

Stochastic and Cost-Benefit Analysis of Two Unit Hot Standby Database System

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Abstract: A discrete state space and continuous time stochastic model for two unit hot standby database system has been developed. The system comprised of one primary unit and single hot standby unit. The primary unit is production unit synchronized with hot standby unit through online transfer of archive redo log files. Different modes of failure of primary database unit are considered. Failure of database unit either primary or standby is dealt by database administrator (DBA). Expressions for various measures of system effectiveness have been obtained by making use of semi-Markov processes and regenerative point technique. Numerical results have been drawn on the basis of data collected. Bounds pertaining the profitability of the system have also been obtained.

Keywords: *Database system, hot standby, semi-Markov process, cost-benefit analysis, regenerative point technique*

1. Introduction

Database system plays a pivotal role locally or globally in handling and securing data for various industries. Promising technological advancements have made enterprises absolutely dependent on automated database systems. A small operational error in these systems may lead to disastrous failures which differ in the severity of their impact depending on the operations of an organization. Few major database system provider companies to mention are Oracle corporation provides RDBMS Oracle and open source DBMS MySQL; Microsoft Corporation provides Microsoft Access and Microsoft SQL Server; IBM provides DB2. Depending upon the volume of company and significance of the data primary database unit and number of standby units are provided by the companies. Such systems maintain the data for various industries working globally in Telecommunication, Automobile, Gas & Oil, Transportation, Education, Medical, Finance, Marketing, Banking, Textile and Garments sectors, etc. Keeping that in view, reliability and availability analysis of these automated database systems is of great importance in the present scenario. The present paper is our attempt to analyze a two unit hot standby database system.

Standby systems have been discussed extensively by various researchers including [1-3]. Beith et al. [4] discussed a two-identical-cold standby system with two types of

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repair persons under arbitrary life and repair times. Shakuntla et al. [5] did the reliability analysis of polytube industry by using supplementary variable technique. Reliability analysis of a seven unit desalination plant with shutdown during winter season and repair/maintenance on FCFS basis was carried out by Rizwan et al. [6]. Manocha and Taneja [7] analyzed a stochastic model for two unit cold standby system with arbitrary distribution for life, repair and waiting times. Kuo and Ke [8] did the comparative analysis of standby systems with unreliable server and switching failures. Kumar and Batra [9] developed a stochastic model for printed circuit boards (PCBs) manufacturing process.

Singh et al. [10] did the reliability and availability analysis of database system with numbers of standby units provided by system provider. Higher number of standby units eventually may lead to increased cost, which is to be paid by the user itself. Two unit hot standby database system has not being discussed so far in the literature. So, the present study deals with database system comprising of one primary production database unit synchronized with single hot standby unit through online transfer and simultaneous archiving redo logs.

Notations

λ	constant failure rate of primary database unit
α	constant failure rate of standby database unit
p_1	probability that standby unit is in synchronous with primary database unit on the failure of primary database unit
p_2	probability that redo log files are not updated in standby database unit on the failure of primary database unit
p_3	probability that redo log files are not created in standby database unit on the failure of primary database unit
$M_i(t)$	probability that primary database unit up initially in regenerative state i is up at time t without passing through any other regenerative state or returning to itself through one or more non-regenerative states
$g(t)/G(t)$	pdf/cdf of the time for repairing the primary database unit
$g_1(t)/G_1(t)$	pdf/cdf of the time for repairing the standby database unit
$h_1(t)/H_1(t)$	pdf/cdf of the time for updating the redo log files in the standby database unit
$h_2(t)/H_2(t)$	pdf/cdf of the time for creating the redo log files in the standby database unit
\otimes/\odot	symbol for Laplace-Stieljes convolution / Laplace convolution

for some standard notations one may refer to [2].

Let us now describe the system considered for modelling along with the assumptions taken for developing the same.

2. System Description and Assumptions

Figure 1 demonstrates the functioning of two unit hot standby database system. The standby database unit is kept in synchronization with primary database unit through online redo log files. Redo log files created at primary site are archived at the standby site. In case of the failure of primary unit the standby unit becomes the production unit and the failed unit goes under repair of database administrator (DBA) immediately.

