International Journal of Performability Engineering, Vol. 3, No. 3, July, 2007, pp. 303 - 318

Printed in India

Reliability Importance Analysis of Generalized Phased-Mission Systems

LIUDONG XING[1]

Electrical and Computer Engineering Department
University of Massachusetts Dartmouth
285 Old Westport Road, North Dartmouth, MA 02747, USA

(*Received on June 8, 2006*)

Abstract – Reliability importance analysis is usually used to detect design weakness, to identify which components contribute the most to the failure of a system, and thus to help find the most cost-effective way for upgrading the system. In this paper, we consider the reliability importance analysis of components in a generalized phased-mission system using the Birnbaum's measure. Our method is based on multi-state binary decision diagrams. In addition, it accounts for two-level modular imperfect coverage that arises from the combination of traditional imperfect coverage and more general than OR-ed combinatorial phase requirements. An example three-phased mission system is analyzed to illustrate the applications and advantages of the proposed approach.

Keywords: *Birnbaum's measure, fault tree, modular imperfect coverage, multi-state binary decision diagram, phased-mission system*

1. Introduction

A phased-mission system (PMS) is a system that is used in a mission characterized by multiple, consecutive, and non-overlapping phases of operation [1]. Traditionally in a PMS, the mission is assumed to fail if the system fails during any phase. Recent work [2] extended such OR-ed phase failure requirements to combinatorial phase requirements (CPR), by allowing the failure of the mission to be expressed as a logical combination of phase failures. The outcome of the generalized PMS (GPMS) may exhibit multiple performance levels between binary outcomes (*success* and *failure*). Thus a phase failure does not necessarily lead to a mission failure in the GPMS; it may just produce degraded performance of the mission.

When adding the consideration of imperfect coverage (IPC) [2], [3], where a malicious or uncovered component fault causes extensive damage (traditionally, the entire system failure), we can model two possibilities for the extent of the damage from an uncovered fault: it may just fail the phase in which the fault occurs, but not the mission, or it can fail the entire mission. Such two-level imperfect fault coverage was named as "modular imperfect coverage" (MIPC), and those two failure cases were named as "phase uncovered failure" and "mission uncovered failure", respectively [4]. Hence, a component in a fault tolerant GPMS can have

[1]Corresponding author's email: lxing@umassd.edu

three failure modes: covered failure, phase uncovered failure, and mission uncovered failure. And different failure modes have distinct effects on the system failure. A covered failure is local to the affected component, it may or may not lead to system failure depending on the remaining redundancy, a phase uncovered failure causes immediate phase failure, while a mission uncovered failure is globally malicious and leads to the failure of the entire system. The consideration of those multiple failure modes introduced by MIPC poses unique challenges to existing analysis methods.

As with any other systems, the reliability of a GPMS depends on the reliability of its components, and some components can be more critical than others in contributing to the entire system reliability. In 1969, Birnbaum introduced the concept of component importance measures [5], which provide information concerning a component's contribution to the entire system reliability. Later on many other component importance measures were proposed, such as criticality measure, Fussell-Veselay's measure, improvement potential, risk achievement worth etc [6]. All those measures were designed to help identify the component whose improvement is most likely to yield the greatest improvement in the system performance, in particular, the system reliability. Because Birnbaum's measure is central to the many other component importance measures, we focus on the analysis of Birnbaum's measure for GPMS in this paper. The proposed approach can be easily extended for analyzing other importance measures.

Though considerable research efforts have been expended in the component importance analysis using Birnbaum's measure (see, e.g., [6], [7], [8]), only little work has focused on the analysis of PMS [2], [9]. And to the best of our knowledge, no work has been done to perform the importance analysis for components in a GPMS with CPR and MIPC. In this paper, we propose an efficient multi-state binary decision diagram based method for the component importance analysis in GPMS with CPR and MIPC. The traditional phase-OR PMS and conventional *kill-all* IPC will appear as special cases of the proposed method.

The reminder of the paper is organized as follows. Section 2 presents the problem to be solved and assumptions used in the analysis. Sections 3 presents an example of GPMS subject to CPR and MIPC to help make tangible the type of system for which our approach is meant as well as the analytical challenge we address in this paper. Component reliability importance analysis is based on the system reliability analysis. Therefore, in Section 4 we present background and preliminary results regarding the reliability analysis of GPMS with CPR and MIPC. Section 5 presents an efficient approach for computing the reliability importance measure of components in the GPMS. Section 6 presents and discusses the analysis results. In the last section, we present our conclusions.

