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SHORT COMMUNICATIONS

Handling Functional Dependence without Using Markov Models

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Abstract: For the reliability analysis of systems with dynamic behavior of functional dependence, state space oriented approaches, in particular, Markov model based methods have been used for the entire system or at least dynamic portions of the system via a modular approach. Unfortunately, Markov models suffer from the well-known state explosion problem, leading to models that are computationally intensive and intractable. In this short communication, a combinatorial transform method is proposed for the reliability analysis of dynamic systems subject to functional dependencies. The method can obviate the use of Markov models, offering exact and computationally efficient solutions to the reliability analysis of large-scale dynamic systems. In addition, the proposed method will not be restricted to exponential time-to-failure distributions. The basics and advantages of the proposed approach are illustrated through an example.

Keywords: *combinatorial approach, dynamic system, functional dependence, reliability analysis, transform method*

1. Introduction

The analytical modeling approaches to the system reliability analysis can be broadly classified into three categories: combinatorial models, state space oriented methods, and modular solutions that combine the former two methods as appropriate [1]. Combinatorial models have been used as an efficient technique for evaluating static systems whose failure criteria can be fully expressed in terms of combinations of component fault events [2]. However, a widely held view among researchers is that combinatorial models are not able to model dynamic and dependent behavior in response to a fault in dynamic systems.

Functional dependence is a typical dynamic behavior existing in many real-world systems. For example, when communication is achieved through a network interface card, the failure of the card makes the connected components inaccessible in the computer network [2]. In general, functional dependence occurs when the failure of one component (also referred to as the trigger event) causes other components to become inaccessible or unusable. The traditional approach for dealing with functional dependence is to apply state space oriented approaches, in particular, continuous time Markov chains (CTMC) based methods to the analysis of the entire system or at least the dynamic portions of the system through the modular approach [3].

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Unfortunately, the Markov model has a significant disadvantage that its size grows exponentially as the size of the system increases. This rapid growth of the number of states often leads to models that are computationally intensive and intractable. Therefore, the Markov model based methods can only solve dynamic systems of very limited size. Furthermore, the CTMC based methods are typically not able to handle non-exponential time-to-failure distributions for dynamic components. Therefore, there is a critical need to find a way that can obviate the use of Markov models, offering exact and computationally efficient solutions to the reliability analysis of large-scale dynamic systems subject to general component failure distributions. The work presented in this short communication will serve as a starting point for addressing the above need by proposing a combinatorial method for the reliability analysis of dynamic systems with functional dependence.

2. The Proposed Combinatorial Method

Mathematicians have been using various transform methods for simplifying the calculation of probabilistic measures. As a simple example, the logarithm function is one of the first transform methods used successfully. Using the identity of log (A * B) = log A + log B, the problem of multiplying two large numbers can be converted to a simpler problem of adding two small numbers. Motivated by this simple, yet insightful idea, we propose to transform the original reliability problem involving functional dependence into multiple reduced problems. The transformation process will be combinatorial and based on total probability theorem. Moreover, the effects of functional dependence will be factored out of the new problems resulting from the transformation. Thus, the new reduced problems can be solved in a combinatorial way. In particular, the proposed method can be applied in the following three steps:

Step 1: Building Dependent Trigger Event (DTE) Space. Assume there are *m* trigger events (denoted by T_i, i=1…m) involved in all the functional dependence of a system, The *m* trigger events form a set of 2^m collectively exhaustive and mutually exclusive events (termed as DTE in this work) that can occur in the system: $DTE_1 = \overline{T_1} \cap \overline{T_2} \cap ... \cap \overline{T_m}$, $DTE_2 = T_1 \cap \overline{T_2} \cap ... \cap \overline{T_m}$, $\cdots$, $DTE_{2^m} = T_1 \cap T_2 \cap ... \cap T_m$. We call this set of DTE as the "DTE space". We further define a functional dependence group (FDG) as a set of components that are functionally dependent on the same elementary trigger event *T*.