The following situations, which are mutually exclusive and exhaustive, may be observed on the failure of primary unit:

- i. Standby unit working in synchronous with primary unit
- ii. Redo log files are not updated in standby unit
- iii. Redo log files are not created in standby unit

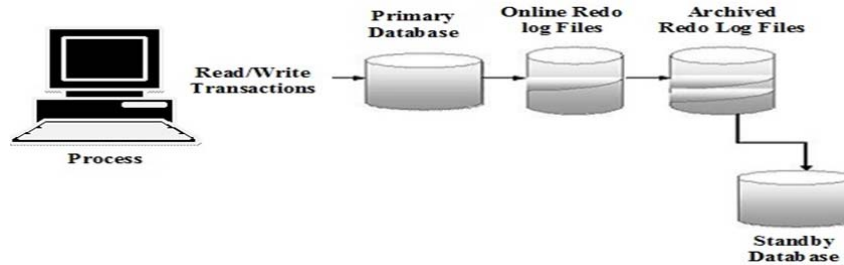


Figure 1: Two Unit Hot Standby Database System

In the situational failure of primary unit all the archived redo log files are updated/created into standby unit by the DBA, before making it production database. Cost for using standby database unit as the primary database unit is to be paid by user itself. Proposed system has a single DBA facility. After each repair, the system works as good as new. All failure times are assumed to have exponential distribution whereas repair times, time for creating and updating redo log files have general distributions. By using semi-Markov process and regenerative point technique expressions for various measures of system effectiveness such as Mean time to system failure, Mean time to failure of primary database unit, Availability of primary database unit, busy period of DBA, Expected number of visits by the DBA and profit function has obtained. Numerical analysis has been done on the basis of data collected for database systems. Bounds concerning the profitability of the system have also been obtained.

All the possibilities have been shown by states and transitions from one state to another along with transition probabilities and mean sojourn times in the following section.

3. Modelling of the System

Symbols for the states of the system are

- P_0 primary database unit is operative
- $H_s / H_r / H_R$ hot standby database/under repair/ repair from previous state
- S_0 hot standby database unit is used as primary database unit
- $S_r / S_w / S_R$ hot standby unit (used as primary database unit) is under repair/waiting for repair/ repair from previous state
- $F_r / F_w / F_R$ failed unit under repair / waiting for repair / repair from previous state
- $P_f H_s \bar{A} \bar{D}$ primary database unit fails and redo log files are not updated in hot standby database
- $P_f H_s \bar{A}$ primary database unit fails and redo log files are not created in hot standby database unit

Considering these symbols, the system can be in any one of the following states

- State 0: (P_0, H_s) State 1: (F_r, S_0) State 2: (P_0, H_r) State 3: (F_w, H_R)
 - State 4: $(P_f H_s \bar{A} \bar{D})$ State 5: $(P_f H_s \bar{A})$ State 6: (F_R, S_w) State 7: (P_0, S_r) State 8: (F_w, S_R)
- Possible transitions of the system are shown in Figure 2:

