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A New Uncertainty Importance Measure in Fuzzy Reliability Analysis

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Abstract: Uncertainty is inevitable in any reliability analysis of complex engineering systems due to uncertainties present in models, parameters of the model, phenomena and assumptions. Uncertainties at the component level are propagated to quantify uncertainty at the system level reliability. It is very important to identify all the uncertainties and treat them effectively to make reliability studies more useful for decision making. Conventional probabilistic approaches adopt probability distributions to characterize uncertainty where as fuzzy reliability models adopt membership functions to characterize uncertainty. Both the approaches are widely used in uncertainty propagation for reliability studies. However, identification of critical parameters based on their uncertainty contribution at the system level is very important for effective management of uncertainty. A method is proposed here in the fuzzy framework to rank the components based on their uncertainty contribution to the over all uncertainty of system reliability. It is compared with probabilistic methods using a practical reliability problem in the literature.

Keywords: *Epistemic uncertainty, fuzzy number, importance measures, Monte Carlo simulation.*

1. Introduction

Analyses for complex systems inevitably involve uncertainties arising from inherent variability (randomness or aleatory) or lack of knowledge (epistemic uncertainty). Uncertainties are present in any reliability calculations due to randomness in the failure/repair phenomena and the limitation in assessing the component parameters of the failure/repair probability density functions [1]. Accuracy of the studies is greatly influenced by methodology, unjustified assumptions, model uncertainty and lack of plant specific data. The impact of these uncertainties must be addressed if it is to serve as a tool in the decision making process. The problem of acknowledging and treating uncertainty is central for the quality and practical usability of quantitative reliability analysis. The uncertainty propagation of system reliability assessment quantifies uncertainty in system characteristic (ex: system unavailability) synthesizing it from the uncertainties in component characteristics. Limitations in exactly assessing the parameters of the random variables are leading to uncertainty in component characteristics. This uncertainty is identified as “epistemic uncertainty” which is knowledge based and can be reduced with more information. There are several methods available in the literature for propagating epistemic uncertainties. There are several methods for propagating

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uncertainties such as: (i) Analytical Methods (Method of Moments), [2] (ii) Discrete Probability Distributions [3], (iii) Sampling Methods [4], (iv) Probability Bounds [5] (v) Interval Arithmetic [6] and (vi) Fuzzy Arithmetic [7,8]. Probabilistic approaches (i-iii) characterize the uncertainty in the parameter by a probability distribution where as interval approach (iv) represents with an interval having lower bound and upper bound. Fuzzy Set Theory based approach (vi) characterizes it by a fuzzy membership function. The different approaches to dealing with uncertainty mentioned above have proved to possess different desirable and undesirable features, making them more or less useful in different situations.

One of the major objectives in performing parameter uncertainty propagation is to rank the parameters with respect to their contribution to the uncertainty in the model prediction. The most obvious reason for this being that such a ranking makes it possible to allocate resources efficiently in case the reduction in the calculated uncertainties in the output prove necessary in order to reach an acceptable degree of confidence in the results. The process of identifying components from uncertainty contribution point of view is called uncertainty importance measures. It is different from functional importance, which denotes the criticality of the component in the successful/failure operation of whole system. The methods required for this kind of ranking will depend upon the type of uncertainty propagation method used. In the probabilistic framework, there are several methods available in the literature for uncertainty importance measures such as non parametric methods [3, 9-10] and variance based methods [11-13]. They are useful in identifying the critical uncertain parameters and further with more information reducing the uncertainty. In fuzzy reliability framework, importance measures from functional (or structural) point of view are available in the literature [14] and work on fuzzy uncertainty importance measure is attempted by Utkin [15]. In this paper, a new approach is proposed in the fuzzy framework where uncertain parameters are ranked based on their contribution of uncertainty of system reliability. The proposed method is validated with comparative studies on a reliability problem and found that proposed fuzzy uncertainty importance measure is matching with probabilistic uncertainty importance measures.

