International Journal of Performability Engineering, Vol. 10, No. 6, September 2014, pp. 605-614.

Printed in India

Dependent Systems Reliability Estimation by Structural Reliability Approach

ERIK KOSTANDYAN[1] and JOHN SØRENSEN[2]

[1]*Post.Doc., Department of Civil Engineering, Aalborg University, Denmark*
[2]*Professor, Department of Civil Engineering, Aalborg University, Denmark*

(Received on December 30, 2013, revised on March 17, 2014, and April 19, 2014)

Abstract: Estimation of system reliability by classical system reliability methods generally assumes that the components are statistically independent, thus limiting its applicability in many practical situations. A method is proposed for estimation of the system reliability with dependent components, where the leading failure mechanism(s) is described by physics of failure model(s). The proposed method is based on structural reliability techniques and accounts for both statistical and failure effect correlations. It is assumed that failure of any component is due to increasing damage (fatigue phenomena) and the component lifetimes follow some continuous and non-negative cumulative distribution functions. An illustrative example utilizing the proposed method is provided, where damage is modeled by a fracture mechanics approach with correlated components and a failure assessment diagram is applied for failure identification. Application of the proposed method can be found in many real world systems.

Keywords*: Dependent/coherent systems reliability; load sharing systems reliability; sequential order statistics; structural systems reliability*

1. Introduction

Correlations between components have a significant influence on system reliability assessment. The classical reliability approaches often assume that components/subsystems are statistically independent and upon failure no influence will be exerted on the surviving components/subsystems. However, in many real world systems these assumptions are not justified; *e.g.*, suspension or cable-stayed bridges, where failure of any cables will increase load on the remaining cables and increase the risk of failure, wire-bonding lift off failure modes on chips or processors, server systems in computer networks, bolted or welded joints subsystems in wind turbines, mooring cable subsystems for floating wind turbines, *etc*. As it is seen, many real world systems require a more detailed approach for the dependent system reliability modeling.

Correlation between components/subsystems can be taken into account through both statistical correlation and physical failure-effect dependency. Many attempts have been made to quantify system lifetime due to physical failure-effect dependency via sequential order statistics models [1-4]. These models assume that the parent distributions at each level are available in priori. In reality, accurate estimation of the parent and/or minimum lifetime distributions at each level can be a challenging task. Two possible experimental designs for failure-effect dependent systems reliability estimation could be used:

- If stress/load level modifications upon any component failure in dependent systems were known in advance, then experiments could be designed to run a group of components at each stress level in order to estimate parent distributions for that level. Based on the estimated parent distributions at each level, minimum lifetime

*Corresponding author’s email: erik.kostandyan@gmail.com

distributions at each level need to be estimated (by considering the uncertainties in the estimated parent distributions),

- Another experimental test could be designed to test dependent systems in its full configuration, in order to estimate minimum lifetime distributions at each level directly. For estimation accuracy, more than one experiment should be run and several measurements at each level would be collected.

Upon availability of sufficient results in either test set up, estimation could be carried out based on non-parametric/parametric survival analysis or based on semi-parametric hazard analysis approaches. It should be mentioned that after justification of hazard proportionality between each level, semi-parametric regression analyses with frailty variable introduction could be used, *e.g.,* multiplicative Cox Proportional Hazard Model (PHM) [5] and/or Additive Hazard Model (AHM) [6]. Limitations on PHM and AHM could arise based on 1) assumption on frailty variable distribution, 2) situations where time-varying covariates for on-field reliability estimation are not in the range of the observed time-varying covariates, 3) hazard rates at each stress level not only depend on time-varying covariates instantaneously but also on their past values, and/or 4) leading failure modes and/or associated mechanisms change by stress level changes, which could violate associated hazard ratios (proportionality or linearity) assumptions. If monotonicity of hazard ratios were justified then a Generalized Linear PHM [7] could be applied as well. It is noted that irrespective of the selected estimation procedure, uncertainties/variations in estimated lifetime distributions heavily depend on the collected test results (sample size) and associated model assumptions.