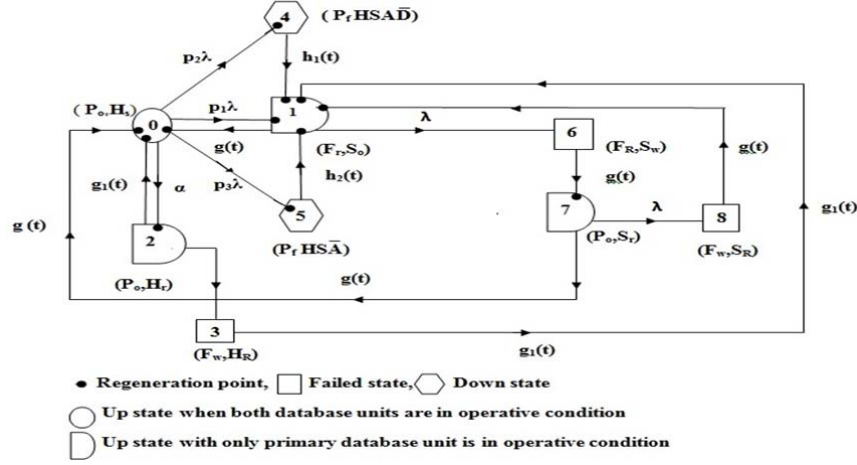


Figure 2: State Transition

Here $E = \{0, 1, 2, 4, 5, 7\}$ is set of regenerative states and $\bar{E} = \{3, 6, 8\}$ is a set of non-regenerative states. States 3, 6 and 8 are failed states, whereas 4 and 5 are down states. Let $T_0 (\equiv 0), T_1, T_2, \dots$ be the epochs at which system enters to any state $i \in E$ and let X_n be the state visited at epoch in T_{n+1} i.e. just after transition at T_n . Then $\{X_n, T_n\}$ is a Markov-renewal process with state space E . Let

$$q_{ij} = P\{X_{n+1} = j, T_{n+1} - T_n \leq t \mid X_n = i\} \quad (1)$$

be the semi-Markov kernel over E . The transition probability matrix (t.p.m.) of the embedded Markov chain is, therefore, given by

$$P = (p_{ij}) = [q_{ij}(\infty) = q(\infty)] \quad (2)$$

where

$$\begin{aligned}
 p_{01} &= \int_0^\infty p_1 \lambda e^{-(\lambda+\alpha)t} dt, & p_{02} &= \int_0^\infty \alpha e^{-(\lambda+\alpha)t} dt, & p_{04} &= \int_0^\infty p_2 \lambda e^{-(\lambda+\alpha)t} dt, \\
 p_{05} &= \int_0^\infty p_3 \lambda e^{-(\lambda+\alpha)t} dt, & p_{10} &= \int_0^\infty e^{-\lambda t} g(t) dt, & p_{20} &= \int_0^\infty e^{-\lambda t} g_1(t) dt \\
 p_{23} &= \int_0^\infty \lambda e^{-\lambda t} \bar{G}_1(t) dt, & p_{41} &= \int_0^\infty h_1(t) dt, & p_{51} &= \int_0^\infty h_2(t) dt, \\
 p_{70} &= \int_0^\infty e^{-\lambda t} g(t) dt, & p_{16} &= p_{78} = \int_0^\infty \lambda e^{-\lambda t} \bar{G}(t) dt, \\
 p_{21}^{(3)} &= \int_0^\infty (\lambda e^{-\lambda t} \otimes 1) g_1(t) dt, & p_{17}^{(6)} &= p_{71}^{(8)} = \int_0^\infty (\lambda e^{-\lambda t} \otimes 1) g(t) dt. & & (3-15)
 \end{aligned}$$

It can be verified that,

$$\begin{aligned}
 p_{01} + p_{02} + p_{04} + p_{05} &= 1, & p_{10} + p_{16} &= 1, & p_{10} + p_{17}^{(6)} &= 1, & p_{20} + p_{23} &= 1, \\
 p_{20} + p_{21}^{(3)} &= 1, & p_{70} + p_{78} &= 1, & p_{41} + p_{51} &= 1, & p_{70} + p_{71}^{(8)} &= 1. & (16-23)
 \end{aligned}$$

Mean Sojourn time (μ_i) in regenerative state $i \in E$, is given as

$$\begin{aligned}
 \mu_0 &= \int_0^\infty e^{-(\lambda+\alpha)t} dt, & \mu_1 &= \mu_7 = \int_0^\infty e^{-\lambda t} \bar{G}(t) dt, & \mu_2 &= \int_0^\infty e^{-\lambda t} \bar{G}_1(t) dt, \\
 \mu_4 &= \int_0^\infty \bar{H}_1(t) dt & \mu_5 &= \int_0^\infty \bar{H}_2(t) dt & & (24-28)
 \end{aligned}$$

