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Performability of a Stochastic-Flow Information Network Considering Maintenance

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Abstract: This paper works on the evaluation of performability for information systems by adopting the stochastic-flow network model. To construct an information system as an information network, each arc (delivery medium) has two parameters, the bandwidth and the lead time. Consider maintenance budget and time limitation, we evaluate the performability such that a given amount of data can be delivered from the source to the sink. The contributions of this paper are twofold: (i) an algorithm integrated branch-and-bound elevating method is proposed to generate all lowest bandwidth vectors satisfying demand, maintenance budget, and time limitation. The performability is computed in terms of such vectors accordingly; (ii) the proposed algorithm is applied to a real case of Taiwan Academic Network to show the applicability and efficiency.

Keywords: *Performability; Stochastic-Flow Information Network; Branch-and-Bound; Elevating Method*

1. Introduction

To guarantee the performance is a critical issue as the growing of complicated composition of systems. In other words, a system should provide a reliable status to meet demands or requirements from customers and users. This paper evaluates the probability that the network can send a given demand from source to sink under both maintenance budget and time limitation; such a probability is termed the performability. We model such an information system as a stochastic-flow network because the bandwidth of delivery media in the network is stochastic. In addition, quickest path problem (QPP) is adopted in this paper to measure the data delivery time.

1.1 Stochastic-Flow Network

Network analysis is widely applied for the performance evaluation in diverse systems, such as information [1-3], logistic [4], and manufacturing systems [5]. In information networks, arcs model the delivery media such as fiber optics or coaxial cables, while nodes represent delivery facilities including routers and switches. In particular, the bandwidth of each arc in an information network is stochastic due to (partial) malfunction or maintenance. For example, a delivery medium may consist of several physical lines and each line has only operation or malfunction state. Hence, the delivery medium performs multiple states (*i.e.*, stochastic bandwidth) according to the number of physical lines that operate successfully. An information network characterized by such arcs also performs stochastic bandwidths and it is a typical stochastic-flow network [1,6-8]. The previous works by Jane *et al.* [6], Lin *et al.* [7], and Lin [1] have been devoted to generating the lowest bandwidths of arcs such that a stochastic-flow network can satisfy a specified

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demand level. In addition, Zuo *et al.* [8] provided an efficient algorithm to derive the probability of demand satisfaction based on the lowest bandwidths of arcs. However, those previous works did not consider the delivery time.

1.2 Quickest Path Problem

In addition to the stochastic bandwidth, the delivery time is another important parameter to be taken into account for performance evaluation of an information network. For an information network, selecting a shortest delayed path is a necessary way to reduce the delivery time [9]. However, Golden and Magnanti [9] focused on the selection of a path without considering the data delivery time by the path.

In order to find a path with minimum delivery time, Chen and Chin [10] proposed a version of the shortest path problem called the quickest path problem (QPP). In the QPP, two deterministic parameters are involved in each arc--the bandwidth and the lead time [10, 11]. Thereafter, diversities of QPP, such as constrained QPP [12,13], the first-*k* QPP [14], and all-pairs QPP [12,15] are proposed. If the data is required to be delivered by a specified path, then such a restricted problem is termed the constrained QPP [13,16]. The QPP was further extended to deliver data by *k* best paths ($k > 1$), which is termed the first-*k* QPP [14]. There were also studies finding the quickest path to connect any possible source and sink in a network and such a problem is all-pairs QPP [12,15]. Several studies [2, 3,14] further devoted to reducing the delivery time from the source to the sink. Those studies suggested that the data can be delivered through several disjoint paths simultaneously to shorten the delivery time. However, the previous literatures considered only demand satisfaction and time limitation without taking the maintenance into account.

1.3 Maintenance Issue

Some studies [5, 17] have defined the maintenance as repairing a network from the malfunction state back to the original state. The malfunction state means that the network provides bandwidths less than the minimal demand. Hence, the arcs (delivery media) must be repaired to their highest bandwidths (original state) when only the minimal demand can be satisfied. A significant amount of research [1-4,8] has been devoted to studying the probability that an information network can send a given amount of data from source to sink. Such a probability is solved in terms of minimal paths (MPs), where an MP is a set of arcs connecting from source to sink without loops. To guarantee the information network performs a good quality of service (QoS), the maintenance is necessary. That is, the information network must be repaired before dropping to a malfunction state.