Thus, a more detailed understanding and practical modeling of the minimum lifetimes at each stress level, which could be statistically correlated with the components damage/degradation states, are essential for failure-effect dependent systems. The aim of this paper is to develop a reliability estimation procedure for the failure-effect and statistically correlated dependent systems, where the leading failure mechanism(s) can be described via physics of failure model(s). Physical failure effect correlation is considered via sequential order failures and combined by structural reliability approaches for estimation of the next failure time. It is assumed that components fail due to fatigue, governed by the accumulated damage during the usage time. Crack propagation failure mechanism is described based on a fracture mechanics model, where statistical correlation within the components is taken into account.

Since notations between several references might be different and mislead the reader, the paper starts by providing short summaries on sequential order statistics and structural reliability theories. The proposed method is then developed and illustrated by an example.

2. Ordinary and Sequential Ordered Random Variables

Ordered random variables are a special class of random variables in statistical and probability theory. They have applications in description of engineering systems and reliability estimation of the so-called ‘*r^th^ out n*’ systems, non-parametric distribution estimations, *etc*. The derivation of distribution functions of ordered random variables assumes identical distributions and independence within the sample (i.i.d.). These assumptions lead to the class of ordinary order statistics [8-9]. The cumulative distribution function (c.d.f.) and probability density function (p.d.f.) of $X_{r:n}$ order random variables from the $X_i = \{X_1, X_2, ..., X_n\}$ random sample, where the X_{i}'s are i.i.d. from the parent distribution $F_X(t)$ c.d.f. and $f_X(t)$ p.d.f., are given by:

$$O_{X_{r:n}}(t) = P[X_{r:n} \le t] = I_{F_X(t)}(r, n-r+1) \tag{1}$$

$$o_{X_{r:n}}(t) = \frac{n!}{(r-1)!(n-r)!} F_X(t)^{r-1}(1-F_X(t))^{n-r} f_X(t) \quad (2)$$

where $I_z(a,b)$ is the regularized incomplete beta function.

Given the information on preceding failure times $X_{i:n} = \{t_i; i = 1\cdots(r-1)\}$, the conditional distribution of $X_{r:n}$ only depends on $X_{r-1:n} = t_{r-1}$. This means that ordinary order statistics form a Markov chain with transition probabilities:

$$P[X_{r:n} > t_r \mid X_{r-1:n} = t_{r-1}] = \left(\frac{1-F_X(t_r)}{1-F_X(t_{r-1})} \right)^{n-r+1} \quad (3)$$

If a failure in the sample influences the remaining component lifetimes, then the concept of sequential order statistics introduced in [1-2] can be used. Consider a random sample of size '*n*' by $X_i^{(1)} = \{X_1^{(1)}, X_2^{(1)}, ..., X_n^{(1)}\}$, where all $X_i^{(1)}$ are i.i.d and continuous random variables with $f_{X^{(1)}}(t)$ p.d.f and $F_{X^{(1)}}(t)$ c.d.f., respectively. After failure of the minimum $X_{1:n}^{(1)} = \min\{X_i^{(1)}; i = 1,\ldots,n\}$ at the first level, new i.i.d. sample of $X_i^{(2)} \mid s_1 = \{X_1^{(2)} \mid s_1, ..., X_{n-1}^{(2)} \mid s_1\}$ with size '*n-1*' from the remaining components lifetimes are composed at the second level, where $s_1 = X_{1:n}^{(1)}$. Assuming, at the second level parent distribution $F_{X^{(2)}}(t)$ of unconditional $X_i^{(2)}$ is known, then a distribution of $X_i^{(2)} \mid s_1$ random variables $F_{X^{(2)}|X_{1:n}^{(1)}}(t \mid s_1)$ is specified by a truncated distribution of $F_{X^{(2)}}(t)$ at $X_{1:n}^{(1)} = s_1$. Thus, influence of the first failure time on the remaining lifetimes is considered. The second failure will correspond to $X_{1:n-1}^{(2)} \mid s_1 = \min\{X_i^{(2)} \mid s_1; i = 1,\ldots,n-1\}$. After the second failure $s_2 = X_{1:n-1}^{(2)} \mid s_1$ is observed, new i.i.d. sample of $X_i^{(3)} \mid s_2 = \{X_1^{(3)} \mid s_2, ..., X_{n-2}^{(3)} \mid s_2\}$ with size '*n-2*' from the remaining components lifetimes are composed at the third level. Random variables $X_i^{(3)} \mid s_2$ are conditional only on s_2 (portraying the Markov property) as far as information from the first failure $s_1 = X_{1:n}^{(1)}$ is accounted based on the fact that $s_2 > s_1$.

