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Machine Learning Techniques Applied to Uniaxial Compressive Strength of Oporto Granite

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Abstract: This article employs two machine learning techniques, *viz.*, Least Square Support Vector Machine (LSSVM) and Multivariate Adaptive Regression Spline (MARS), for determination of Uniaxial Compressive Strength (σ_c) of oporto granite. LSSVM uses a quadratic cost function. MARS is a nonparametric regression technique. Free porosity (N_{48}), dry bulk density (d) and ultrasonic velocity (v) have been used as input of the LSSVM and MARS models. The output of LSSVM and MARS is σ_c. The developed LSSVM and MARS give equations for prediction of σ_c. A comparative study has been carried out between the developed LSSVM, MARS, Support Vector Machine (SVM) and Artificial Neural Network (ANN) models. The results show that the developed LSSVM and MARS models are efficient tools for determination of σ_c of Oporto granite.

Keywords: *Uniaxial compressive strength, oporto granite, Least Square support vector machine, multivariate adaptive regression spline; artificial neural network, support vector machine.*

1. Introduction

Uniaxial Compressive Strength (σ_c) is a key parameter for determination of deformation behaviour of rock mass. So, the determination of σ_c is an imperative task in civil engineering. Civil engineers use different methods for determination of σ_c of rock [1-5]. The available methods are not so reliable Martins *et al.* [6]. Martins et al. [6] successfully applied Artificial Neural Network (ANN) and Support Vector Machine (SVM) for determination of σ_c of Oporto granite. Fuqing *et al.* (2013) [7] presented a comparative study between ANN and SVM. However, ANN has various limitations such as black box approach, low generalization capability, and over training problem [8, 9]. SVM has the following limitations [10]:

- SVM makes unnecessarily liberal use of basis functions since the number of support vectors required typically grows linearly with the size of the training set
- Predictions are not probabilistic
- The determination of design value of capacity factor(C) and error insensitive zone (ε) is a complicated task
- The kernel function must satisfy Mercer's condition.

This article adopts two machine learning techniques {Least Square Support Vector Machine (LSSVM) and Multivariate Adaptive Regression Spline (MARS)} for prediction of σ_c of Oporto granite. Oporto granite is located in Oporto and surrounding areas. It is a light grey, two-mica, medium-grained, hypidiomorphic granite. It contains quartz, microcline (k-feldspar), plagioclase muscovite and biotite.

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The LSSVM is a statistical-learning method which has a self-contained basis of statistical-learning theory [11]. There are lots of applications of LSSVM in literatures [12-14]. MARS is developed by Friedman [15]. It is nonparametric regression technique. It has been successfully adopted for solving different problems in engineering [16-18]. LSSVM and MARS are developed by using the database collected from the work of Martins et al. [6]. The database contains information about free porosity (N_{48}), dry bulk density (d), ultrasonic velocity (v) and σ_c. The developed LSSVM and MARS give equations for determination of σ_c of Oporto granite. A comparative study has been carried out between the developed LSSVM, MARS, ANN and SVM models.

2. Details of LSSVM

In the following, we briefly introduce LSSVM for prediction of σ_c of Oporto granite. Consider the following dataset.

$$\{x_k, y_k\}_{k=1}^{N} \; x \in R^n, y \in R \tag{1}$$

where x is input, y is output, N is number of data points, R^n is n-dimensional vector space and R is one dimensional vector space. This study uses N_{48}, d, and v as input variables of the LSSVM. The output of LSSVM is σ_c. So, $x = \lfloor N_{48,} d, v \rfloor$ and $y = [\sigma_c]$.

In feature space LSSVM models take the form:

$$y(x) = w^T \varphi(x) + b \tag{2}$$

where the nonlinear mapping $\varphi(.)$ maps the input data into a higher dimensional feature space, w is weight and b is bias.

LSSVM use the following optimization problem for determination w and b [11].

$$\text{Min: } \frac{1}{2} w^T w + \gamma \frac{1}{2} \sum_{i=1}^{N} e_i^2$$

$$s.t. \; y_i = w^T \varphi(x_i) + b + e_i \tag{3}$$

where γ and e_i are the regularization and error variables, respectively.

The following equation has been obtained by solving the above optimization problem and it has been used for prediction of σ_c [19, 20].

$$\sigma_c = y = \sum_{k=1}^{N} \alpha_k K(x, x_k) + b \tag{4}$$

where $K(x, x_k)$ is kernel function.

To develop the LSSVM, the dataset have been divided into the following two groups:

Training Dataset: This is required to develop the LSSVM model. This study uses 39 out of 55 datasets as training dataset.