2. Probabilistic Approach to Ranking Uncertain Parameters in System Reliability Models

The expression of the system reliability (or availability), Y, as a function of the component reliabilities (X_i) is written as:

$$Y = f(X_1, X_2, ..., X_n) \tag{1}$$

This relation can be obtained from fault tree analysis technique which denotes system failure logic with various failure combinations of one or more components. Due to scarcity or lack of data, it is not possible to exactly give a fixed value to the reliability of each of the components. In case of probabilistic approach, reliability of components is treated as a random variable represented by a probability distribution. Uncertainty in system reliability is obtained by propagating component uncertainties through Monte-Carlo simulation. But it is equally important to identify which component is contributing more uncertainty to system reliability as this information is required for effective management of uncertainty. This helps in identifying the components for which more information should be collected so that the uncertainty in the calculated system reliability can be reduced. In the probabilistic framework, various methods used for uncertainty importance measures are just briefed here which will be subsequently used for comparison with the proposed method.

a. Correlation Coefficient Method

One fairly simple and straightforward method of ranking uncertain parameters is to calculate the sample correlation coefficient of the model prediction and each of the uncertain parameters, using the sample of output values and the corresponding sample of values for each input. Consider ‘m’ samples from the output and a single input, denoted as $y_j, x_j,$ for j =1 to m. The sample (or Pearson) correlation coefficient is computed from[6]:

$$r_{XY} = \frac{\sum_{j=1}^{m}(x_j-\overline{x})(y_j-\overline{y})}{\sqrt{\sum_{j=1}^{m}(x_j-\overline{x})^2 \times \sum_{j=1}^{m}(y_j-\overline{y})^2}} \tag{2}$$

The correlation coefficient provides an estimate of the degree of linear relationship between the sample values of the model output and the input parameter. The sign of the coefficient tells us the direction of the relationship, and the absolute value of the coefficient indicates the strength of the relationship where -1 indicates a completely negative linear relation and +1 a completely positive linear relation.

b. Variance Based Method

Variance based techniques explain V_Y, i.e. the variance of Y, in terms of variances of the individual parameters or parameter groups. They identify the parameters that contribute to over all uncertainty in Y the most, as follows. V_Y can be written in terms of individual parameter and parameter group contribution as [13]:

$$V_Y = \sum_i V_i + \sum_{i<j} V_{ij} + \sum_{i<j<m} V_{ijm} + \ldots + V_{12\ldots n} \tag{3}$$

where n is the number of parameters, X_i denotes the ith parameter, $E(Y|X_i=x_i^*)$ denotes the expectation of Y conditional on X_i having a fixed value x_i^*. $V_i = V(E(Y|X_i=x_i^*))$ stands for the variance over all possible values of x_i, and analogous definitions hold for the higher order terms.

First order global sensitivity indexes can be expressed using Eq. (3) as [12]:

$$S(x_i) = V_i/V_Y \tag{4}$$

Parameters that have a higher contribution to the variance will have higher conditional variances V_i, and therefore will have higher $S(x_i)$ is then taken as the uncertainty importance measure of the individual parameter x_i.

3. Proposed Method

In fuzzy set theory based uncertainty analysis, component reliability is treated as a fuzzy number and the variability is characterized by the membership function. The membership function is usually assumed to be a triangular function (a, b, c) and is treated as a possibility distribution. Several authors worked extensively in applying fuzzy set theory to system reliability analysis in assessing uncertainty in reliability models [7,8,14] However, one of the major objectives in performing parameter uncertainty propagation is to rank the parameters with respect to their contribution to the uncertainty in the model output. Many measures are available in probabilistic approaches which are explained in the previous section. In the context of fuzzy reliability models, an algorithm is proposed here for characterizing uncertainty importance measures which is shown in Fig. 1.

Fuzzy uncertainty importance measure is introduced as:

$$FUIM_i = \frac{Y_i^R}{Y_i^L}, \tag{5}$$

where Y_i^R is system model output, system unavailability (for repairable engineering systems unavailability is appropriate measure of reliability) with i[th] component parameter value at the most pessimistic value (for α=0, upper value - c) and remaining components are as per the given membership functions. Y_i^L is unavailability with i[th] component parameter value at the most optimistic value (for α=0, lower value - a) and remaining components are as per the given membership functions. Parameters that have higher value of above measure will contribute more uncertainty to the system unavailability.