The sequential failures are continued until the last component $X_{1:1}^{(n)} \mid s_{n-1}$ fails at level '*n*', which is the lifetime of the initial '*n*' component failure depended system. Let us denote by $X_*^{(i)} = X_{1:n-i+1}^{(i)} \mid X_{1:n-i+2}^{(i-1)}$ the revealed ordered sequence, where $i = 1,\ldots,n;\ X_{1:n+1}^{(0)} = s_0 = -\infty$. Then, the random variables $\left\{X_*^{(1)}, X_*^{(2)}, ..., X_*^{(i)}, ..., X_*^{(n-1)}, X_*^{(n)}\right\}$ are denoted as sequential order statistics based on $\left(F_{X^{(1)}}(t), F_{X^{(2)}}(t), ..., F_{X^{(i+1)}}(t), ..., F_{X^{(n-1)}}(t), F_{X^{(n)}}(t)\right)$ parent distributions.

A time truncated c.d.f. and p.d.f. in sequential order representation will be given by:

$$F_{X^{(i)}|X_{1:n-i+2}^{(i-1)}}(t \mid s_{i-1}) = \frac{F_{X^{(i)}}(t) - F_{X^{(i)}}(s_{i-1})}{1 - F_{X^{(i)}}(s_{i-1})} \quad (4)$$

$$f_{X^{(i)}|X_{1:n-i+2}^{(i-1)}}(t \mid s_{i-1}) = \frac{f_{X^{(i)}}(t)}{R_{X^{(i)}}(s_{i-1})} \quad (5)$$

where, $i = 1,\ldots,n;\ X_{1:n+1}^{(0)} = s_0 = -\infty;\ F_{X^{(1)}}(s_0) = 0;\ s_{i-1} < t;\ R(\cdot) = 1 - F(\cdot)$.

Thus, the c.d.f. and p.d.f. of the corresponding sequential order statistics $X_*^{(i)}$ will be given by:

$$F_{X_{1:n-i+1}^{(i)}|X_{1:n-i+2}^{(i-1)}}(t\mid s_{i-1}) = 1-\left(\frac{1-F_{X^{(i)}}(t)}{1-F_{X^{(i)}}(s_{i-1})}\right)^{n-i+1} \tag{6}$$

$$f_{X_{1:n-i+1}^{(i)}|X_{1:n-i+2}^{(i-1)}}(t\mid s_{i-1}) = (n-i+1)\left(\frac{R_{X^{(i)}}(t)}{R_{X^{(i)}}(s_{i-1})}\right)^{n-i}\frac{f_{X^{(i)}}(t)}{R_{X^{(i)}}(s_{i-1})} \tag{7}$$

where, $i=1,\dots,n;\ X_{1:n+1}^{(0)} = s_0 = -\infty;\ F_{X^{(1)}}(s_0)=0;\ s_{i-1}<t; R(\cdot)=1-F(\cdot)$

3. Series Systems Reliability Estimation by Structural Reliability Approach

A series system with 'n' components is a system, which fails if at least one of the components fails. Its time-dependent unreliability model will be given by:

$$F_s(t) = P\left[\bigcup_{i=1}^{n} E_i(t)\right] \tag{8}$$

where $F_s(t)$ is the series system unreliability by the time 't' (from zero until time 't'), $E_i(t)$ is the event that i^{th} component operates unsuccessfully by the time 't'.

If the lifetime of the i^{th} component Y_i follows some c.d.f. defined by $F_{Y_i}(t) = P(Y_i \le t) = P\left[E_i(t)\right]$, and two or more components comprise the series system, then the series system reliability by the time 't' will be given based on the components survival joint distribution function, defined as:

$$\overline{F_{Y_1,\cdots,Y_n}(t,\dots,t)} = P\left[\bigcap_{i=1}^{n} Y_i > t\right] \tag{9}$$

Unreliability of a series system consisting of 'n' such components by the time 't' will be given by:

$$F_s(t) = 1-P\left[\bigcap_{i=1}^{n}\overline{E_i(t)}\right] = 1-\overline{F_{Y_1,\cdots,Y_n}(t,\dots,t)} \tag{10}$$

where $\overline{E_i(t)}$ is the event that i^{th} component operates successfully by the time 't'.

