(\alpha).$$

Then it follows from the continuity of $f(\cdot)$ that, for any ε_4, there exists a neighborhood I_{ε_4} of α^* within which

$$f(\alpha)\ge f(\alpha^*)-\varepsilon_4. \tag{37}$$

We can then write

$$\begin{gathered}A_2\ge K_1R^qN_1^{\Sigma\Sigma t_{i,j}}(1-2^{-\varepsilon_3R})^{MN+\frac{\Sigma\Sigma t_{i,j}}{2}}\left(\frac{1}{\rho}\right)^{MN+\frac{\Sigma\Sigma t_{i,j}}{2}}\\ 2^{(f(\alpha^*)-\varepsilon_4-c(k))R}\mathrm{Vol}\left\{\mathcal{A}_{\varepsilon_3}\cap I_{\varepsilon_4}\right\}.\end{gathered} \tag{38}$$

From (35) and (38),

$$\begin{gathered}\frac{A_1}{A_2}\le\left(1-(2^{\varepsilon R})^{-\frac{\varepsilon_3}{\varepsilon}}\right)^{-\left(MN+\frac{\Sigma\Sigma t_{i,j}}{2}\right)}(2^{\varepsilon R})^{\frac{f_1-f(\alpha^*)+\varepsilon_4}{\varepsilon}}\\ \mathrm{Vol}^{-1}\left\{\mathcal{A}_{\varepsilon_3}\cap I_{\varepsilon_4}\right\}\end{gathered} \tag{39}$$

then

$$\limsup_{\rho\to\infty}\frac{\log(1+A_1/A_2)}{\log\rho}=0. \tag{40}$$

Note that (40) holds as $\rho\to\infty$. To characterize the second term on the right-hand side of (33), we write that

$$A_2\le K_1R^qN_1^{\frac{\Sigma\Sigma t_{i,j}}{2}}\left(\frac{1}{\rho}\right)^{MN+\frac{\Sigma\Sigma t_{i,j}}{2}}2^{(f_2-c(k))R} \tag{41}$$

where

$$f_2\triangleq\sup_{\mathcal{A}_2}f(\alpha).$$

Performing operations similar to those outlined in (24) through (26) yield

$$\begin{aligned}\frac{\log A_2}{\log\rho}\le\frac{\log\left(K_1R^qN_1^{\Sigma\Sigma t_{i,j}}\right)}{\log\rho}-g(k)+\\ \left(MN+\frac{1}{2}\Sigma\Sigma t_{i,j}-g(k)\right)\frac{\varepsilon R}{\log\rho}+\left(\tilde{c}(k)-c(k)\right)\frac{R}{\log\rho}\end{aligned} \tag{42}$$

With (40) and (42) we look back at (33) to get

$$\limsup_{\rho\to\infty}\frac{\log P_{otg}-c(k)R}{\log\rho}\le-g(k). \tag{43}$$

Thus completes the proof of Theorem 1.

B. *Proof of Theorem 2*

The proof of Theorem 2 is similar to proof of [5, Theorem 3]. However, the proof given here is for $l\ge M+N-1+\frac{\Sigma\Sigma t_{i,j}}{2}$ whereas the proof given in [5] for $l\ge M+N-1$.

Proof:

In particular, we first have to show that

$$\liminf_{\rho\to\infty}\frac{\log P_e-c(k)R}{\log\rho}\ge-g(k) \tag{44}$$

To prove equation (44), we need to follow [Equation (49)- (54), Theorem 3, 5]. The second step is to prove that

$$\limsup_{\rho\to\infty}\frac{\log P_\varepsilon(R,\rho)-c(k)R}{\log\rho}\le-g(k) \tag{45}$$

for the codeword length $l\ge M+N-1+\frac{\Sigma\Sigma t_{i,j}}{2}$ has little deviations than [Theorem 3, 5]. It is easy to realize that the maximum-likelihood error probability conditioned on certain channel realizations is upper bounded by

$$\begin{aligned}P_{E|\mathbf{G}}(R,\rho)&\le 2^{Rl}\det\left(\mathbf{I}_q+\frac{\rho}{2M}\mathbf{G}\mathbf{G}^H\right)^{-l}\\ &=2^{Rl}\prod_{i=1}^{q}\left(1+\frac{R}{2M}\lambda_i\right)^{-l}\end{aligned} \tag{46}$$

where $\lambda_q\ge\cdots\ge\lambda_1\ge0$ represents the ordered eigenvalues of $\mathbf{G}\mathbf{G}^H$. Introducing change of variables

$$\gamma_i=\frac{\log\left(1+\frac{\rho}{2M}\lambda_i\right)}{R}$$

we can write (46) as

$$P_{E|\mathbf{G}}(R,\rho)\le 2^{(1-\Sigma_{i=1}^{q}\gamma_i)Rl} \tag{47}$$

Next, we define $\boldsymbol{\mathcal{D}}$ as

$$\boldsymbol{\mathcal{D}} \triangleq \left\{\boldsymbol{\gamma}|\gamma_q \geq \cdots \geq \gamma_1 \geq 0, 1 - \sum_{i=1}^{q} \gamma_i \geq 0\right\}. \tag{48}$$