1.4 Focused Problem

This paper focuses on evaluate the performability that the information network can send a given demand from source to sink under both maintenance budget and time limitation. The contributions of this paper are twofold. First, an algorithm is proposed to generate all lowest bandwidth vectors (LBVs) fulfilling the demand *d*, maintenance budget B, and time limitation T. In other words, the LBVs are the minimal requirement that the stochastic-flow information network has to provide. The performability is calculated in terms of LBVs by the Recursive Sum of Disjoint Products (RSDP) algorithm afterwards. In particular, the maintenance budget cannot be formulated by the previous works [4,17], in which the previous works delete unqualified bandwidth vectors whose total cost is over the maintenance budget. Hence, a branch-and-bound elevating method is integrated in the proposed algorithm to adjust those unqualified bandwidth vectors instead of deleting them.

2. Model Formulation

Information network of $G(\mathbf{N}, \mathbf{A}, \mathbf{M}, \mathbf{C}, \mathbf{L})$ is a stochastic-flow network with a source s and a sink t where $\mathbf{N}$ is the set of nodes (servers or switches), $\mathbf{A} = \{a_i|i = 1, 2, \ldots, n\}$ represents the set of arcs, $\mathbf{M} = \{M_i|i = 1, 2, \ldots, n\}$ with M_i representing the maximal bandwidth of a_i, $\mathbf{C} = \{c_i|i = 1, 2, \ldots, n\}$ with c_i representing per unit maintenance cost of a_i, and $\mathbf{L} = \{l_i|i = 1, 2, \ldots, n\}$ with l_i denoting the lead time of a_i. The notation P_κ denotes the κth MP for $\kappa = 1, 2, \ldots, m$ where m is the numbers of MPs. The bandwidth vector $X = (x_1, x_2, \ldots, x_n)$ is defined as the system state of G where x_i represents the bandwidth of arc a_i. To measure the performability, this paper takes the demand d, maintenance budget B, and time limitation T into consideration. Assumptions for such a G are described as follows:

I. Each node will not malfunction.

II. The (partial) malfunctions of different arcs are statistically independent.

2.1. Maintenance Cost

Maintenance action is essential for the real-world information network. The calculation of maintenance cost is determined by the repaired amount of bandwidth at each arc. The total cost to repair the bandwidth of arcs from the state X to the original state is

$$F(X) = \sum_{i:a_i \in \bigcup_{\kappa=1}^{m} P_\kappa} c_i(M_i - x_i)\,. \tag{1}$$

The term $c_i(M_i - x_i)$ describes the maintenance cost to repair a_i from the bandwidth x_i to the highest bandwidth M_i with unit repair cost c_i. Only the arcs on the MPs are needed to be repaired. Take a maintenance budget B into account when repairing the information network, the following constraint is necessary,

$$F(X) \le \text{B}. \tag{2}$$

2.2. Time Limitation

For an information network, the bandwidth is the delivery rate of data sent through the medium (arc, node, or MP) per unit of time. It implies that the delivery time is calculated according to the bandwidth. Each arc a_i contains M_i identical physical lines, suggesting that $(M_i + 1)$ bandwidth states are possible, $i = 1, 2, \ldots, n$. The lowest level (0) corresponds to complete malfunction, while M_i will be the highest level of operation. Based on the number of physical lines, the bandwidth of each arc takes values from $\{0, 1, 2, \ldots, M_i\}$ according to a given probability distribution. With respect to each MP $P_\kappa = \{a_{\kappa 1}, a_{\kappa 2}, \ldots, a_{\kappa n_\kappa}\}$, $\kappa = 1, 2, \ldots, m$, the maximal bandwidth of P_κ is $\min_{1\le k\le n_\kappa}(M_{\kappa k})$. Similarly, under the bandwidth vector X, the bandwidth of P_κ is $\min_{1\le k\le n_\kappa}(x_{\kappa k})$. The delivery time to send d units of data through P_κ under the bandwidth vector X, $\Gamma(d, X, P_\kappa)$, is