A limit state function can be formulated to describe $E_i(t)$ and is defined as a function that separates the space into two distinct subspaces, termed the failure and survival subspaces. It is typically formulated based on loads, resistances and mechanisms of failure by taking into account the physical, geometrical, mechanical, *etc*., properties of the component and by describing component failure by a physical phenomenon (or phenomena if many failure mechanisms are present). The limit state function becomes a function of both time 't' and the stochastic variables $\mathbf{X}^{\mathbf{T}} = (X_1,\dots,X_m)$, and is denoted by $g_i(\mathbf{X},t)$ such that the failure subspace is defined whenever $g_i(\mathbf{X},t)\le 0$. The limit state function $g_i(\mathbf{X},t)$ might be transformed into the standardized Normal domain by some transformation function $\mathbf{X} = T(\mathbf{Z})$, where $\mathbf{Z}$ is a vector of the mutually independent standard normal random variables. Depending on the linearity of $g_i(T(\mathbf{Z}),t)$, the exact or an approximate probability of failure might be computed or estimated.

For a series system consisting 'n' components described by (10), the time-dependent limit state function for each component will be defined by $g_i(\mathbf{X},t)$ where $i=1,\dots,n$, and it is assumed to describe fatigue failure with a non-decreasing damage function. The series system unreliability by the time 't' can be written as:

$$F_s(t) = P\left[\bigcup_{i=1}^{n}\{g_i(T(\mathbf{Z}),t) \le 0\}\right] \tag{11}$$

If $g_i(T(\mathbf{Z}),t)$ is non-linear then first order or second order reliability methods (FORM or SORM) can be used and $F_s(t)$ can be estimated approximately. By applying linearization in standardized domain, $g_i(T(\mathbf{Z}),t)$ becomes a hyperplane and can be written:

$$g_i(T(\mathbf{Z}),t) \approx \beta_i(t) - \boldsymbol{\alpha}_\mathbf{i}^\mathbf{T}(t)\mathbf{Z} \tag{12}$$

where $\beta_i(t)$ is termed as reliability index, and $\boldsymbol{\alpha}_\mathbf{i}(t)$ is a unit normal column vector directed towards to the failure subset, so that $\|\boldsymbol{\alpha}_\mathbf{i}(t)\| = 1$. So, (11) will be given by:

$$F_s(t) \approx 1 - P\left[\bigcap_{i=1}^{n}\{\boldsymbol{\alpha}_\mathbf{i}^\mathbf{T}(t)\mathbf{Z} \le \beta_i(t)\}\right] \tag{13}$$

By introducing $\boldsymbol{\beta}(t) = [\beta_1(t), \cdots, \beta_n(t)]^T$ and $\underline{\boldsymbol{\alpha}(t)} = [\boldsymbol{\alpha}_\mathbf{1}(t), \cdots, \boldsymbol{\alpha}_\mathbf{n}(t)] = \begin{bmatrix} \alpha_{1,1}(t) & \cdots & \alpha_{1,n}(t) \\ \vdots & \ddots & \vdots \\ \alpha_{m,1}(t) & \cdots & \alpha_{m,n}(t) \end{bmatrix}$, the correlation matrix of the '*n*' linearized limit state functions can be estimated by:

$$\boldsymbol{\rho}\left(\boldsymbol{\alpha}_\mathbf{i,j}(t)\right) = \underline{\boldsymbol{\alpha}^\mathbf{T}(t)}\,\underline{\boldsymbol{\alpha}(t)} \tag{14}$$

Estimation of series system unreliability by the time '*t*' defined in (13) becomes:

$$F_s(t) \approx 1 - \Phi_m\left(\boldsymbol{\beta}(t), \boldsymbol{\rho}\left(\boldsymbol{\alpha}_\mathbf{i,j}(t)\right)\right) \tag{15}$$

$$\beta_G^s(t) \approx -\Phi^{-1}\left[1 - \Phi_m\left(\boldsymbol{\beta}(t), \boldsymbol{\rho}\left(\boldsymbol{\alpha}_\mathbf{i,j}(t)\right)\right)\right] \tag{16}$$

where, $\beta_G^s(t)$ is the corresponding generalized series system reliability index.