The unconditional time taken by system to transit for any regenerative state j when it (time) is counted from the epoch of entrance into state i is given as

$$\begin{aligned}
 m_{01} + m_{02} + m_{04} + m_{05} &= \mu_0, & m_{10} + m_{16} &= \mu_1, & m_{20} + m_{23} &= \mu_2, & m_{70} + m_{78} &= \mu_7, \\
 m_{41} &= \mu_4, & m_{51} &= \mu_5, & m_{10} + m_{17}^{(6)} &= m_{70} + m_{71}^{(8)} = -g^{*(0)} = K_1 \text{ (say)}, \\
 m_{20} + m_{21}^{(3)} &= -g_1^{*(0)} = K_2 \text{ (say)}. & & & & & & (29-36)
 \end{aligned}$$

Now, to find the reliability and to give cost-benefit analysis, various measures of system effectiveness are required to be obtained. The succeeding section determines all such measures.

4. Measures of System Effectiveness

4.1 Mean Time to System Failure (MTSF)

To determine mean time to system failure (MTSF) we regard the failed state as absorbing state. Let $\pi_i(t)$, where $i \in E$, be c.d.f. of the first passage time from regenerative state i to a failed state, employing the arguments used for regenerative process, we have the following recursive relations for ' $\pi_i(t)$ '

$$\begin{aligned}\pi_0(t) &= Q_{01}(t) \otimes \pi_1(t) + Q_{02}(t) \otimes \pi_2(t) + Q_{04}(t) \otimes \pi_4(t) + Q_{05}(t) \otimes \pi_5(t) \\ \pi_1(t) &= Q_{16}(t) + Q_{10}(t) \otimes \pi_0(t) \\ \pi_2(t) &= Q_{23}(t) + Q_{20}(t) \otimes \pi_0(t) \\ \pi_4(t) &= Q_{41}(t) \otimes \pi_1(t) \\ \pi_5(t) &= Q_{51}(t) \otimes \pi_1(t) \\ \pi_7(t) &= Q_{78}(t) + Q_{70}(t) \otimes \pi_0(t)\end{aligned}\quad (37-42)$$

Taking Laplace-Stieljes transformation of the above equations and solving them for $\pi_0^{**}(s)$, We obtain $\pi_0^{**}(s) = N(s)/D(s)$

$$\begin{aligned}\text{where, } N(s) &= Q_{01}^{**}(s) Q_{16}^{**}(s) + Q_{02}^{**}(s) Q_{23}^{**}(s) + Q_{04}^{**}(s) Q_{41}^{**}(s) Q_{16}^{**}(s) \\ &\quad + Q_{05}^{**}(s) Q_{51}^{**}(s) Q_{16}^{**}(s) \\ \text{and } D(s) &= 1 - Q_{01}^{**}(s) Q_{10}^{**}(s) - Q_{02}^{**}(s) Q_{20}^{**}(s) - Q_{04}^{**}(s) Q_{41}^{**}(s) Q_{10}^{**}(s) \\ &\quad - Q_{05}^{**}(s) Q_{51}^{**}(s) Q_{10}^{**}(s)\end{aligned}\quad (43-45)$$

The reliability of the system at time t is given by,

$$R(t) = L^{-1} \{ [1 - \pi_0^{**}(s)] / s \} \quad (46)$$

Now the mean time to system failure (MTSF) when the system starts from state '0' is

$$MTSF = \int_0^{\infty} R(t) dt = N/D$$

where, $N = \{ \mu_0 + \mu_1 (p_{01} + p_{04} + p_{05}) + p_{02} \mu_2 + p_{04} \mu_4 + p_{05} \mu_5 \}$

$$\text{and } D = \{ p_{16} (p_{01} + p_{04} + p_{05}) + p_{02} p_{23} \} \quad (47-49)$$