$$x_0 = \frac{\int_{-\infty}^{+\infty} xf(x)dx}{\int_{-\infty}^{+\infty} f(x)dx} = \frac{\int_a^b xf^L(x)dx + \int_b^c xf^R(x)}{\int_a^b f^L(x)dx + \int_b^c f^R(x)} \tag{6}$$

$$y_0 = \frac{\int_0^1 y(g^R(y) - g^L(y))dy}{\int_0^1 (g^R(y) - g^L(y))dy} \tag{7}$$

$$D = \sqrt{x_0^2 + y_0^2} \tag{8}$$

But $FUIM_i$ is a fuzzy set and based on this it is difficult to rank the components as per their shape of membership function. Hence ranking of fuzzy numbers is required to compare the fuzzy uncertainty importance measures. Method proposed based on the centroid calculation and distance between origin and centroid is adopted here as it is more efficient than other methods [11, 16]. Centroid for a triangular fuzzy number can be calculated using Eqs. (6) and (7). The distance between centroid and origin denoted with D_i, provides a measure of uncertainty importance as shown in Eq. (8). The distance D_i provides a measure of uncertainty importance. D_i has to be calculated for all the components of the system. Component having highest value of D_i is the most critical uncertain parameter. Now components will be ranked based on the value of D_i in the decreasing order as it is a crisp value.

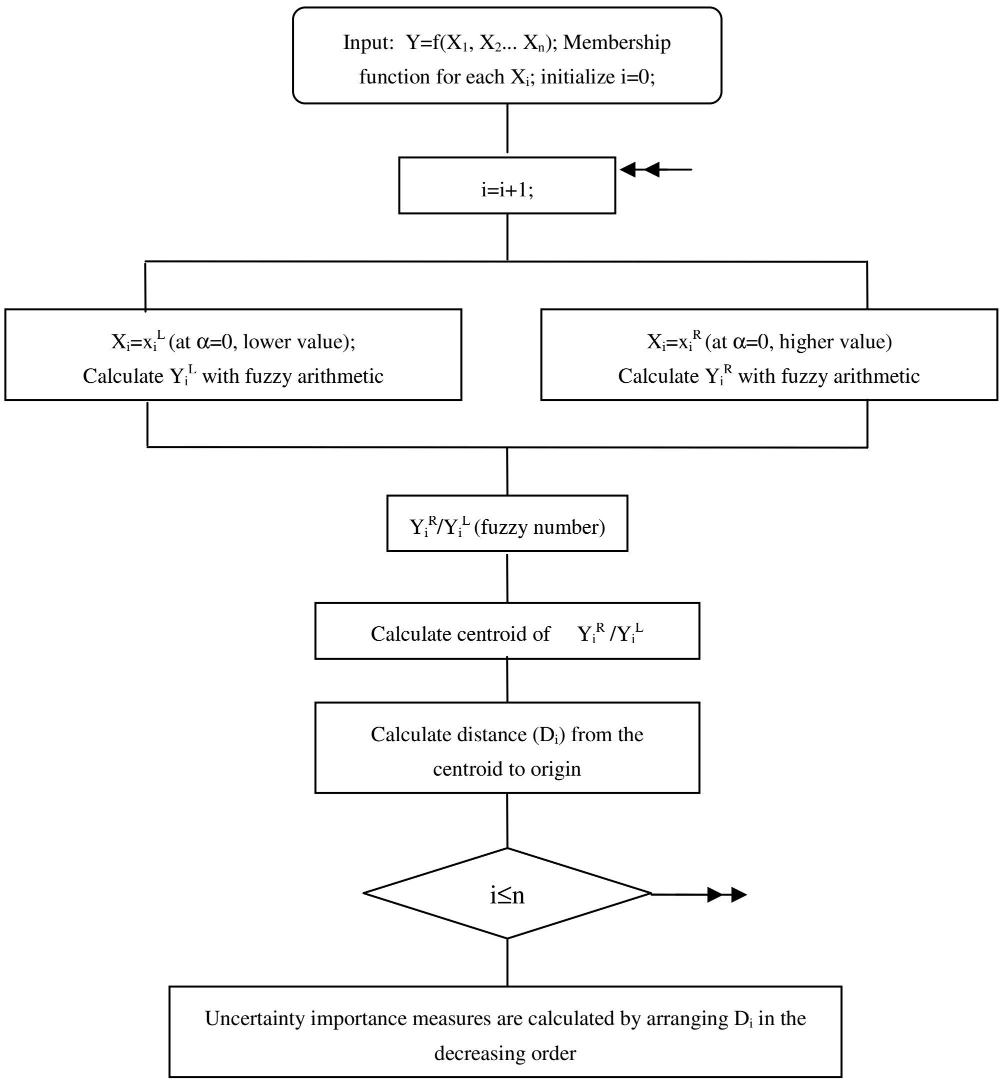

Fig. 1: Algorithm for calculation of fuzzy uncertainty importance measures

4. Application to a Practical System

This section applies the proposed algorithm to a practical reliability problem [12]. The expression for the top event (failure probability or unavailability of the system) of the fault tree is the sum of minimal cut-sets as expressed in equation (9). All basic events of fault tree (components of system) are assumed to be mutually independent and log-normally distributed in probabilistic calculations and triangular membership functions in fuzzy framework with the same median and tail values (90% confidence bounds) as that of probability distribution. The component data is shown in Table 1.