2. Problem Statement

This paper considers the problem of evaluating the reliability importance of components in GPMS with CPR and the two-level MIPC in an efficient manner. The input of the problem include: number of phases (m), duration of each phase (T_i), combinatorial phase requirements and phase failure criteria in the fault tree model, and failure parameters & imperfect coverage factors for each component in each phase.

In the model for the component importance analysis of GPMS, the following assumptions are made:

- Component failures are *s*-independent within each phase. Dependencies arise among different phases and different failure modes.

- Fault occurrence probability for each component at each phase is given a) as a fixed probability for a specified mission time, or b) in terms of a lifetime distribution.
- The system is not maintained during the mission; once a component transfers from the operation mode to a failure mode (*covered*, or *phase uncovered*, or *mission uncovered*), it will remain in that failure mode for the rest of the mission time.
- Phase durations are deterministic.
- The system is coherent, which means that each component contributes to the system state, and the system state worsens (at lease does not improve) with an increasing number of component failures [10].

3. An Illustrative Example GPMS

An example GPMS from [4] will be used to illustrate our methodology for generalized phased-mission importance analysis. The system consists of 3 non-repairable components *A*, *B*, and *C*, which are used for *3* consecutive phases. Each phase has different system configuration and failure criteria as shown in the system fault tree model (Figure 1). In addition, the failure and coverage parameters for each component may be different from phase to phase (refer to Tables 1 & 2 in Section 6). The entire mission fails if the system fails in phase 1 or fails in both phase 2 and phase 3.

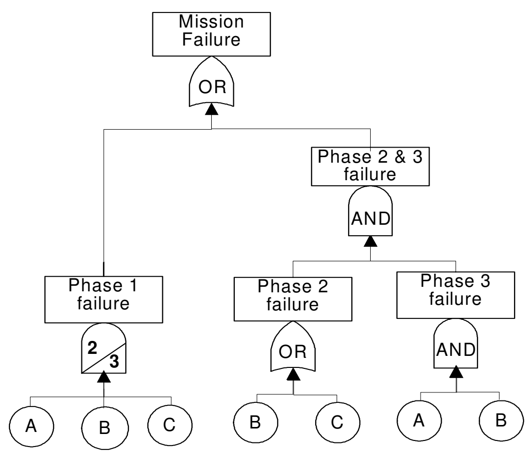

Fig. 1: System Configuration & Phase Requirements in Fault Tree

4. Preliminary Results

This section presents the modular imperfect coverage model (MIPCM), a model for describing the MIPC behavior of a component in a GPMS phase, and the approach to the reliability analysis of GPMS with CPR and MIPC.

4.1 Modular Imperfect Coverage Model

Figure 2 shows the general structure of the MIPCM [4]. Similar to traditional imperfect coverage model [3], the MIPCM is a black box with a single entry and multiple exits. It models in detail the fault type (transient or permanent), and the system's response to a component fault. The model is activated when a fault occurs, and is exited when the fault is successfully handled or when the fault causes a phase failure or the entire system failure. The following gives the explanation of each exit.