4.2 Mean Time to Failure of Primary Database Unit (MTFP)

Let $TP_i(t)$, where $i \in E$, be the c.d.f. of the first passage time of primary database unit from regenerative state i to the state where it is failed. By using arguments of theory of regenerative process, we have the following recursive relations for ' $TP_i(t)$ '

$$\begin{aligned}TP_0(t) &= Q_{01}(t) + Q_{04}(t) + Q_{05}(t) + Q_{02}(t) \otimes TP_2(t) \\ TP_2(t) &= Q_{23}(t) + Q_{20}(t) \otimes TP_0(t)\end{aligned}\quad (50-51)$$

Taking Laplace-Stieljes transformation of above equations and solving them for $TP_0^{**}(s)$, We obtain

$$\begin{aligned}TP_0^{**}(s) &= N_1(s)/D_1(s) \\ \text{where, } N_1(s) &= Q_{01}^{**}(s) + Q_{04}^{**}(s) + Q_{05}^{**}(s) + Q_{02}^{**}(s) Q_{23}^{**}(s) \\ \text{and } D_1(s) &= 1 - Q_{02}^{**}(s) Q_{20}^{**}(s)\end{aligned}\quad (52-54)$$

Now, the mean time to failure of primary database unit when the system start from state '0' is

$$MTFP = \lim_{s \rightarrow 0} \{ [1 - TP_0^{**}(s)] / s \} = N_1/D_1$$

where, $N_1 = (\mu_0 + p_{02} \mu_2)$

$$\text{and } D_1 = (1 - p_{02} p_{20}) \quad (55-57)$$

4.3 Availability of Primary Database Unit (AP₀)

Let AP_i(t), where i ∈ E, be the probability that the primary database unit is in upstate at instant t given that the system entered regenerative state i at t = 0, and using arguments the theory of regenerative process, AP_i(t) is seen to satisfy the following recursive relations

$$AP_0(t) = M_0(t) + q_{01}(t) \odot AP_1(t) + q_{02}(t) \odot AP_2(t) + q_{04}(t) \odot AP_4(t) + q_{05}(t) \odot AP_5(t)$$

$$AP_1(t) = q_{10}(t) \odot AP_0(t) + q_{17}^{(6)}(t) \odot AP_7(t)$$

$$AP_2(t) = M_2(t) + q_{20}(t) \odot AP_0(t) + q_{21}^{(3)}(t) \odot AP_1(t)$$

$$AP_4(t) = q_{41}(t) \odot AP_1(t)$$

$$AP_5(t) = q_{51}(t) \odot AP_1(t)$$

$$AP_7(t) = M_7(t) + q_{70}(t) \odot AP_0(t) + q_{71}^{(8)}(t) \odot AP_1(t)$$

$$\text{where, } M_0(t) = e^{-(\lambda+\alpha)t}, \quad M_2(t) = e^{-\lambda t} \bar{G}_1(t), \quad M_7(t) = e^{-\lambda t} \bar{G}(t) \quad (58-66)$$

Taking Laplace transform of the above equations and solving them for AP₀^{*}(s), We obtain

$$AP_0^{**}(s) = N_2(s)/D_2(s)$$

$$\text{Where, } N_2(s) = (1 - q_{17}^{(6)*}(s) q_{71}^{(8)*}(s)) (q_{02}^*(s) M_2^*(s) + M_0^*(s)) \\ + q_{17}^{(6)*}(s) (q_{01}^*(s) + q_{05}^*(s) q_{51}^*(s) + q_{04}^*(s) q_{41}^*(s) + q_{02}^*(s) q_{21}^*(s)) M_7^*(s)$$

$$\text{and } D_2(s) = (1 - q_{02}^*(s) q_{20}^*(s)) (1 - q_{17}^{(6)*}(s) q_{71}^{(8)*}(s)) - q_{04}^*(s) q_{41}^*(s) (q_{10}^*(s) \\ + q_{70}^*(s) q_{17}^{(6)*}(s)) - q_{10}^*(s) (q_{02}^*(s) q_{21}^{(3)*}(s) + q_{01}^*(s) + q_{05}^*(s) q_{51}^*(s)) \\ - q_{17}^{(6)*}(s) (q_{05}^*(s) q_{51}^*(s) q_{70}^*(s) + q_{01}^*(s) q_{70}^*(s) + q_{21}^{(3)*}(s) q_{02}^*(s) q_{70}^*(s)) \quad (67-69)$$