$$\text{lead time of } P_\kappa + \left\lceil d/\text{the capacity of } P_\kappa \right\rceil = \sum_{k=1}^{n_\kappa} l_{jk} + \left\lceil d \big/ \min_{1\le k\le n_\kappa} x_{\kappa k} \right\rceil, \tag{3}$$

where $\lceil x \rceil$ is a ceiling function to find the smallest integer no less than x. The lead time is the processing time required to pass through a path, which is generally determined by the length of each arc [10]. For any demand pair (d_1, d_2) assigned to two MPs P_1 and P_2, the delivery time $\theta(d_1, d_2, X)$ is $\max\{\Gamma(d_1, X, P_1), \Gamma(d_2, X, P_2)\}$. The minimum delivery time to send d units of data under X is $\lambda(d, X) = \min_{\text{all}(d_1, d_2):d_1+d_2=d}\{\theta(d_1, d_2, X)\}$. If $\lambda(d, X) > \text{T}$, the time limitation is violated. The following lemma shows the relationship between bandwidth vector and delivery time.

Lemma 1: *If $X \le Y$, then $\Gamma(d, X, P_\kappa) \ge \Gamma(d, Y, P_\kappa)$.*

2.3. Lowest Bandwidth Vectors and Performability

The performability $\Psi_{d,\mathrm{B},\mathrm{T}}$ to fulfill demand d, maintenance budget B, and time limitations T is $\Pr\{X|\lambda(d, X) \le \mathrm{T}$ and $F(X) \le \mathrm{B}\}$. Any bandwidth vector X with $F(X) \le \mathrm{B}$ and $\lambda(d, X) \le \mathrm{T}$ means that X can send d units of data from s to t under B and T. Thus, Ω is the set of such X. However, it would be computationally inefficient to find all X such that $\lambda(d, X) \le \mathrm{T}$ and $F(X) \le \mathrm{B}$ and then sum their probabilities to obtain $\Psi_{d,\mathrm{B},\mathrm{T}}$. We define a set of the minimal X fulfilling d, B, and T, say $\Omega_{\min}$ and $X \in \Omega_{\min}$ is termed an LBV. The LBVs constitute a more effective approach to compute $\Psi_{d,\mathrm{B},\mathrm{T}}$.

Definition: *X is an LBV if and only if (i) $\lambda(d, X) \le T$, (ii) $F(X) \le B$, and (iii) $\lambda(d, Y) > T$ or $F(Y) > B$ for any bandwidth vector Y with $Y < X$.*

The following lemma shows the property for LBVs.

Lemma 2: *If X is an LBV, then $Y \in \Omega$ for any $Y \ge X$.*

3. The Branch-and-Bound Elevating Method

Suppose m MPs are given [20], say $P_1, P_2, \ldots,$ and P_m, all LBVs are generated as follows.

Step 1. Determine the maximal assigned demand $\overline{d_\kappa}$ such that $\sum_{i:a_i \in P} l_i + \left\lceil \overline{d_\kappa} / \min_{i:a_i \in P_\kappa} M_i \right\rceil \le \mathrm{T}$, $\kappa = 1, 2, \ldots, m$.

Step 2. [Search for all feasible demand assignments] Generate all non-negative integer solutions of $\sum_{\kappa=1}^{m} d_\kappa = d$ where $d_\kappa \le \overline{d_\kappa}$, $\kappa = 1, 2, \ldots, m$.

Step 3. [Generation of LBVs satisfying d and T] For each demand assignment $(d_1, d_2, \ldots, d_m)$, do the following steps.