More details on system reliability estimation by structural reliability methods can be found in [10]. Also, computation of (15) or (16) requires a computer software or some approximation method (*e.g.*, build in MATLAB function "mvncdf" might be used with the developed codes of FORM or SORM at [11], or other software).

4. Proposed Method for Dependent Systems Reliability Estimation

In sequential order representation, knowledge of the parent distributions at each level is required in order to estimate the minimum distribution at the level, latter is necessary in order to estimate sequential order pattern and evaluate subsequent failure times. The minimum distribution at each level is suggested to be estimated directly by series system structural reliability methods, and then integrated with sequential order pattern estimation methods. When the leading failure mechanism(s) is described by a physics of failure model(s), the advantages of using series system structural reliability methods in a context of sequential order pattern estimation are: 1) the capability to consider statistical correlations of the physics of failure model(s) (emerged from the associated limit state functions and stochastic models of the uncertain parameters) and 2) reduction of the computational time by directly estimating the minimum distribution at each level. Thus, the procedure should be defined such that the outcomes from sequential order statistics and series system structural reliability methods have to be identical.

It is observed that (3) with 'r = 1' is comparable with (6), if the sample size in (3) is changed with each observed sequential ordered failure. This leads to the formulation: if an i.i.d. sample $\{Y_1^{(i)},\ldots,Y_{n-i+1}^{(i)}\}$ with size '*n-i+1*' is selected from the $F_{Y^{(i)}}(t)$ distribution at level '*i*', such that $P[Y_k^{(i)}=Y_m^{(i)}]=0$ where $k\neq m$, $k=1,\ldots,n-i+1$ and $m=1,\ldots,n-i+1$, then the distribution function of the sequential order statistic $X_*^{(i)}$ at level '*i*' based on the $F_{X^{(i)}}(t)$ parent distribution, such that $F_{X^{(i)}}(t)=F_{Y^{(i)}}(t)$, and given that failure at level '*i-1*' is s_{i-1} ($s_0=-\infty$), can be written as:

$$\begin{aligned}F_{X^{(i)}_{1:n-i+1}|X^{(i-1)}_{1:n-i+2}}(t\,|\,s_{i-1}) &= P[X^{(i)}_{1:n-i+1}\leq t\,|\,X^{(i-1)}_{1:n-i+2}=s_{i-1}]\\ &=P[Y^{(i)}_{1:n-i+1}\leq t\,|\,Y^{(i)}_{1:n-i+1}>s_{i-1}]=F_{Y^{(i)}_{1:n-i+1}/Y^{(i)}_{1:n-i+1}}(t\,|\,s_{i-1})\end{aligned} \tag{17}$$

Illustrated relationship by (17) has an important representation in methodical sense and its application from structural reliability viewpoint is explored in the following. The first row of (17) is the minimum distribution function of the truncated parent distribution $F_{X^{(i)}}$ at level '*i*' (as it is described in sequential order statistics, see left side of Figure 1), whereas the second row of (17) is the truncated distribution function of the minimum distribution at level '*i*' with $F_{Y^{(i)}}$ parent distribution (see right side of Figure 1). Furthermore, the series systems reliability estimation by structural reliability approaches is used for the minimum distribution estimation at level '*i*' (see Figure 1).

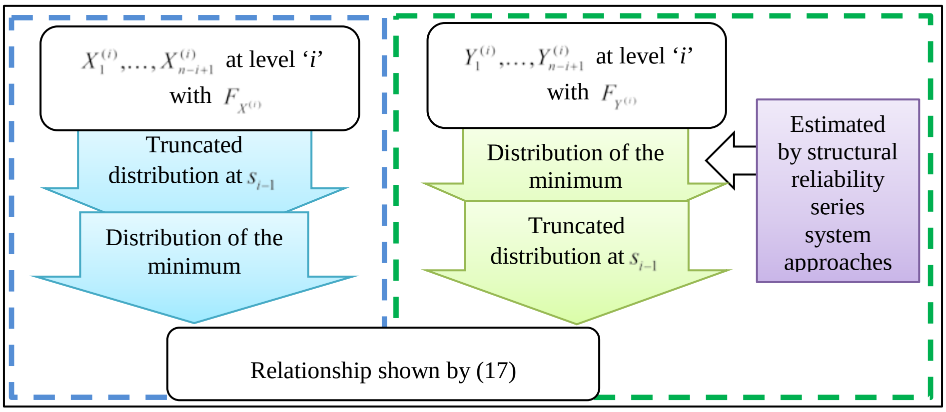

Figure 1: Illustration of the Suggested Procedure Described by (17)

This estimation procedure is non-destructive and robust, as far as it is based on limit state functions, describing each component failure behavior and allows estimating reliability in engineering designs by parameters/variables modification or introduction of new parameters. This result is useful in failure dependent system applications as far as estimation of the minimum c.d.f. is practically observable.