In steady-state the availability of primary database unit, is given by

$$AP_0 = \lim_{t \rightarrow \infty} AP_0(t) = \lim_{s \rightarrow 0} s AP_0^*(s) = N_2/D_2$$

$$\text{where, } N_2 = (1 - p_{17}^{(6)} p_{71}^{(8)}) (\mu_1 + p_{02} \mu_2) + (1 - p_{02} p_{20}) p_{17}^{(6)} \mu_7$$

$$\text{and } D_2 = (1 - p_{17}^{(6)} p_{71}^{(8)}) (\mu_0 + p_{04} \mu_4 + p_{05} \mu_5 + p_{02} K_2) + (1 - p_{02} p_{20}) (1 + p_{17}^{(6)}) K_1 \quad (70-72)$$

Proceeding in the similar manner and employing the arguments used for regenerative process, the other measures of system effectiveness are:

$$\text{Expected time for which standby database unit worked as primary database unit (S}_0\text{)} \\ = N_3/D_2$$

$$\text{Expected time for updating the redo log files in standby database unit (AU}_0\text{)} = N_4/D_2$$

$$\text{Expected time for creating redo log files in standby database unit (AC}_0\text{)} = N_5/D_2$$

$$\text{Expected time for repairing primary database unit (BP}_0\text{)} = N_6/D_2$$

$$\text{Expected time for repairing standby database unit (BH}_0\text{)} = N_7/D_2$$

$$\text{Expected number of visits by DBA (V}_0\text{)} = N_8/D_2$$

$$\text{where, } N_3 = (1 - p_{02} p_{20}) \mu_1 ; \quad N_4 = (1 - p_{17}^{(6)} p_{71}^{(8)}) p_{04} \mu_4$$

$$N_5 = (1 - p_{17}^{(6)} p_{71}^{(8)}) p_{05} \mu_5 ; \quad N_6 = (1 - p_{02} p_{20}) (K_1 + p_{17}^{(6)}) K_1$$

$$N_7 = (1 - p_{17}^{(6)} p_{71}^{(8)}) p_{02} K_2 ; \quad N_8 = (1 - p_{17}^{(6)} p_{71}^{(8)}) \quad (73-84)$$

and D₂ is already specified in equation (72).

On the basis of the above measures, the cost-benefit analysis can be carried out as in the following section.

5. Cost-Benefit Analysis

Needless to say, that the profit is most important aspect for a company/firm/organization using any type of system and hence, it is significant to carry out cost-benefit analysis for the system under consideration.

The profit is excess of revenue over the various costs involved and hence profit function takes the form

$$P(t) = \text{Expected revenue in } (0, t] - \text{Expected total cost in } (0, t] \quad (85)$$

For the model discussed in the present paper, the expected profit per unit time incurred to the system, in steady-state, is given by

Profit (P) = (Revenue generated by the system) – (Cost incurred when the secondary database is used as primary database + cost for updating the redo log files in standby database unit + cost for creating the redo log files in standby database unit + cost for which DBA is busy for repairing primary database unit + cost for which DBA is busy for repairing standby database unit + cost per visit of DBA + cost of Initial Installation)

$$\text{i.e. } P = (C_0AP_0) - (C_1S_0 + C_2AU_0 + C_3AC_0 + C_4BP_0 + C_5BH_0 + C_6V_0 + 2CI) \quad (86)$$

where, C_0 = Revenue per unit uptime

C_1 = Cost per unit time for which standby database unit worked as primary database unit

C_2 = Cost per unit time for updating the redo log files in standby database unit

C_3 = Cost per unit time for creating the redo log files in standby database unit

C_4 = Cost per unit time for which DBA is busy for repairing primary database unit

C_5 = Cost per unit time for which DBA is busy for repairing standby database unit

C_6 = Cost per visit of DBA

CI = Cost per unit time of Initial Installation

Now, one may be interested in knowing as to how the MTSF, availability of primary database unit and profit are affected with changes in various rates /cost /revenue. The authors have made an attempt to find such effects by performing various numerical calculations as done in the following section.