3.1 Find the lowest bandwidth v_κ of P_κ such that d_κ units of data can be sent through P_κ under T. That is, find the smallest integer v_κ such that

$$\sum_{i:a_i \in P_\kappa} l_i + \lceil d_\kappa / v_\kappa \rceil \le \mathrm{T},\ \kappa = 1, 2, \ldots, m. \tag{4}$$

3.2 $j = j + 1$. $X_j = (x_1, x_2, \ldots, x_n)$ is obtained according to

$$x_i = \begin{cases} \text{minimal capacity } u \text{ of } a_i \text{ such that } u \ge v_\kappa \\ \qquad\qquad \text{if } a_i \in P_\kappa \text{ for a } \kappa \in \{1, 2, \ldots, m\}, \\ 0 \qquad\qquad\qquad \text{otherwise.} \end{cases} \tag{5}$$

3.3 For $k = 1$ to $j - 1$, if $X_j \ge X_k$, go to Step 3.5; if $X_j < X_k$, $\Omega_\mathrm{T} = \Omega_\mathrm{T} \setminus X_k$.

3.4 $\Omega_\mathrm{T} = \Omega_\mathrm{T} \cup \{X_j\}$.

3.5 Next $(d_1, d_2, \ldots, d_m)$.

Step 4. [Generation of LBVs] For each $X_j \in \Omega_\mathrm{T}$, do the following steps.

4.1 Calculate the maintenance cost $F(X_j)$.

4.2 If $F(X_j) \le \mathrm{B}$, $\Omega_{\min} = \Omega_{\min} \cup \{X_j\}$ and $\Omega_\mathrm{T} = \Omega_\mathrm{T} \setminus X_j$. Then, go to Step 4.4.

4.3 For the X_j such that $F(X_j) > \mathrm{B}$, do the following steps:

4.3.1 $\Omega_\mathrm{T} = \Omega_\mathrm{T} \setminus X_j$.

4.3.2 For each $i : a_i \in \bigcup_{\kappa=1}^{m} P_\kappa$, let $X_{j,i} = X_j + e_i$. If the bandwidth of a_i in $X_{j,i}$ is $M_i + 1$, then remove $X_{j,i}$. //e_i is the standard basis vector for elevating the unqualified X_j such that $x_i = 1$ for $i : a_i \in \bigcup_{\kappa=1}^{m} P_\kappa$ and 0 for others.

4.3.3 Compare each $X_{j,i}$ with $X \in \Omega_{\min}$. If $X_{j,i} \ge X$ in $\Omega_{\min}$, delete $X_{j,i}$; otherwise, $X_{j,i} \in \Omega_\mathrm{T}$.

If $X_{j,i}$ is less than any X in Ω_{min}, delete that X from Ω_{min}.

4.3.4 Treat each $X_{j,i} \in \Omega_T$ as the role of X_j and go to Step 4.1.

4.4 Next $X_j \in \Omega_T$.

Step 5. All $X_j \in \Omega_{min}$ are the LBVs fulfilling d, B, and T.

4. Illustrative Examples

4.1. Example 1

A bridge network is shown in Figure 1 to illustrate the solution procedure. The bandwidth, lead time, and per unit maintenance cost of each arc are shown in Table 1.

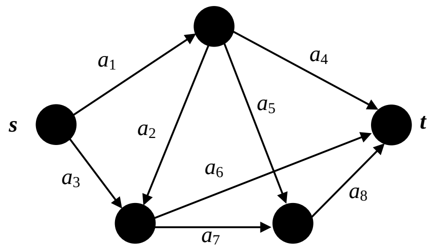

Figure 1: A bridge network [15, 18, 22]

Table 1: The Arc Data of Figure 1

Arc	Cost	Lead time (sec)	Bandwidth (Gbps) 0	1	2	3	4	5
a_1	250	2	0.000659	0.020731	0.217562	0.761048	0.000000	0.000000
a_2	150	1	0.000405	0.015212	0.190360	0.794023	0.000000	0.000000
a_3	250	3	0.000405	0.015212	0.190360	0.794023	0.000000	0.000000
a_4	400	3	0.087000	0.913000	0.000000	0.000000	0.000000	0.000000
a_5	200	1	0.007569	0.158862	0.833569	0.000000	0.000000	0.000000
a_6	150	2	0.000057	0.002405	0.037856	0.264845	0.694837	0.000000
a_7	400	2	0.000002	0.000139	0.003475	0.043481	0.272048	0.680855
a_8	300	1	0.000030	0.001501	0.028173	0.235031	0.735265	0.000000