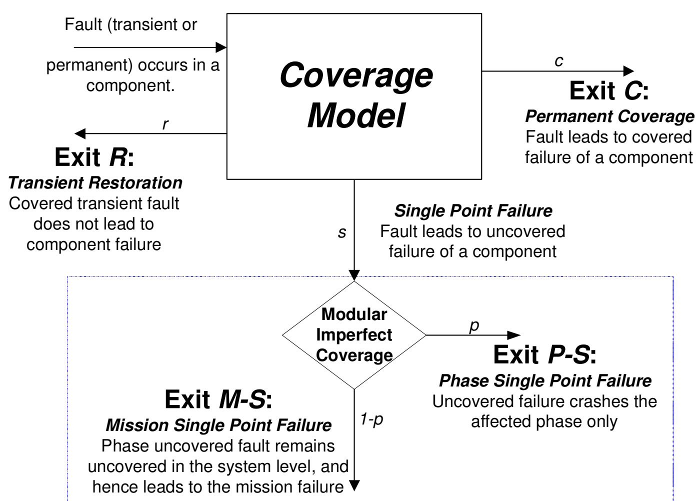

Fig. 2: General Structure of MIPCM for GPMS

- If the component fault is transient and can be handled without discarding any component, then the transient restoration exit *R* is taken. When *R* exit is reached, the component is still in the operational state.
- If the fault is determined to be permanent, and the offending component can be successfully removed or isolated, then the permanent coverage exit *C* is taken. When the *C* exit is reached, a covered component failure is said to occur.
- If a single fault by itself brings down a phase to which the fault belongs even in the presence of fault-tolerant mechanisms, then the single-point failure (or uncovered failure) is said to occur. Further, if this uncovered fault is covered at the higher system level, the phase single-point failure exit (labeled *P-S*) is reached, then a phase uncovered component failure occurs; if the uncovered fault remains uncovered or untolerated at the system level, the mission single-point failure exit (labeled *M-S*) is reached, and then a mission uncovered failure is said to occur.

The exit probabilities are required for our analysis. To make the MIPCM backward compatible with the traditional coverage model, we define [*r*, *c*, *s*] to be the probability of taking the [*R*, *C*, single-point failure] exit, given that a fault occurs, and $r+c+s=1$. The two-level effects from an uncovered fault is defined by a new coverage factor *p,* which is a conditional probability that an uncovered fault fails a single phase not the mission conditioned on an uncovered fault occurring in that phase. Conversely, $(1-p)$ is the conditional probability that an uncovered fault leads to the immediate mission failure conditioned on an uncovered fault occurring in the phase. Then $s*p$ will be the probability of taking *P-S* exit, and $s*(1-p)$ will be the probability of taking *M-S* exit. When $p=0$, the MIPCM is reduced to the conventional coverage model. The equation $r+c+s*p+s*(1-p)=1$ means that the four exits *R*, *C*, *P-S*, and *M-S* are mutually exclusive and complete. The values of those exist probabilities can be determined using an appropriate fault coverage model [11], [12]. An efficient separable approach for incorporating the MIPCM into the reliability analysis of GPMS is presented in the next subsection.

4.2 Reliability Analysis of GPMS with MIPC

In a GPMS, the system has to accomplish a specified task during each mission phase. Since the tasks can differ from phase to phase, the system may be subject to different stresses as well as different reliability requirements. Thus, system configuration, failure criteria, and component failure parameters may also change from phase to phase. This dynamic behavior usually requires a distinct model for each phase of the mission in analysis. In addition, there exist statistical-dependencies across the phases for a given component. For example, the state of a component at the beginning of a new phase is identical to its state at the end of the previous phase. The consideration of these dynamics and dependencies poses challenges to the existing analysis methods [2], [13]-[17]. Further complicating the analysis of GPMS are the multiple failure modes introduced by MIPC. Constructing an overall Markov model is a feasible way to address MIPC in the reliability analysis of GPMS. However, Markov model has the biggest disadvantage that its size grows exponentially as the size of the system increases. This rapid growth of states may lead to intractable models. Also, Markov-based approaches usually require exponential component failure distribution. In [18], an efficient method based on the multi-state concept was proposed to deal with the conventional imperfect coverage for a single-phase, distributed computer system with exponential component failure behavior. Reference [4] extended the multi-state concept to deal with MIPC for the efficient reliability analysis of GPMS with arbitrary component failure distribution. We present and refine the basics of this efficient approach as follows.

4.2.1 Multi-state Variable and Multi-state Binary Decision Diagrams (MBDD)

According to the previous discussion, each component A in the phase j of a GPMS can exist in one of the four states: operation (denoted by O), covered failure (C), phase uncovered failure (PU), and mission uncovered failure (MU). Each of those states is represented using a Boolean variable (called multi-state variable) A_{js}, $s \in \{O, C, PU, MU\}$. And A_{js}=1 means that A in phase j is in the corresponding state s. As in [14], each component A in each phase j is denoted by a mini-component a_j. Let $q_{a_j}(t)$ be the conditional failure probability of a_j conditioned on the success of a_{j-1}, which can be obtained from the input failure parameters. Note that the time t is measured from the beginning of the phase j so that $0 \le t \le T_j$. Let $n[a_j]$, $c[a_j]$, $pu[a_j]$, and $mu[a_j]$ denote the probability that the mini-component a_j is in O, C, PU, and MU state, respectively. Based on the MIPCM (Figure 2), those four mini-component state probabilities can be calculated as: $n[a_j] = 1 - q_{a_j}(t) + r_{a_j} \cdot q_{a_j}(t)$, $c[a_j] = c_{a_j} \cdot q_{a_j}(t)$, $pu[a_j] = s_{a_j} \cdot p_{a_j} \cdot q_{a_j}(t)$, $mu[a_j] = s_{a_j} \cdot (1 - p_{a_j}) \cdot q_{a_j}(t)$, and $n[a_j] + c[a_j] + pu[a_j] + mu[a_j] = 1$.