6. Numerical Calculations, Results and Discussion

6.1 Input Variables

For numerical calculations we consider repair times, time for updating and creating redo log files follows exponential distribution i.e.

$$g(t) = \eta e^{-\eta t}, g_1(t) = \alpha_1 e^{-\alpha_1 t}, h_1(t) = \gamma_1 e^{-\gamma_1 t} \text{ and } h_2(t) = \gamma_2 e^{-\gamma_2 t}. \quad (87)$$

The estimates of various probabilities, rates and costs for primary database unit as well as for standby database unit on the basis of the data collected are given as follows:

Probability that standby unit is in synchronous with primary unit on the failure of primary database unit (p_1) = 0.1628

Probability that redo log files are not updated in standby database unit on the failure of primary database unit (p_2) = 0.0465

Probability that redo log files are not created in standby database unit on the failure of primary database unit (p_3) = 0.7907

Constant failure rate of primary database unit (λ) = 0.00205 per hr

Constant repair rate of primary database unit (η) = 0.6529 per hr

Constant failure rate of standby database unit (α) = 0.00087 per hr

Constant repair rate of standby database unit (α_1) = 0.8533 per hr

Cost per unit time for which DBA is busy for repairing primary database unit (C_4) = 7326₹

Cost per unit time for which DBA is busy for repairing standby database unit (C_5)=8750₹
The values of above mentioned input variables have been taken as fixed as obtained from the collected data. However, for some parameters/variable, the data were not available and hence their values have been assumed as mentioned in the sub-sections 6.3 to 6.5.

6.2 Output Variables

The output variables here are various measures of system effectiveness including MTSF, Availability of primary database unit and Profit function. Results of the effects of various input variables on the output variables have been obtained and have been shown in the following sub-sections:

6.3 Effect of Rate (γ_1) on MTSF and Availability (AP_0) for Different Values of Rate(γ_2)

MTSF and steady-state availability of primary database (AP_0) are calculated on varying the rate (γ_1) for different values of rate (γ_2). The results are shown in Table 1.

Table 1: Values of MTSF and Availability (AP_0) w.r.t. Rate (γ_1) for Different Values of Rate (γ_2)

γ_1	MTSF (In hrs)			Availability (AP_0)		
	$\gamma_2=2.98$	$\gamma_2=2.9$	$\gamma_2=3$	$\gamma_2=2.98$	$\gamma_2=2.9$	$\gamma_2=3$
5	118178.5	118178.3	118178.1	0.996309	0.996311	0.996313
6	118178.1	118177.9	118177.7	0.996312	0.996314	0.996316
7	118177.9	118177.7	118177.4	0.996314	0.996316	0.996318
8	118177.7	118177.5	118177.2	0.996316	0.996318	0.99632
9	118177.5	118177.3	118177.1	0.996317	0.996319	0.996321
10	118177.4	118177.2	118177	0.996318	0.99632	0.996322

It is observed that

- MTSF decreases with the increase in the values of rate (γ_1) and it has lower values for higher values of rate (γ_2).
- Availability of primary database (AP_0) increases with the increase in the rate (γ_1) and has higher values for higher values of rate (γ_2).

6.4 Effect of Revenue (C_0) on Profit (P) for Different Values of Cost (C_1)

Behavior of profit (P) with respect to revenue (C_0) per unit uptime for different values of cost (C_1) per unit time for which standby database worked as primary unit is shown in Figure 3. Profit increases with increase in the revenue and has higher values for lower values of cost (C_1).

Moreover, for $\gamma_1 = 12, \gamma_2 = 3, C_2 = 50, C_3 = 150, C_6 = 100, CI = 5$, it is also observed that

- For $C_1 = 50$, the profit is positive, zero or negative according to $C_0 >$ or $=$ or $<$ 42.55. Hence for $C_1 = 50$, the revenue should be more than 42.55 to get the profit.