$$\begin{aligned} Y &= f(X_1, X_2 \ldots X_n) \\ &= X_1X_3X_5 + X_1X_3X_6 + X_1X_4X_5 + X_1X_4X_6 + X_2X_3X_4 + X_2X_3X_5 + X_2X_4X_5 + X_2X_5X_6 + X_2X_4X_7 + X_2X_6X_7 \end{aligned} \tag{9}$$

Table 1: Component data

Component	Prob. approach (lognormal)		Fuzzy approach (triangular)		
	Median Probability/frequency	90% error factor	Low	Median	High
1	2(initiat. event freq.)	2	1	2	4
2	3(initiating event)	2	1.5	3	6
3	1e-3	2	5e-4	1e-3	2e-3
4	2e-3	2	1e-3	2e-3	4e-3
5	4e-3	2	2e-3	4e-3	8e-3
6	5e-3	2	2.5e-3	5e-3	1e-2
7	3e-3	2	1.5e-3	3e-3	6e-3

In the probabilistic framework, two techniques as explained in section 2.1 and 2.2 are adopted here. In the first method, Monte Carlo simulation with 10^6 iterations have been carried out which gave 10^6 sample of inputs (x_1, x_2... x_7) and associated system output (y_i), where 'i' denotes iteration number. Pearson correlation coefficient has been calculated with the Eq. (2) for each component. This coefficient provides a measure of how much each input contributes to the output uncertainty, larger the value higher the contribution. They are shown in Table 2 and ranked in the decreasing order.

In the second method also, simulation has been carried out for 10^6 iterations and calculated variance V_Y from the sample of y_i. As per Eq. (4), V_i has to be calculated for each component. For all the components, first order global sensitivity index (Eq. (4)) has been calculated. Larger value of the index, uncertainty contribution will be larger.

Table 2: Comparison of results for uncertainty importance measures

Component	Correlation Coefficient		Variance based Method		Proposed Method	
	r_{XiY}	Rank	V_i/V_Y	Rank	D_i	Rank
1	0.187	6	0.036	6	4.1403	6
2	0.574	1	0.34	1	5.9193	1
3	0.1238	7	0.017	7	3.9248	7
4	0.296	4	0.088	4	4.5357	4
5	0.4147	3	0.178	3	5.0833	3
6	0.4631	2	0.22	2	5.3404	2
7	0.2247	5	0.049	5	4.2539	5

In the fuzzy framework, a new algorithm proposed in section 3 has been applied and compared with the probabilistic methods. D_i has to be calculated for each component, which gives uncertainty importance measure. For components having higher value of D_i, uncertainty contribution will be larger. Ranks have been obtained based on the calculated D_i values of all components. They are shown graphically in Fig. 2. Ranking based on the proposed approach is exactly matching (see Table 2) with the conventional probabilistic approaches. The proposed method is very simple and also computational effort required is less compared with the probabilistic methods. Thus, in the fuzzy reliability models, the proposed algorithm is able to rank the components based on their uncertainty contribution.