Suppose '*n*' components at time '*0*' were in parallel system at the level '*1*' and components dependency is described by the failure of either one. Right-after '*i-1*' total failures, a new parallel system will be formed from '*n-i+1*' surviving components at level '*i*'. Failure time at level '*i*' is modeled by the truncated distribution of the series system from '*n-i+1*' components at level '*i*', whereas estimation of the series system probability of failure is carried out by structural reliability approaches. The failure time at level '*i*' is predicted at the desired percentile and used to estimate the subsequent truncated distribution and desired percentile of failure time at level '*i+1*'. The procedure is continued until the truncated distribution at level '*n*' is estimated, which corresponds to the lifetime distribution of the ‘n’ component parallel system with failure-effect correlated components.

5. Illustrative Example

Let us assume that 3 identical components form a 3-component parallel system and are loaded with constant cyclic tension loading at level 1 with stress range 110 MPa (Load 1). After failure of either component, a new 2-component parallel system from the surviving components is formed and undergoes to cyclic tension loading at level 2 with stress range 120 MPa (Load 2). Next, after failure of either component in the 2-component parallel system, the remaining single component undergoes cyclic tension loading at level 3 with stress range 130 MPa (Load 3). The leading failure mode in either component is assumed to be governed by the fatigue cracking mechanism in opening mode. Physics of failure is modeled based on fracture mechanics approaches, where the double edge cracking model is used. It is assumed that components are made from steel and failure might happen due to either brittle failure (fracture) or ductile failure (plastic failure). Thus, a failure assessment diagram is used to assess failure criteria. Paris’s equation in the form $da/dN = C(\Delta K)^m$ is used to estimate the number of cycles of failure. Material properties, Paris’s equation constants, loading profiles and failure assessment diagram criteria are summarized in Table 1 and Table 2. Refer to [12-13] for more details on fracture mechanics approaches.

In addition, it is assumed that variabilities and statistical correlations exist in the physics of failure model and they are described based on the $\{C, m, a_0, K_{IC}, S_{yeild}\}$ stochastic variables. The assumed correlation matrix, coefficient of variations (COV) and distributions are summarized in Table 3, while expected values are provided in Table 1. In addition, it is assumed that $\{K_{IC}, S_{yield}\}$ stochastic variables are common for each component in order to describe the assumption that components are made from the same steel material, with the same manufacturer and with the same manufacturing process (thus describing correlations within the components).

The component cumulative probability of failure for the given number of cycles (N) is estimated based on the following limit state function:

$$g_i(C^i, m^i, a_0^i, K_{IC}, S_{yeild}, dS^i, N) = 1 - \frac{N}{N_f} \tag{18}$$

where $i = 1, \cdots, n_{Load}$ and N_f is the random number of cycles to failure estimated via fracture mechanics model and the failure assessment diagram.

Monte Carlo Simulation (5000 replications), FORM and SORM (difference quotient in numerical differentiation and error for reliability index estimation were set to 10^{-5} and 10^{-2}, respectively) were performed for a single component at each level, where Nataf transformation with Cholesky decomposition was used to transform correlated variables into the standardized Normal domain.