Testing Dataset: This is used to verify the performance of LSSVM. The remaining 16 datasets have used as testing dataset.

The data is normalized between 0 and 1. Radial basis function ($K(x, x_k) = \exp\left[-\frac{(x_k - x)(x_k - x)^T}{2\sigma^2}\right]$, where σ is width of radial basis function) has been adopted as kernel function. The program of LSSVM has been developed by using MATLAB.

3. Details of MARS

MARS is a nonlinear and non-parametric regression [15]. This section will describe the MARS model for prediction of σ_c of Oporto granite.
In MARS, the relation between input(x) and output(y) is given by the following relation

$$y = c_0 + \sum_{m=1}^{M} c_m B_m(x) \tag{5}$$

where x is input, y is output and c_0 is constant, $B_m(x)$ is basis function and $c_m(x)$ is coefficient of $B_m(x)$. MARS uses N_{48}, d, and v as input variables. The output of MARS is σ_c. So, $x = [N_{48}, d, v]$ and $y = [\sigma_c]$.

MARS uses two-sided truncated power functions as basis function. The expression of basis function is given below:

$$[-(x-t)]_+^q = \begin{cases} (t-x)^q & \text{If } x < t \\ 0 & \text{otherwise} \end{cases}$$

$$[+(x-t)]_+^q = \begin{cases} (t-x)^q & \text{If } x \geq t \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where t is called knot.

To develop MARS, the following two steps are involved.

Forward step: Basis functions are introduced to define equation (5). The developed MARS can show overfitting due to large number of basis functions. So, it will not give good prediction for new input.

Backward step: In this step, redundant basis functions are deleted. Basis functions are deleted base on generalized cross-validation (GVC) value [21]. The value of GVC has been determined by using the following relation:

$$GCV = \frac{\frac{1}{N}\sum_{i=1}^{N}[y_i - f(x_i)]^2}{\left[1 - \frac{C(B)}{N}\right]^2} \tag{7}$$

where N is the number of data and C(B) is a complexity penalty that increases with the number of basis function in the model and which is defined as:

$$C(B) = (B+1) + dB \tag{8}$$

where d is a penalty for each basis function included into the model and B is number of basis functions in equation (5). The details about d are given by Friedman [15].

Analysis of Variance (ANOVA) decomposition of the MARS model is given by the following expression [15]:

$$f(x) = \beta_0 + \sum_{B=1} f_i(x_i) + \sum_{B=2} f_{ij}(x_i, x_j) + \sum_{B=3} f_{ijk}(x_i, x_j, x_k) + \ldots \tag{9}$$

$\sum_{B=1} f_i x_i$ is over all basis functions that involve only a single variable; $\sum_{B=2} f_{ij}(x_i, x_j)$ is over all basis functions that involve exactly two variables; and $\sum_{B=3} f_{ij}(x_i, x_j, x_k)$ represents (if present) the contributions from three variable interactions and so on.
MARS uses the same training dataset, testing dataset and normalization technique as used by LSSVM model. The program of MARS has been developed by using MATLAB.

4. Results and Discussion

The performance of LSSVM depends on the value of γ and σ. The design values of γ and σ have been determined by trial and error approach. The developed LSSVM gives best performance at γ=200 and σ=0.1. The performance of training and testing dataset has been determined by using γ=200 and σ=0.1.

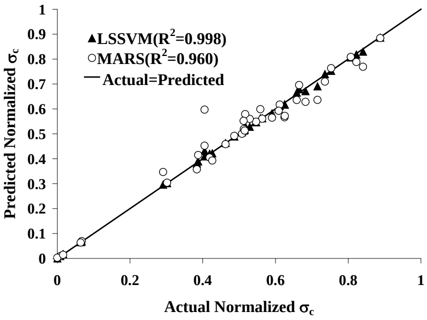

Figure 1: Performance of Training Dataset.

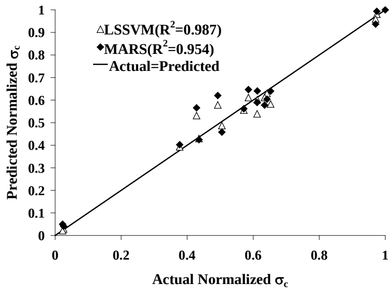

Figure 2: Performance of Testing Dataset.