The source has to send 6 giga bits of data to the sink through $P_1 = \{a_1, a_4\}$ and $P_2 = \{a_3, a_7, a_8\}$ within 9 seconds and under maintenance budget 3200. That is, the network should be repaired from state 6. The performability $\Psi_{6,3200,9}$ is derived as follows.

Step 0. $\Omega_{min} = \emptyset$, $\Omega_T = \emptyset$, $j = 0$.

Step 1. The maximal demand $\overline{d_1}$ such that $(l_1 + l_4)+\lceil \overline{d_1}/\min\{M_1, M_4\}\rceil \le 9$ is $\overline{d_1} = 4$.

Similarly, $\overline{d_2} = 9$.

Step 2. Generate all non-negative integer solutions of $d_1 + d_2 = d$ where $d_1 \le \overline{d_1}$ and $d_2 \le \overline{d_2}$. The feasible (d_1, d_2) are (4, 2), (3, 3), (2, 4), (1, 5), and (0, 6).

Step 3. For $(d_1, d_2) = (4, 2)$, do the following steps.

3.1 For P_1, $l_1 + l_4 = 5$; $v_1 = 1$ is the smallest integer such that $(5+\lceil 4/v_1\rceil) \le 9$. Similarly, $v_2 = 1$ is the smallest integer such that $(6+\lceil 2/v_2\rceil) \le 9$.

3.2 $X_1 = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (1, 0, 1, 1, 0, 0, 1, 1)$.

3.4 $\Omega_T = \Omega_T \cup \{X_1\} = \{X_1\}$.

3.5 Next (d_1, d_2).

⋮

3.4 $\Omega_T = \Omega_T \cup \{X_5\} = \{X_1, X_5\}$.
The results are shown in Table 2.

Table 2: Results of Step 3 in Example 1

(d_1,d_2)	(v_1,v_2)	X	$X_j \in \Omega_T$ or not	Remark
(4,2)	(1,1)	X_1 = (1,0,1,1,0,0,1,1)	Yes	-
(3,3)	(1,1)	X_2 = (1,0,1,1,0,0,1,1)	No	$X_2 \geq X_1$
(2,4)	(1,2)	X_3 = (1,0,2,1,0,0,2,2)	No	$X_3 \geq X_5$
(1,5)	(1,2)	X_4 = (1,0,2,1,0,0,2,2)	No	$X_4 \geq X_5$
(0,6)	(0,2)	X_5 = (0,0,2,0,0,0,2,2)	Yes	-

Step 4. For each $X_j \in \Omega_T$, do the following steps.

4.1 For X_1, $F(X_1) = \sum_{i=1,3,4,7,8} c_i(M_i - x_i)$ = 3500.

4.2 $F(X_1)$ = 3500 > B = 3200, so X_1 is needed to be elevated.

4.3 Elevate X_1 by the following step.

4.3.1 $\Omega_T = \Omega_T \setminus X_1 = \{X_5\}$.

4.3.2 For i = 1, 3, 4, 7, 8, let $X_{1,i} = X_1 + e_i$. We get all $X_{1,i}$ as follows:
$X_{1,1} = X_1 + e_1$ = (1 + 1, 0, 1, 1, 0, 0, 1, 1) = (2, 0, 1, 1, 0, 0, 1, 1);
$X_{1,3} = X_1 + e_3$ = (1, 0, 1 + 1, 1, 0, 0, 1, 1) = (1, 0, 2, 1, 0, 0, 1, 1);
$X_{1,4} = X_1 + e_4$ = (1, 0, 1, 1 + 1, 0, 0, 1, 1) = (1, 0, 1, 2, 0, 0, 1, 1);
$X_{1,7} = X_1 + e_7$ = (1, 0, 1, 1, 0, 0, 1 + 1, 1) = (1, 0, 1, 1, 0, 0, 2, 1);
$X_{1,8} = X_1 + e_8$ = (1, 0, 1, 1, 0, 0, 1, 1 + 1) = (1, 0, 1, 1, 0, 0, 1, 2).
The bandwidth x_4 in $X_{1,4} > M_4 = 1$, so $X_{1,4}$ should be removed.