Table 1: Paris's Equation Constants, Material Properties and Failure Assessment Diagram

Paris' law intercept (C)	$1.35*10^{-10}$ m/cycle
Paris' law slop (m)	2.25
Width (W)	$20*10^{-3}$ m
Thickness (B)	$20*10^{-3}$ m
Initial crack length (a_0)	$0.2*10^{-3}$ m
Yield strength (S_{yield})	1315 MPa
Fracture toughness (K_{IC})	50 MPa(m)$^{0.5}$
Stress intensity factor ($K_I(a)$)	$K_I(a) = S\sqrt{a}Y(a)$ MPa(m)$^{0.5}$ where S is the applied stress in MPa
Geometry factor ($Y(a)$)	$Y(a) = 1.98 + 0.36\left(\frac{2a}{W}\right) - 2.12\left(\frac{2a}{W}\right)^2 + 3.42\left(\frac{2a}{W}\right)^3$ where a is the crack size (meters) in double-edge cracked plate in tension
$L_r = S_{crack\max}(a) / S_{yeild}$	$S_{crack\max}(a) = \frac{F_{\max}}{B(W-2a)}$, where $F_{\max}$ is the actual load and $S_{crack\max}(a)$ is the maximal stress at crack tip
$K_r = K_{I\max}(a) / K_{IC}$	$K_{I\max}(a) = S_{\max}\sqrt{a}Y(a)$
Failure Assessment Diagram Equation for $0 \le L_r \le 1.2$ and $0 \le K_r \le 1$	$K_r = (1 - 0.14L_r^{\,2}) * (0.3 + 0.7e^{-0.65L_r^{\,6}})$

Table 2: Loading Profiles

	Load 1	*Load 2*	*Load 3*
Stress range ($dS = S_{\max} - S_{\min}$)	110 MPa	120 MPa	130 MPa
Stress ratio ($\mathrm{R} = K_{I\min}(a) / K_{I\max}(a) = S_{\min} / S_{\max}$)	0.5	0.5	0.5
Number of components (n_{Load})	3	2	1

Table 3: Parameters for Stochastic Variables

Stochastic variable	*Distribution*	*COV*	*Correlation matrix*				
			C	m	a_0	K_{IC}	S_{yeild}
C	LogNormal	0.02	1	-0.8	0	0	0
m	Normal	0.02	-0.8	1	0	0	0
a_0	Exponential	1	0	0	1	0	0
K_{IC}	Normal	0.05	0	0	0	1	-0.5
S_{yeild}	Normal	0.02	0	0	0	-0.5	1

Based on the results (see Figure 2), parent distributions at each load level were estimated for each reliability estimation method (Simulation, FORM, SORM). As it is expected, the probability of failure increases when the load increases, which is a typical behavior in failure dependent systems. It is observed that SORM has closer estimation to the Simulation results than FORM. Thus, reliability indexes $\beta_i(N)$ and unit normal vectors $\boldsymbol{\alpha}_\mathbf{i}(N)$ from SORM output are selected for the series system reliability estimation.

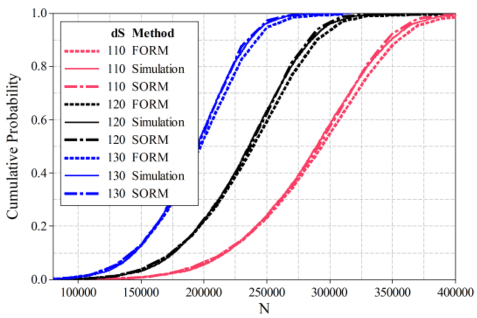

Figure 2: Parent Distributions

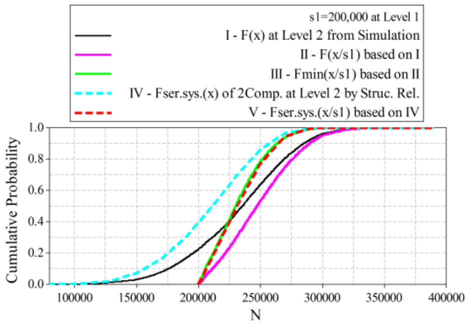

Figure 3: Sequential Order Statistics Approach vs. Structural Reliability Series System Approach

Suppose that the weakest component (1^{st} failure) in the 3-component parallel system fails at 200,000 cycles at level 1. Then, the 2-component parallel system, composed of the surviving 2 components, undergoes to level 2 and it is required to estimate the 2^{nd} failure distribution function at load level 2.

First, the sequential order statistics approach was implemented. The parent distribution function at load level 2 was chosen (see I in Figure 3), and the associated truncated minimum distribution function was estimated based on (4) and (6) (see II and III in Figure 3). Second, the series system structural reliability approach was implemented based on the SORM output and 2-component series system distribution was estimated based on (15), and truncated at 1^{st} failure time (see IV and V in Figure 3). As it is seen III and V in Figure 3 are identical, and this is due to the illustrated relationship by (17) and suggested procedure described in Figure 1. By estimating 2^{nd} failure time for the desired percentile, the procedure can be continued and 3^{rd} failure distribution function can be estimated.