The relation between a component A in phase j and its mini-components can be described as: $A_j = a_1 a_2 \ldots a_j$, meaning that A is operational in phase j (represented by $A_j = 1$ or $\overline{A}_j = 0$) if and only if it has functioned in all the previous phases [14]. According to this relationship, the four state occupation probabilities for A in phase j (i.e., $P(A_{jO})$, $P(A_{jC})$, $P(A_{jPU})$, $P(A_{jMU})$) can be calculated as follows:

$P(A_{jO})$=Pr{A hasn't failed before the end of phase j} = Pr{all a_i ($i=1\ldots j$) are not failed}

$$= n[a_1]*n[a_2]*n[a_{j-1}]*n[a_j] = \prod_{i=1}^{j} n[a_i] \quad (1)$$

$P(A_{jC})$=Pr{A has failed covered before the end of phase j} = Pr{any a_i ($i=1 \ldots j$) fails covered}$= c[a_1] + n[a_1]\cdot c[a_2] + \cdots + n[a_1]\cdot n[a_2]\cdots n[a_{j-1}]\cdot c[a_j]$ (2)

$P(A_{jPU})$=Pr{A has failed phase uncovered before the end of phase j}
= Pr{any a_i ($i=1 \ldots j$) fails phase uncovered}
$= pu[a_1] + n[a_1]\cdot pu[a_2] + \cdots + n[a_1]\cdot n[a_2]\cdots n[a_{j-1}]\cdot pu[a_j]$ (3)

$P(A_{jMU})$=Pr{A has failed mission uncovered before the end of phase j}
= Pr{any a_i ($i=1 \ldots j$) fails mission uncovered}
$= mu[a_1] + n[a_1]\cdot mu[a_2] + \cdots + n[a_1]\cdot n[a_2]\cdots n[a_{j-1}]\cdot mu[a_j]$ (4)

It is easy to verify that $P(A_{jO})+ P(A_{jC})+P(A_{jPU})+P(A_{jMU})=1$. Since if A in phase j is not in O state, it must be in one of the three mutually exclusive failure states (C, PU, and MU), the relation among the four multi-state variables for component A in phase j can be expressed as: $\overline{A}_{jO} = A_{jC} + A_{jPU} + A_{jMU}$. Figure 3 shows the binary decision diagram (BDD) [19] format of this relation.

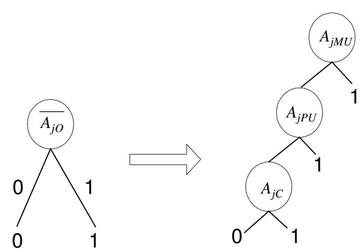

Fig. 3: BDD Format of Equation: $\overline{A}_{jO} = A_{jC} + A_{jPU} + A_{jMU}$

MBDD is an outcome of the combination of conventional BDD and the multi-state concept described above. Similar to the conventional BDD, a MBDD has two sink nodes labeled with constants "0" and "1", representing the system is operational and failed, respectively. The non-sink nodes of MBDD are labeled with multi-state variables and have two outgoing edges: the left branch called "0-edge" indicates the component represented by the node is not in the corresponding state; the right branch called "1-edge" indicates the component is in the corresponding state. Similar to the ordinary BDD, the ordering strategy is critical for the MBDD generation because the size of MBDD heavily depends on the input variable ordering. However, the problem of determining the best way of ordering input variables is itself a NP-complete problem [19]. A set of heuristics can be used to produce an adequate ordering [20]. In GPMS, because each component A corresponds to a set of variables that represent this component in each phase: A_i ($i=1\ldots m$), m is the total number of phases. Further, each A_i corresponds to three failure state indicator variables (A_{iC}, A_{iPU}, A_{iMU}), the following 3-step strategy can be used for putting all the multi-state variables in an ordered list: 1) Order components in GPMS using heuristics [20]. 2) Replace each component indicator variable A in the ordered list with a set of variables that represent this component in each phase, i.e., A_i ($i=1\ldots m$). The variable order will be the reverse of the phase order, that is, $A_m < A_{m-1} < \ldots < A_1$. This ordering strategy is referred to as backward order. Conversely, the ordering strategy in which the variable order is the same as the phase order, that is, $A_1 < A_2 < \ldots < A_m$ is referred to as forward order. It has been shown that the backward order can generate smaller BDD than the forward order [17]. Therefore, we use backward order in our analysis. 3) Replace each A_i with the three corresponding failed multi-state variables in the order of $A_{iMU} < A_{iPU} < A_{iC}$.