From (47) it is seen that $\boldsymbol{\mathcal{D}}$ consists of channel realizations for which $P_{E|\gamma}$ cannot be made smaller even through the usage of infinitely long codewords. We upper bound $P_{E|\gamma}$ by 1 for those channel realizations, i.e.,

$$\begin{gathered} P_{\varepsilon}(R,\rho) = P_{E,\mathcal{D}^C}(R,\rho) + P_{E,\mathcal{D}}(R,\rho) \\ P_{\varepsilon}(R,\rho) \leq P_{E,\mathcal{D}^C}(R,\rho) + P_{\mathcal{D}}(R,\rho) \end{gathered} \tag{49}$$

where $\boldsymbol{\mathcal{D}}^{\mathrm{C}}$ denotes the complement of $\boldsymbol{\mathcal{D}}$**.** Focusing on $P_{\boldsymbol{\mathcal{D}}}(R,\rho)$ we realizing that $\boldsymbol{\mathcal{D}}$ is precisely same set as $\boldsymbol{\mathcal{A}}$ and, $p(\gamma)$ is identical to $p(\alpha)$, then

$$\limsup_{\rho\to\infty} \frac{\log P_{\mathcal{D}}\,(R,\rho) - c(k)R}{\log \rho} \leq -g(k). \tag{50}$$

Now, turning our attention to $P_{E,\mathcal{D}^C}(R,\rho)$ we realize that

$$\lim_{\rho\to\infty} P_{E,\mathcal{D}^C}(R,\rho)\, 2^{-c(k)R} = 0 \tag{51}$$

which means that

$$\limsup_{\rho\to\infty} \frac{\log P_{E,\mathcal{D}_1^C}\,(R,\rho) - c(k)R}{\log \rho} = \limsup_{\rho\to\infty} \frac{\log P_{E,\mathcal{D}_2^C}\,(R,\rho) - c(k)R}{\log \rho} \tag{52}$$

where

$$\mathcal{D}_1^C = \left\{\gamma \notin |\gamma_q > \frac{\log \rho}{R}\right\}$$
$$\mathcal{D}_2^C = \left\{\gamma \notin |\gamma_q \leq \frac{\log \rho}{R}\right\}.$$

Notice that (51) holds exactly as (40) does. By utilizing (47) we have

$$\begin{gathered} P_{E,\mathcal{D}_2^C}(R,\rho) 2^{-c(k)R} = 2^{-c(k)R} \int_{\mathcal{D}_2^C} P_{E|\gamma}(R,\rho)\, p(\gamma) d\gamma \\ \leq K_1 R^q N_1^{\Sigma\Sigma t_{i,j}} \rho^{-\left(MN + \frac{\Sigma\Sigma t_{i,j}}{2}\right)} \int_{\mathcal{D}_2^C} 2^{[f + l(1 - \Sigma_{i=1}^{q} \gamma_i)]R}\, d\gamma. \end{gathered} \tag{53}$$

Thus

$$P_{E,\mathcal{D}_2^C}(R,\rho) 2^{-c(k)R} \leq K_1 R^q N_1^{\Sigma\Sigma t_{i,j}} \rho^{-\left(MN + \frac{\Sigma\Sigma t_{i,j}}{2}\right)} 2^{(f_2 - c(k))R} \mathrm{Vol}\{\mathcal{D}_2^C\} \tag{54}$$

where

$$f_2 \triangleq \sup_{\mathcal{D}_2^C} f + l\left(1 - \sum_{i=1}^{q} \gamma_i\right). \tag{55}$$

Realizing that for $l \geq M + N - 1 + \frac{\Sigma\Sigma t_{i,j}}{2}$, the supremum occurs at $\gamma = \gamma^*$, such that $1 - \Sigma\gamma_i^* = 0$, f_2 can be easily derived from (24) and (25) by simply plugging in $\varepsilon = 0$

$$\limsup_{\rho\to\infty} \frac{\log P_{E,\mathcal{D}_2^C}\,(R,\rho) - c(k)R}{\log \rho} \leq -g(k). \tag{56}$$

Thus completes the proof.

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S. Rajarajan is currently pursuing Ph. D. at Reliability Engineering Centre, Indian Institute of technology Kharagpur, West Bengal, India. He did his masters in Communication Systems from SRM University, Chennai, Tamilnadu, India in the year 2012. His B.E is in Electronics and Communication from Anna University, Chennai, Tamilnadu, India in the year 2009. His research interests include physical layer design and analysis of wireless communication systems and information theory. He is an associate editor to International Journal of Performability Engineering.

S. K. Chaturvedi is currently working as Associate Professor at Reliability Engineering Centre, Indian Institute of Technology, Kharagpur (WB) India. He has research interest in the area of reliability modeling and analysis, network reliability, life-data analysis, maintenance and optimization. He has published papers in several international journals such as IEEE Transactions on Reliability, Performability Engineering (IJPE), Quality and Reliability Management, Quality, Reliability and Safety Engineering, Uncertain Systems, Risk and Reliability Part ‘O’, Quality Technology and Quantitative Management, Quality and Reliability International etc. and executed several consultancy projects of private and Govt. organizations. He’s two books to his credits and is also on the review panel of IEEE transactions on Reliability, IJPE, IJQRM, IJ System Science, IJ Failure Analysis. He’s on the editorial board of International Journal of Mathematical, Engineering and Management Sciences (IJMEMS). He’s also a senior member to IEEE and Co-Editor-in-Chief to IJPE. Google scholar: https://scholar.google.co.in/citations?user=ULGIT6UAAAAJ&hl=en .