6. Conclusion

A method for dependent systems reliability estimation was developed, where the leading failure mechanism(s) was described based on physics of failure model(s). Dependency was considered for both statistical and failure effect correlations. These aspects are the main deficiency in classical reliability methods for system reliability estimation. An important representation in methodical sense was revealed, which expresses a subsequent failure distribution function in a dependent system by the truncated distribution of the series system distribution, estimated by structural reliability methods. The parent distribution at the given load level is not necessary to be known, even though it is possible to estimate. The proposed method could be used for calibration of limit state functions based on the test data availability and can easily be extended for dependent systems reliability estimation with non-identical components. It could also be used in various decision-making problems, where decision depends on the system’s residual lifetime. It could be implemented for the optimal operation and maintenance strategy determination based on the severity/consequence of failure. In addition, based on the proposed method, it is possible to develop an “early warning” system at the selected level to prevent system collapse. Limitation of the suggested method might be dictated by the fact that the estimated series system distributions via structural reliability methods for non-linear limit state functions are approximations. However, simulation techniques might be used for more complicated problems in order to justify structural reliability methods suitability.

Acknowledgment: This work has been partially funded by the Norwegian Centre for

Offshore Wind Energy (NORCOWE) under grant 193821/S60 from Research Council of Norway (RCN). NORCOWE is a consortium with partners from industry and science, hosted by Christian Michelsen Research.

References

[1] Kamps, U. *A Concept of Generalized Order Statistics*. B.G. Teubner, Stuttgart, 1995.

[2] Kamps, U. *A Concept of Generalized Order Statistics*. Journal of Statistical Planning and Inference, 1995; 48(1):1-23.

[3] Navarro J., and Burkschat M. *Coherent Systems Based on Sequential Order Statistics*. Naval Research Logistics, 2011; 58(2):123-135.

[4] Burkschat M. *Systems with Failure-Dependent Lifetimes of Components*. Journal of Applied Probability, 2009; 46(4):1052-1072.

[5] Cox D.R. *Regression Models and Life Tables*. Journal of the Royal Statistical Society, 1972; 34(2):187-220.

[6] Lin, D.Y., and Z. Ying. *Semiparametric Analysis of the Additive Risk Model*. Biometrika, 1994; 81(1):61-71.

[7] Bagdonavicius, V.B., Gerville-Reache, L., and M.S. Nikulin. *On Parametric Inference for Step-Stress Models.* IEEE Transactions on Reliability, 2002; 51(1): 27-31.

[8] David, H.A., and H.N. Nagaraja. *Order Statistics*. Wiley, New Jersey, 2004.

[9] Balakrishnan N., and A.C. Cohen. *Order Statistics and Inference: Estimation Methods*. Academic Press, New York, 1991.

[10] Madsen H.O., S. Krenk and N.C. Lind. *Methods of Structural Safety*. Englewood Cliffs, London, 1986.

[11] http://reliability.co.nf/

[12] Stephens R.I., A. Fatemi, R.R. Stephens and H.O. Fuchs. *Metal Fatigue in Engineering*. Wiley, 2000.

[13] BS 7910:2005. *Guide to Methods for Assessing the Acceptability of Flaws in Metallic Structures*. British Standards Institution, London, 2005.

Erik Kostandyan is a Post. Doc. in the Department of Civil Engineering at Aalborg University, Denmark and involved in NORCOWE project. He is an author to many scientific papers and active researcher in developing non-destructive reliability estimation models/methods, applicable for Wind Turbines O&M strategies development. He holds a Ph.D. in Civil Engineering from Aalborg University, Denmark and a Ph.D. in Industrial Engineering from Western Michigan University, USA.

John Dalsgaard Sørensen is a professor in the Department of Civil Engineering at Aalborg University, Denmark. He is a researcher and scholar in the area of structural reliability theory, author to many scientific papers and serving as an editorial board member to structural reliability journals and conferences. His current research areas of interest are Wind Turbines structural subsystems reliability assessment and maintenance planning.