4.3.3 Since $\Omega_{min} = \varnothing$, no $X_{1,i}$ is deleted in this step. So, $\Omega_T = \{X_{1,1}, X_{1,3}, X_{1,7}, X_{1,8}, X_5\}$.

4.3.4 Treat $X_{1,1}$, $X_{1,3}$, $X_{1,7}$, and $X_{1,8}$ as X_j and go to Step 4.1.

4.1a For $X_{1,1}$, $F(X_{1,1})$ = 3250.

4.2a $F(X_{1,1})$ = 3250 > B = 3200, so $X_{1,1}$ is needed to be elevated.

4.3a Elevate $X_{1,1}$ by the following step.

4.3.1a $\Omega_T = \Omega_T \setminus X_{1,1} = \{X_{1,3}, X_{1,7}, X_{1,8}, X_5\}$.

4.3.2a For i = 1, 3, 4, 7, 8, let $X_{1,1,i} = X_{1,1} + e_i$. We get all $X_{1,1,i}$ as follows: $X_{1,1,1}$ = (3, 0, 1, 1, 0, 0, 1, 1), $X_{1,1,3}$ = (2, 0, 2, 1, 0, 0, 1, 1), $X_{1,1,4}$ = (2, 0, 1, 2, 0, 0, 1, 1), $X_{1,1,7}$ = (2, 0, 1, 1, 0, 0, 2, 1), and $X_{1,1,8}$ = (2, 0, 1, 1, 0, 0, 1, 2). The bandwidth x_4 in $X_{1,1,4} > M_4 = 1$, so $X_{1,1,4}$ should be removed.

4.3.3a Since $\Omega_{min} = \varnothing$, no $X_{1,1,i}$ is deleted in this step. So, $\Omega_T = \{X_{1,1,1}, X_{1,1,3}, X_{1,1,7}, X_{1,1,8}, X_{1,3}, X_{1,7}, X_{1,8}, X_5\}$.

4.3.4a Treat $X_{1,1,1}$, $X_{1,1,3}$, $X_{1,1,7}$, and $X_{1,1,8}$ as X_j and go to Sep 4.1.

4.1b For $X_{1,1,1}$, $F(X_{1,1,1})$ = 3000.

4.2b $F(X_{1,1,1})$ = 3000 < B = 3200, $\Omega_{min} = \Omega_{min} \cup \{X_{1,1,1}\} = \{X_{1,1,1}\}$ and $\Omega_T = \Omega_T \setminus X_{1,1,1} = \{X_{1,1,3}, X_{1,1,7}, X_{1,1,8}, X_{1,3}, X_{1,7}, X_{1,8}, X_5\}$. Then, go to Step 4.4.

4.4b Next $X_j \in \Omega_T$.

⋮

The results are concluded in Table 3.