Applying the above strategy to the example system (Figure 1), we generate the ordering of variables that belong to the same component (for example, component A) in the three phases as: $A_{3MU} < A_{3PU} < A_{3C} < A_{2MU} < A_{2PU} < A_{2C} < A_{1MU} < A_{1PU} < A_{1C}$. In addition, we assume the order of different components for the example GPMS is $A < B < C$.

4.2.2 Separable MBDD based Approach to the Reliability Analysis of GPMS

References [2], [21] presented a Simple and Efficient Algorithm, abbreviated as SEA, for incorporating the conventional IPC into the reliability analysis of a combinatorial model. Because in the two-level MIPC, a mission uncovered failure leads to the entire system failure as an uncovered failure in the conventional IPC does, we apply the SEA idea to separate the consideration of all the mission uncovered failures from the combinatorics of the solution, for reducing the complexity and difficulty of the overall solution.

Specifically, consider two mutually exclusive and complete events E_1 (1 or more components in GPMS fail mission uncovered) and E_2 (no component fails mission uncovered). Define E as the event of the entire GPMS failure, $P(E_1)$ as P_M, and $P(E|E_2)$ as Q_M. According to the Total Probability Theorem, we can calculate the unreliability of GPMS as:

$$\begin{aligned} U_{GPMS} = P(E) &= P(E \mid E_1) \bullet P(E_1) + P(E \mid E_2) \bullet P(E_2) \\ &= 1 \bullet P_M + Q_M \bullet (1 - P_M) \\ &= P_M + (1 - P_M) \bullet Q_M \end{aligned} \tag{5}$$

Equation (5) decouples all the mission uncovered failures from the combinatorics of the solution. Then, we use MBDD method to evaluate Q_M. Both covered failures and phase uncovered failures will be incorporated into the generation and thus the evaluation of MBDD automatically [4]. We summarize the main steps of the separable MBDD-based approach to the reliability analysis of GPMS with CPR and MIPC as follows. For more details about this approach, see [4] and [22].

1) For each component, obtain the state occupation probability $P(A_{js})$, $j \in \{1, \ldots, m\}$, $s \in \{C, PU, MU\}$ using (2)-(4).
2) For each component, obtain the modified state occupation probability $\tilde{P}(A_{js})$, $j \in \{1, \ldots, m\}$, $s \in \{C, PU\}$ using (6).
$$\tilde{P}(A_{js}) = \frac{P(A_{js})}{1 - P(A_{mMU})} \tag{6}$$
3) Order multi-state variables using the three-step strategy depicted in Section 4.2.1.
4) Find the system mission uncovered failure probability, i.e., P_M in (5), using (7).
$$P_M = 1 - \prod_{\forall A} [1 - P(A_{mMU})] \tag{7}$$
5) For each phase, generate the MBDD that ignores all mission uncovered failures. Then according to the specified CPR, combine the MBDD of each phase to obtain the final

GPMS MBDD excluding the mission uncovered failures. For addressing the *s*-dependencies existing among variables of the same component across different phases (for example, if a component fails covered in one phase, it will remain in that state for all subsequent phases, and it cannot be failed uncovered (mission or phase) either in all subsequent phases or in any of the previous phases), new phase algebra and phase-dependent operations have been developed for combining the single-phase MBDD to generate the GPMS MBDD. Figures 4 and 5 shows the MBDD for each phase and the entire GPMS of Figure 1, respectively.