Step 2: Generating and Solving Reduced Problems. Based on the DTE space built in the first step and the total probability theorem, we can evaluate the occurrence probability of the event of the system failure as:

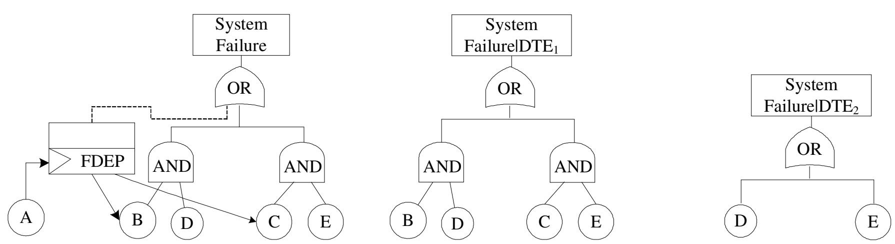
$$U_{sys} = \sum_{i=1}^{2^m} \Pr[(\text{system fails} \mid DTE_i) \bullet \Pr(DTE_i)] \tag{1}$$

$\Pr(DTE_i)$ in (1) can be easily calculated based on the occurrence probability of elementary trigger events. The conditional probability Pr(system fails|DTE_i) in (1) is actually a reduced reliability problem in which the set of components affected by DTE_i (denoted by S_{DTE_i}) does not appear. Note that S_{DTE_i} can be obtained based on the related FDGs, as illustrated in Section 3. Specifically, if the system model is represented in a fault tree, each basic event that is affected by DTE_i will be replaced by a constant logic value '1' (*True*). After the replacement, a Boolean reduction is applied to the system fault tree to generate a simpler fault tree in which all the components affected by DTE_i do not appear. Most importantly, the evaluation of the reduced fault tree can proceed without further consideration the effects of functional dependencies. Hence, these reduced problems can be solved using efficient combinatorial methods.

Step 3: Integrating for System Unreliability. Lastly, using (1) the results for all the reduced problems will be integrated with the occurrence probabilities of DTEs to obtain the final unreliability of the system subject to functional dependence.

3. An Illustrative Example

Consider a dynamic system in Figure 1(a), where *B* and *C* are functionally dependent on *A*, i.e., FDG_A = {B, C}. Using the traditional approach, we must solve a compact Markov chain (after merging all failure states and related transitions) with 11 states and 25 transitions. In contrast, using the proposed method, we only need to analyze two static systems with 4 and 2 components, respectively. Specifically, because there is only one trigger event *A* in this example, the DTE space consists of 2 DTEs, i.e., *A* not occurring (DTE_1) and *A* occurring (DTE_2). And, $S_{DTE_1} = \phi$, $S_{DTE_2} = FDG_A = \{B, C\}$. Applying (1), two reduced problems will be generated: Pr(System fails| DTE_1) and Pr(System fails| DTE_2). Figure 1(b) and (c) show the fault tree models for these two reduced problems, respectively. Apparently, the reduced fault trees can be solved using the efficient combinatorial methods, e.g., binary decision diagrams (BDD) [2]. Using the failure rates of 0.00001/hour for all the 5 components in the example, we obtained the system unreliability for the mission time of 10000hours as 0.0335641, which is consistent with the result obtained using the Markov solution. The method has also been tested using several other examples, which are not shown here due to the space limitation.



(a) Original Fault Tree (b) Reduced Fault Tree (c) Reduced Fault Tree

Fig. 1: System Fault Tree Models for the Illustrative Example

4. Conclusions and Future Work

We proposed an efficient combinatorial approach to the reliability analysis of dynamic systems subject to functional dependence. The method is applicable for general time-to-failure distributions for system components. Our future work includes the investigation of combinatorial methods for addressing other dynamic behaviors such as priorities of failure events, and sequence dependence.

References

[1] Misra, K. B. (Ed), *Handbook of Performability Engineering*, Springer, U.K.,2008.

[2] Dugan, J. B., and S. A. Doyle, *New Results in Fault-Tree Analysis*, Tutorial Notes of The Annual Reliability & Maintainability Symposium, January 1996.

[3] Gulati, R., and J. B. Dugan, *A Modular Approach for Analyzing Static and Dynamic Fault Trees*, Proc. of The Annual Reliability & Maintainability Symposium, 1997.