Figures 1 and 2 depict the performance of training and testing dataset, respectively. The performance of developed LSSVM and MARS has been evaluated by using Coefficient of Determination (R^2) value. For a good model, the value of R^2 should be close to one. It is observed from figures 1 and 2 that the value of R^2 is close to one. So, the developed LSSVM proves his ability predicting σ_c of Oporto granite. The following equation (by putting $K(x, x_k) = \exp\left[-\frac{(x_k - x)(x_k - x)^T}{2\sigma^2}\right]$, σ=0.1, b=-0.117 and N=39 in equation (4) has been obtained from the developed LSSVM.

$$\sigma_c = \sum_{k=1}^{39} \alpha_k \exp\left[-\frac{(x_k - x)(x_k - x)^T}{0.02}\right] - 0.117 \tag{10}$$

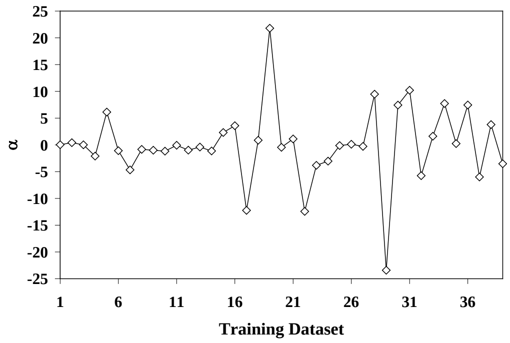

Figure 3: Values of α.

The value of α has been given by Figure 3.

For developing MARS, eight basis functions have been introduced in forward step. In backward step, two basis functions have been deleted. So, the final MARS contains six basis functions. The final equation (by putting y=σ_c, M=6 and c_0=0.325 in equation 1) of MARS is given below:

$$\sigma_c = 0.325 + \sum_{m=1}^{6} c_m B_m(x) \tag{11}$$

The performance of training and testing dataset has been determined by using equation (11). Figure 1 and 2 illustrates the performance of training and testing dataset. It is clear from Figures 1 and 2 that the value of R^2 is close to one. Therefore, the developed MARS predicts σ_c reasonable well. ANOVA decomposition has been done on the developed MARS.

Table 1: Details of Basic Functions

Basis function($B_m(x)$)	Equation	Coefficient(c_m)
$B_1(x)$	$\max(0, N_{48} - 0.470)$	-0.638
$B_2(x)$	$\max(0, 0.470 - N_{48})$	1.530
$B_3(x)$	$\max(0, d - 0.678)$	-1.534
$B_4(x)$	$\max(0, v - 0.497)$	1.31
$B_5(x)$	$B_4 * \max(0, d - 0.678)$	-2.411
$B_6(x)$	$B_4 * \max(0, 0.678 - d)$	-4.397

Table 2: Result of ANOVA Decomposition

Variables	GCV	Standard Deviation	Basis Function	Functions
N_{48}	0.257	0.18	2	1
d	0.113	0.046	1	2
v	0.14	0.051	1	3
d,v	0.078	0.019	2	4

Table 2 shows the results of ANOVA decomposition. It is observed from table 2 that the value of GCV is maximum for N_{48}. So, N_{48} is most important parameter for prediction of σ_c.

A comparative study has been carried out between the developed LSSVM, MARS, SVM, and ANN models. ANN and SVM models developed by Martins *et al.* [6].

Figure 4 shows the bar chart of R^2 values of the different models for testing dataset. The developed LSSVM and MARS outperform the ANN and SVM models. LSSVM, MARS and ANN models do not produce sparse solution, whereas SVM produces sparse solution.

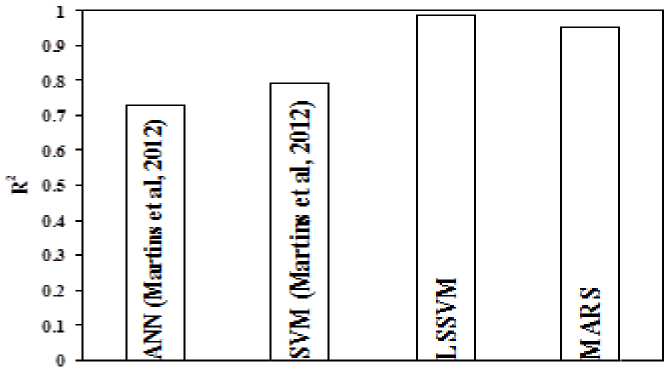

Figure 4: Bar Chart of R^2 of the Different Models

5. Conclusion

This article describes LSSVM and MARS for prediction of σ_c of Oporto granite. 55 datasets have been utilized to develop the LSSVM and MARS models. The performance of developed LSSVM and MARS is encouraging. The developed LSSVM and MARS give better performance than the SVM and ANN models. The performance of LSSVM and MARS is comparable. The developed equations can be used for determination of σ_c of Oporto granite. In summary, it can be concluded that LSSVM and MARS can be used to solve different problems in civil engineering.

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