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Control Charts with Runs Rules for Poisson Process Data

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Abstract: We use Markov chains to compare run lengths of Poisson process individuals control charts with and without runs rules. Evidence quantifies the advantage of runs rules for certain cost structures $z = C_\alpha / C_\beta$, where C_α is the cost of a Type I error, and C_β is the cost of a Type II error, and different shifts from in-control parameter λ_1 to out-of-control parameter λ_2.

Keywords: *counting process, economic design, Markov chain, quality control*

1. Introduction

Quality engineers have considered runs rules since Western Electric increased a Shewhart chart's sensitivity to drifts in the mean. Champ and Woodall used Markov chains to compare run lengths of Shewhart charts with and without supplementary runs rules, and the cumulative sum chart [1]. We extended that concept to optimized limits for theoretically appropriate charts assuming individual observations of a Poisson process. Others have concentrated on run lengths for count data [2], and runs rules in general [3]. More recent literature addresses categorical data and performability engineering in general [4-7].

2. Methods for an Upward Shift

We compared Poisson process individuals control charts with those employing runs rules. That is a traditional limit CL_2 compared with an additional limit CL_1, where two-out-of-three consecutive observations greater than CL_1 indicate an out-of-control signal.

Rules are generally defined by the possible states.

- AA: Two consecutive observations between 0 and CL_1
- BA: An observation between 0 and CL_1, followed by one between CL_1 and CL_2
- AB: An observation between CL_1 and CL_2, followed by one between 0 and CL_1
- OOC: The absorbing state, where two-out-of-three consecutive observations are between CL_1 and CL_2, or any one observation is greater than CL_2

Table 1 is a Markov transition matrix Q. In each row are probabilities p of moving from existing state x to future state x', where p_A and p_B are cumulative Poisson probabilities of observations between 0 and CL_1, and CL_1 and CL_2, respectively.

The first passage of time for an in-control process was the following, based on the in-control Poisson parameter λ_1.

$$ARL_0 = N_{14} = (-1 - p_B - p_A p_B) / (-1 + p_A + p_A^2 p_B)$$

The expected value of the first passage time for an out-of-control process was calculated by solving the following function group.

$$N_{14} = 1 + p_A N_{14} + p_B N_{24}$$

$$N_{24} = 1 + p_A N_{34}$$

$$N_{34} = 1 + p_A N_{14}$$

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The corresponding first passage of time for the shifted process was ARL1 = ARL0, where the Poisson parameter was changed to λ_2 (instead of λ_1).

Table 1: Markov Transition Matrix.

	x_{AA}'	x_{BA}'	x_{AB}'	x_{OOC}'
x_{AA}	p_A	p_B		$1 - p_A - p_B$
x_{BA}			p_A	$1 - p_A$
x_{AB}	p_A			$1 - p_A$
x_{OOC}				1

The objective was to minimize a linear combination of average run lengths according to the cost ratio z = C_α / C_β, where C_α is the cost of a Type I error, and C_β is the cost of a Type II error. In other words if z = 1, we have equal cost errors

$$\min z / ARL_0 + 1 - 1 / ARL_1$$

ARL_0 is the average run length to a Type I error (measured in number of observations), and ARL_1 is the average run length to detect a true change. To minimize we enumerated reasonably large spaces with respect to control limits and estimated the objective function value by Markov processes.

3. Example

We demonstrated the method for upward shifts through a simple example shown in Table 2. First we arbitrarily set the cost ratio z = 5 and evaluated select combinations of shifts from λ_1 to λ_2. Columns are the following.

- λ_1 is the in-control Poisson parameter
- λ_2 is the out-of-control Poisson parameter
- CL is the optimized control limit without runs rules
- CL_1 is the additional optimized control limit employing runs rules
- CL_2 is the greater optimized control limit employing runs rules
- G is the ratio of obj function values greater than one if runs rules are preferred

Table 2: Cost Comparisons between Traditional Chart and Runs Rules.

λ_1	λ_2	CL	CL_1	CL_2	G
0.5	0.7	5	2	5	1.000
0.5	1.0	3	1	3	1.010
0.5	2.0	2	1	2	1.037
1.0	1.2	9	5	9	1.000
1.0	2.0	3	2	4	1.02

In Table 3 we include objective function ratios G for multiple cost structures z, and the same shifts from λ_1 to λ_2, to show how relative costs of errors Type I and II may affect the usefulness of runs rules.

Table 3: The Effect of z on Runs Rules Performance (G).

λ_1	λ_2	z = 1	z = 2	z = 3	z = 5
0.5	0.7	1	1.004	1.001	1.000
0.5	1.0	1	1	1	1.010
0.5	2.0	1	1	1.004	1.037
1.0	1.2	1	1.001	1	1.000
1.0	2.0	1	1	1.025	1.02

4. Conclusions

We demonstrated a Markov process in order to achieve optimal control limits and consider the cost savings associated with runs rules. Evidence quantifies the increasing advantage (as z increases from 1 to 5) of runs rules for certain cost structures and different shifts, but does not consider what is lost in efficiency. Future work would do this for many combinations of cost structure and shift. Beyond that one may hope to automate the process for deciding among charts by fitting a good (meta) statistical model of results. If simple runs rules are thought to be advantageous, future work should include research into more complicated ones.

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