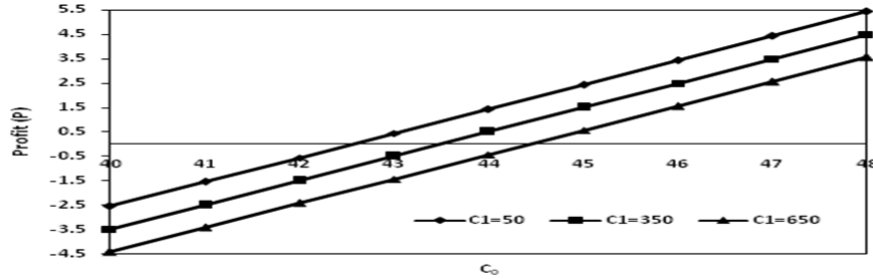


Figure 3: Profit (P) Versus Revenue (C₀) for Different Values of Cost (C₁)

- ii. For C₁ =350, the profit is positive, zero or negative according to C₀ > or = or < 43.49. Hence for C₁=350, the revenue should be more than 43.49 to get the profit.
- iii. For C₁ = 650, the profit is positive, zero or negative according to C₀ > or = or < 44.43. Hence for C₁ = 650, the revenue should be more than 44.43 to get the profit.

6.5 Effect of Cost (C₃) on Profit (P) for Different Values of Cost (C₆)

Profit (P) decreases with increase in the cost (C₃) per unit time for creating the redo log files (archiving) and has higher values for lower value of cost (C₆) per visit of the DBA. Also, when $\gamma_1 = 12, \gamma_2 = 3, C_0 = 43, C_1 = 50, C_2 = 50, CI = 5$, it is interpreted from Figure 4 that

- i. For C₆ =100, the profit is positive, zero or negative according to C₃ < or = or > 980.6. Hence for C₆ =100, the cost for creating the redo log files should not be more than 980.6 to get the profit.

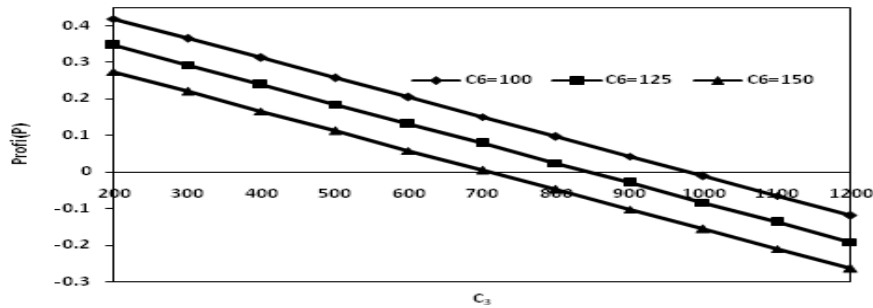


Figure 4: Profit (P) Versus Revenue (C₃) for Different Values of Cost (C₆)

- ii. For C₆ = 125, the profit is positive, zero or negative according to C₃ < or = or > 845.49. Hence for C₆=125, the cost for creating the redo log files should not be more than 845.49 to get the profit.
- iii. For C₆ =150, the profit is positive, zero or negative according to C₃ < or = or > 710.38. Hence for C₆=150, the cost for creating the redo log files should not be more than 710.38 to get the profit.

On the basis of the analysis done above, we have come to the following conclusion.

7. Conclusion

In this paper, we have successfully done the stochastic modeling of two unit hot standby database system. Expressions for various measures of system effectiveness are obtained. Cost/revenue always remains a considerable and vital factor while developing a reliability model for a particular system and hence bounds (lower/upper) for revenue and the cost per unit time for creating the redo log files have been obtained. These ensure that whether the system is functioning in profit or running in loss. Bounds for many other parameters as per the specified requirement of the users can also be obtained in the similar manner.

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