Table 3: Results of Step 4 in Example 1

X	$X_{j,i} \in \Omega_{min}$ or not	Total Cost	Remark
$X_{1,1,1}$ = (3,0,1,1,0,0,1,1)	Yes	3000	-
$X_{1,1,3}$ = (2,0,2,1,0,0,1,1)	Yes	3000	-
$X_{1,1,4}$ = (2,0,1,2,0,0,1,1)	No	-	$x_4 > M_4$
$X_{1,1,7}$ = (2,0,1,1,0,0,2,1)	No	-	$X_{1,1,7} \geq X_{1,7}$
$X_{1,1,8}$ = (2,0,1,1,0,0,1,2)	No	-	$X_{1,1,8} \geq X_{1,8}$
$X_{1,3,1}$ = (2,0,2,1,0,0,1,1)	No	-	$X_{1,3,1} \geq X_{1,1,3}$
$X_{1,3,3}$ = (1,0,3,1,0,0,1,1)	Yes	3000	-
$X_{1,3,4}$ = (1,0,2,2,0,0,1,1)	No	-	$x_4 > M_4$
$X_{1,3,7}$ = (1,0,2,1,0,0,2,1)	No	-	$X_{1,3,7} \geq X_{1,7}$
$X_{1,3,8}$ = (1,0,2,1,0,0,1,2)	No	-	$X_{1,3,8} \geq X_{1,8}$
$X_{1,4}$ = (1,0,1,2,0,0,1,1)	No	-	$x_4 > M_4$
$X_{1,7}$ = (1,0,1,1,0,0,2,1)	Yes	3100	-
$X_{1,8}$ = (1,0,1,1,0,0,1,2)	Yes	3200	-
X_5 = (0,0,2,0,0,0,2,2)	Yes	3200	-

Step 5. $\Omega_{min} = \{X_{1,1,1}, X_{1,1,3}, X_{1,3,3}, X_{1,7}, X_{1,8}, X_5\}$.

Six (6,3200,9)-MPs are then generated by Algorithm I. Let $Q_{1,1,1} = \{X|X \geq X_{1,1,1}\}$, $Q_{1,1,3} = \{X|X \geq X_{1,1,3}\}$, $Q_{1,3,3} = \{X|X \geq X_{1,3,3}\}$, $Q_{1,7} = \{X|X \geq X_{1,7}\}$, $Q_{1,8} = \{X|X \geq X_{1,8}\}$, and $Q_5 = \{X|X \geq X_5\}$, the performability $\Psi_{6,3200,9} = \Pr\{Q_{1,1,1} \cup Q_{1,1,3} \cup Q_{1,3,3} \cup Q_{1,7} \cup Q_{1,8} \cup Q_5\} = 0.99808897$ can be derived by applying the RSDP algorithm.

4.2. Example 2

We employ the TANET with 32 arcs shown in Figure 2 [21] to demonstrate the utility of the approach for assessing the real-world information network. The bandwidth, lead time, and maintenance cost of each arc are provided in Table 4. For the case where the TANET has to preserve a minimal level of QoS that delivers at least 128 giga bits of data from NTU (source) to NSYSU (sink) within 20 seconds through four disjoint MP, say $P_1 = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}\}$, $P_2 = \{a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}\}$, $P_3 = \{a_{21}, a_{22}, a_{23}\}$, and $P_4 = \{a_{24}, a_{25}, a_{26}, a_{27}, a_{28}, a_{29}\}$. Given a maintenance budget of \$20,000, two-hundred (128,20000,20)-MPs are generated by the algorithm and the $\Psi_{128,20000,20} = 0.99302603$.

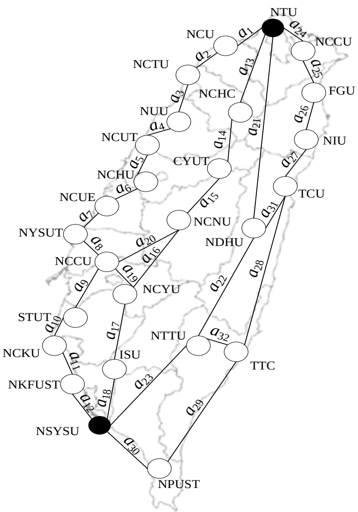

Figure 2: Taiwan Academic Network (TANET) [21]