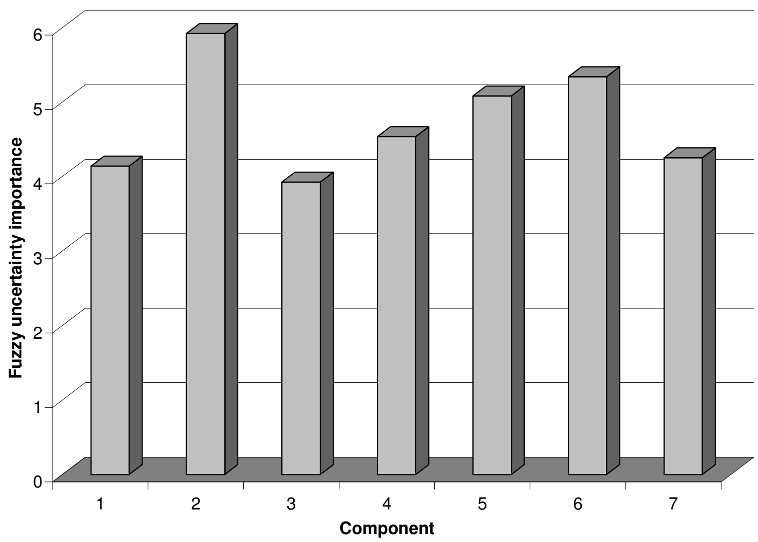

Fig. 2: Fuzzy uncertainty importance measure

5. Conclusions

Uncertainty importance measures play an important role in the management of uncertainty by identifying those sources of uncertainty having greatest impact on system reliability. By collecting more information for critical uncertain parameters, uncertainty in system reliability can be reduced. Reduction of uncertainty in system reliability results will improve confidence in the reliability studies for decision making. In this paper, a method has been proposed in the fuzzy framework to rank the components based on their uncertainty contribution to the over all uncertainty of system reliability. It has been compared with probabilistic methods (Pearson correlation coefficient and variance based methods) with the help of a practical system in the literature. It is found that proposed method is in good agreement with probabilistic methods and also involves very simple calculations.

References

[1]. IAEA, *Procedure for conducting Probabilistic Safety Assessment of Nuclear Power Plants (level 1),* Safety Series No. 50-P-4, International Atomic Energy Agency, Vienna, 1992.

[2]. Rushdi, A.M. and Kafrawy, K.F., *Uncertainty propagation in fault tree analyses using an exact method of moments*, Microelectronics and Reliability, Vol. 28, pp. 945-965, 1988.

[3]. Morgan, M.G. and Henrion, M., *Uncertainty – A guide to dealing uncertainty in Quantitative Risk and policy analysis*, Cambridge University Press, London, 1992.

[4]. Jackson, P.S., Hockenbury, R.W. and Yeater, M.L., *Uncertainty analysis of system reliability and availability assessment,* Nuclear Engineering and Design Vol. 68, pp. 5-29, 1981.

[5]. Scott, F. and Hajago, J.G., *Arithmetic with uncertain numbers: rigorous and often best possible answers,* Reliability Engineering and System Safety, Vol. 85, pp. 135-152, 2004.

[6]. Abrahamsson, M., *Uncertainty in Quantitative Risk Analysis*, Report 1024, Lund University, 2002.

[7]. Misra, K.B. and Weber, G.G., *A New Method for Fuzzy Fault Tree Analysis*, Micr. Rel. Vol.29, No.2, pp.195-216, 1989.

[8]. Tanaka, H., Fan, L.T., Lai, F.S. and Toguchi, K., *Fault tree analysis by fuzzy probability,*

IEEE Transactions on Reliability, Vol. 32, pp. 453-457, 1983.
[9]. Borgonovo, E., *Measuring Uncertainty Importance: Investigation and Comparison of Alternative Approaches,* Risk Analysis, Vol. 26, pp.1349-1361, 2006.
[10]. Saltelli, A. and Marivoet, J., *Non-parameter Statistics in Sensitivity Analysis for Model Output: A Comparison of Selected Techniques,* Reliability Engineering and System Safety, Vol. 28, pp. 229-253, 1990.
[11]. Cheng, C.H., *A New Approach for Ranking Fuzzy Numbers by Distance Method,* Fuzzy Sets and Systems, Vol.95, pp.307-317, 1998.
[12]. Iman, R.L., and Conover, W.J., *A Matrix Based Approach to Uncertainty and Sensitivity Analysis for Fault trees.* Risk Analysis, Vol. 7, No. 1, pp.21-33, 1987.
[13]. Iman, R.L. and Hora, S.C., *A Robust Measure of Uncertainty Importance for use in Fault Tree System Analysis,* Risk Analysis, Vol.10, No. 3, pp. 401-406, 1990.
[14]. Furuta H. and Shiraishi N., *Fuzzy Importance in Fault Tree Analysis,* Fuzzy Sets and Systems, Vol. 12, pp.205-213, 1984.
[15]. Utkin, L.V., *Uncertainty Importance of System Components by Fuzzy and Interval Probability,* Microelectronics and Reliability, Vol. 33, No. 9, pp. 1357-1364, 1993.
[16]. Ying-Ming, W., Jian-Bo, Y., Dong-Ling, X. and Kwai-Sang, C., *On the Centroids of Fuzzy Numbers,* Fuzzy Sets and Systems, Vol.157, pp.919-926, 2006.

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