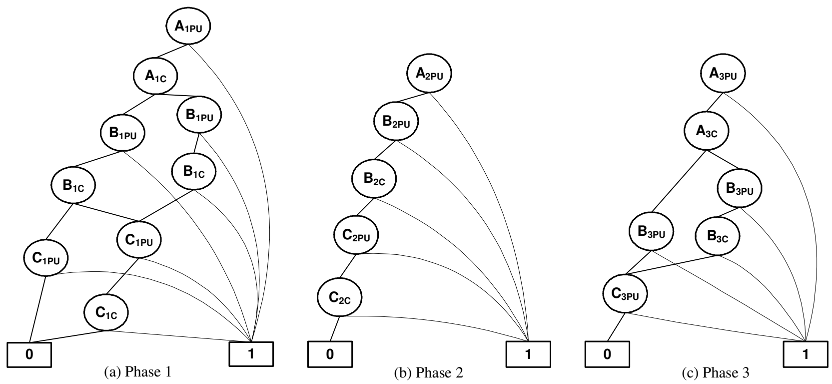

Fig. 4: Single-Phase MBDD for the Example GPMS

6) Evaluate Q_M in (5) recursively from the final GPMS MBDD generated in Step 5). The probability associated with each non-sink node in the GPMS MBDD is the modified state occupation probability $\widetilde{P}(A_{js})$ calculated in Step 2). The recursive algorithm for evaluating GPMS MBDD is given in Section 5.1, along with the recursive algorithm for component importance analysis.

7) Find the unreliability of GPMS with MIPC and CPR using (5), i.e., $U_{GPMS} = P_M + Q_M * (1 - P_M)$.

5. MBDD-Based Approach to Component Reliability Importance Analysis

Based on the MBDD-based approach to the reliability analysis of GPMS, we present a method to compute all system components' Birnbaum's measure and the system unreliability simultaneously with only a single-pass traversal of the MBDD.

The Birnbaum's measure of a component represents the probability that a system is initially in a good state and the failure of that component causes the failure of the system. It is defined as the partial derivative of the system unreliability with respect to the failure probability of that component. According to (5), we obtain the importance vector for the GPMS in (8).

$$\nabla_q(U_{GPMS}) = \nabla_q(P_M + Q_M * (1 - P_M)) = (1 - Q_M)\nabla_q(P_M) + (1 - P_M)\nabla_q(Q_M) \tag{8}$$

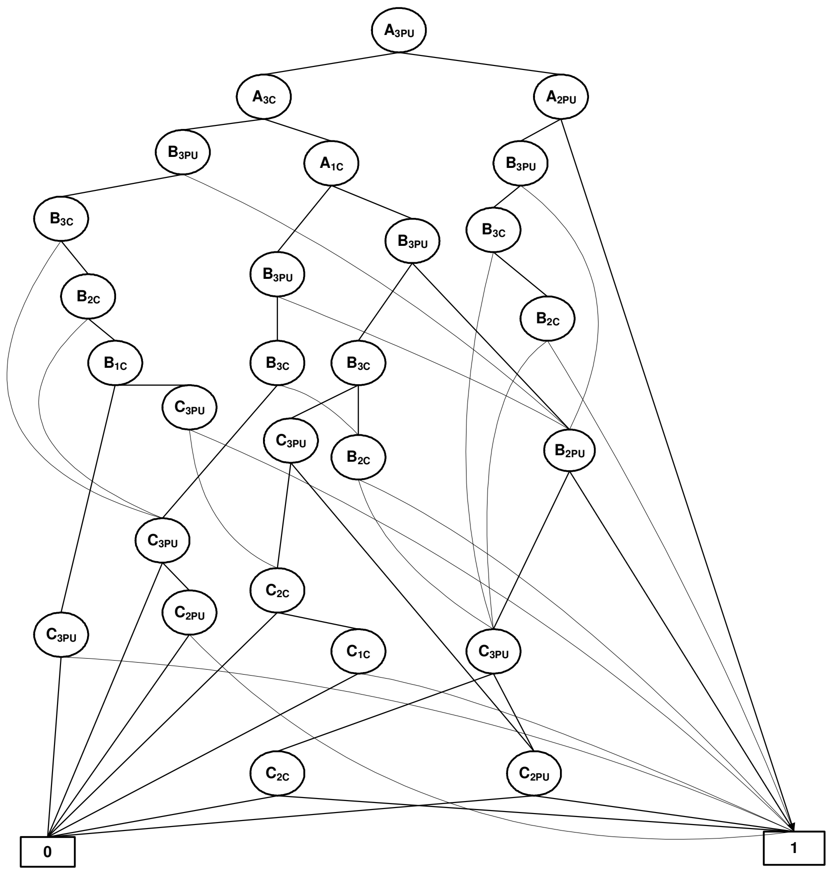

Fig. 5: GPMS MBDD for the Example GPMS

According to (4) and (7), we have:

$$P_M = 1-\prod_{\forall A}(1-P(A_{mMU})) = 1-\prod_{\forall a}(1-mu[a_1]-\sum_{i=2}^{m}(\prod_{k=1}^{i-1} n[a_k])\cdot mu[a_i])$$

$$= 1-\prod_{\forall a}[1-s_{a_1}\cdot(1-p_{a_1})\cdot q_{a_1}(t)-\sum_{i=2}^{m}(\prod_{k=1}^{i-1}[1-q_{a_k}(t)+r_{a_k} * q_{a_k}(t)])\cdot s_{a_i}\cdot(1-p_{a_i})\cdot q_{a_i}(t)]$$

Then, $\nabla_q(P_M)$ can be determined as:

$$\frac{\partial P_M}{\partial q_{a_j}} = \frac{\partial P(A_{mMU})}{\partial q_{a_j}}\prod_{\forall B\neq A}[1-P(B_{mMU})] \tag{9}$$

where $\partial P(A_{mMU})/\partial q_{a_j}$ is evaluated as:

$$\begin{cases} s_{a_1}\cdot(1-p_{a_1})+(r_{a_1}-1)\cdot(mu[a_2]+\sum_{i=3}^{m}(\prod_{k=2}^{i-1}n[a_k])\cdot mu[a_i]) & j=1, \\ s_{a_j}\cdot(1-p_{a_j})\cdot\prod_{i=1}^{j-1}n[a_i]+(r_{a_j}-1)\cdot(\sum_{i=j+1}^{m}(\prod_{k=1,k\neq j}^{i-1}n[a_k])\cdot mu[a_i]) & m>j>1, \\ s_{a_m}\cdot(1-p_{a_m})\cdot\prod_{i=1}^{m-1}n[a_i] & j=m. \end{cases} \tag{10}$$

The next step is to find $\nabla_q(Q_M)$ in (8). Because Q_M can be evaluated from GPMS MBDD composed of nodes that represent covered failure states or phase uncovered failure states, Q_M is actually a function of $\tilde{P}(A_{iC})$ and $\tilde{P}(A_{iPU})$. Therefore, according to the "Chain Rule", we have

$$\frac{\partial Q_M}{\partial q_{a_j}}=\sum_{i=1}^{m}[\frac{\partial Q_M}{\partial \tilde{P}(A_{iPU})}*\frac{\partial \tilde{P}(A_{iPU})}{\partial q_{a_j}}+\frac{\partial Q_M}{\partial \tilde{P}(A_{iC})}*\frac{\partial \tilde{P}(A_{iC})}{\partial q_{a_j}}] \tag{11}$$

According to (4) and (6), we can obtain $\partial\tilde{P}(A_{iC})/\partial q_{a_j}$ of (11) as follows:

$$\frac{\partial\tilde{P}(A_{iC})}{\partial q_{a_j}}=\frac{\partial\dfrac{P(A_{iC})}{1-P(A_{mMU})}}{\partial q_{a_j}}=\frac{(1-P(A_{mMU}))\cdot\dfrac{\partial P(A_{iC})}{\partial q_{a_j}}-P(A_{iC})\cdot\dfrac{\partial(1-P(A_{mMU}))}{\partial q_{a_j}}}{(1-P(A_{mMU}))^2}$$

$$=\frac{(1-P(A_{mMU}))\cdot\dfrac{\partial P(A_{iC})}{\partial q_{a_j}}+P(A_{iC})\cdot\dfrac{\partial P(A_{mMU})}{\partial q_{a_j}}}{(1-P(A_{mMU}))^2}$$

where, $\partial P(A_{mMU})/\partial q_{a_j}$ can be obtained using (10). And according to (2), $\partial P(A_{iC})/\partial q_{a_j}$ can be calculated as:

$$\frac{\partial P(A_{iC})}{\partial q_{a_j}}=\begin{cases} 0 & i<j, \\ c_{a_1} & i=j=1, \\ c_{a_1}+(r_{a_1}-1)\cdot(c[a_2]+\sum_{l=3}^{i}(\prod_{k=2}^{l-1}n[a_k])\cdot c[a_l]) & i>j=1, \\ c_{a_j}\cdot\prod_{l=1}^{j-1}n[a_l]+(r_{a_j}-1)\cdot(\sum_{l=j+1}^{i}(\prod_{k=1,k\neq j}^{l-1}n[a_k])\cdot c[a_l]) & i>j>1, \\ c_{a_i}\cdot\prod_{l=1}^{i-1}n[a_l] & i=j>1. \end{cases}$$