Table 4: The Arc Data TANET

Arc	Cost	Lead time (sec)	Bandwidth (Gbps)						
			0	1	2	3	4	5	6
a_1	220	1	0.000000	0.000002	0.000085	0.002143	0.030544	0.232134	0.735092
a_2	180	1	0.000000	0.000002	0.000085	0.002143	0.030544	0.232134	0.735092
a_3	250	1	0.000000	0.000030	0.001128	0.021434	0.203627	0.773781	0.000000
a_4	200	1	0.000000	0.000030	0.001128	0.021434	0.203627	0.773781	0.000000
a_5	240	1	0.000000	0.000002	0.000085	0.002143	0.030544	0.232134	0.735092
a_6	200	1	0.000000	0.000002	0.000085	0.002143	0.030544	0.232134	0.735092
a_7	260	1	0.000000	0.000030	0.001128	0.021434	0.203627	0.773781	0.000000
a_8	360	2	0.000000	0.000030	0.001128	0.021434	0.203627	0.773781	0.000000
a_9	280	1	0.000000	0.000030	0.001128	0.021434	0.203627	0.773781	0.000000
a_{10}	220	1	0.000000	0.000030	0.001128	0.021434	0.203627	0.773781	0.000000
a_{11}	250	1	0.000000	0.000030	0.001128	0.021434	0.203627	0.773781	0.000000
a_{12}	220	1	0.000000	0.000002	0.000085	0.002143	0.030544	0.232134	0.735092
a_{13}	280	1	0.000000	0.000002	0.000085	0.002143	0.030544	0.232134	0.735092
a_{14}	220	1	0.000000	0.000002	0.000085	0.002143	0.030544	0.232134	0.735092
a_{15}	240	2	0.000006	0.000475	0.013538	0.171475	0.814506	0.000000	0.000000
a_{16}	270	2	0.000006	0.000475	0.013538	0.171475	0.814506	0.000000	0.000000
a_{17}	320	1	0.000000	0.000030	0.001128	0.021434	0.203627	0.773781	0.000000
a_{18}	220	3	0.000000	0.000030	0.001128	0.021434	0.203627	0.773781	0.000000
a_{19}	380	1	0.000000	0.000002	0.000085	0.002143	0.030544	0.232134	0.735092
a_{20}	240	3	0.000000	0.000002	0.000085	0.002143	0.030544	0.232134	0.735092
a_{21}	300	2	0.000006	0.000475	0.013538	0.171475	0.814506	0.000000	0.000000
a_{22}	200	3	0.000000	0.000002	0.000085	0.002143	0.030544	0.232134	0.735092
a_{23}	350	1	0.000000	0.000030	0.001128	0.021434	0.203627	0.773781	0.000000
a_{24}	270	2	0.000000	0.000002	0.000085	0.002143	0.030544	0.232134	0.735092
a_{25}	250	1	0.000000	0.000030	0.001128	0.021434	0.203627	0.773781	0.000000
a_{26}	340	2	0.000000	0.000030	0.001128	0.021434	0.203627	0.773781	0.000000
a_{27}	210	3	0.000000	0.000030	0.001128	0.021434	0.203627	0.773781	0.000000
a_{28}	330	1	0.000000	0.000002	0.000085	0.002143	0.030544	0.232134	0.735092
a_{29}	260	1	0.000000	0.000002	0.000085	0.002143	0.030544	0.232134	0.735092
a_{30}	420	3	0.000000	0.000002	0.000085	0.002143	0.030544	0.232134	0.735092
a_{31}	420	2	0.000000	0.000030	0.001128	0.021434	0.203627	0.773781	0.000000
a_{32}	380	1	0.000000	0.000030	0.001128	0.021434	0.203627	0.773781	0.000000

5. Conclusions

From the perspective of performability evaluation, maintenance should be taken before the information network drops to the malfunction state. Thus, we treat the performability as a performance index to evaluate the probability that an information network can deliver required demand from source to sink through several disjoint MPs under maintenance budget and time limitations. The system supervisor can conduct a sensitivity analysis to investigate the most important arc (delivery medium) in an information network to improve the performability. For instance, the sensitivity analysis could be conducted by increasing the bandwidth of a delivery medium at a time (the others are retained as the same conditions). Once the bandwidth of a delivery medium is increased, the performability is also improved. Eventually, the system supervisor can find the most sensitive delivery medium that increases the performability most, and such a delivery medium is the most important part of the information network.

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