In the similar way, we can obtain $\partial\tilde{P}(A_{iPU})/\partial q_{a_j}$ of (11). Let us revist (11), the items left unknown for finding $\partial Q_M/\partial q_{a_j}$ are $\partial Q_M/\partial\tilde{P}(A_{iPU})$ and $\partial Q_M/\partial\tilde{P}(A_{iC})$. Both items, in general $\nabla_{\tilde{P}}(Q_M)$, can be evaluated with Q_M concurrently during the one-pass recursive traversal of GPMS MBDD. The folowing gives the recursive algorithm.

Begin-of-Recursive-Algorithm

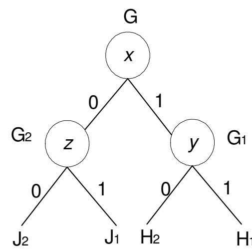

Fig. 6: A Sub-MBDD Used in the Recursive Algorithm

- Consider a sub-MBDD in Figure 6. The if-then-else (*ite*) format [17] is: G = *ite*(x, G_1, G_2), G_1 = *ite*(y, H_1, H_2), G_2 = *ite*(z, J_1, J_2), then

⇒ If x, y belong to the same component, for example, $x = A_{js}$, $y = A_{is}$ ($i < j$), but z doesn't, then

$$P(G) = P(G_1) + [1 - \tilde{P}(x)] \cdot [P(G_2) - P(H_2)]$$

If a state variable being considered is $v \neq x$, then

$$\frac{\partial P(G)}{\partial \tilde{P}(v)} = \frac{\partial P(G_1)}{\partial \tilde{P}(v)} + [1 - \tilde{P}(x)] \cdot (\frac{\partial P(G_2)}{\partial \tilde{P}(v)} - \frac{\partial P(H_2)}{\partial \tilde{P}(v)})$$

Otherwise,

$$\frac{\partial P(G)}{\partial \tilde{P}(x)} = P(H_2) - P(G_2)$$

⇒ If x, z belong to the same component, for example, $x = A_{jPU}$, $z = A_{lC}$ ($l <= j$), but y doesn't, then

$$P(G) = \tilde{P}(x) \cdot [P(G_1) - P(J_2)] + P(G_2)$$

If a state variable being considered is $v \neq x$, then

$$\frac{\partial P(G)}{\partial \tilde{P}(v)} = \tilde{P}(x)[\frac{\partial P(G_1)}{\partial \tilde{P}(v)} - \frac{\partial P(J_2)}{\partial \tilde{P}(v)}] + \frac{\partial P(G_2)}{\partial \tilde{P}(v)})$$

Otherwise,

$$\frac{\partial P(G)}{\partial \tilde{P}(x)} = P(G_1) - P(J_2)$$

⇒ If x, y, z belong to the same component, then

$$P(G) = P(G_1) + P(G_2) - [1 - \tilde{P}(x)] \cdot P(H_2) - \tilde{P}(x) \cdot P(J_2)$$

If a state variable being considered is $v \neq x$, then

$$\frac{\partial P(G)}{\partial \tilde{P}(v)} = \frac{\partial P(G_1)}{\partial \tilde{P}(v)} + \frac{\partial P(G_2)}{\partial \tilde{P}(v)} - [1 - \tilde{P}(x)] \cdot \frac{\partial P(H_2)}{\partial \tilde{P}(v)} - \tilde{P}(x) \frac{\partial P(J_2)}{\partial \tilde{P}(v)}$$

Otherwise,

$$\frac{\partial P(G)}{\partial \tilde{P}(x)} = P(H_2) - P(J_2)$$

⇒ If x, y, z belong to different components, then

$$P(G) = \tilde{P}(x) \cdot P(G_1) + [1 - \tilde{P}(x)] \cdot P(G_2)$$

If a state variable being considered is $v \neq x$, then

$$\frac{\partial P(G)}{\partial \tilde{P}(v)} = \tilde{P}(x)\frac{\partial P(G_1)}{\partial \tilde{P}(v)} + [1-\tilde{P}(x)]\cdot\frac{\partial P(G_2)}{\partial \tilde{P}(v)}$$

Otherwise,

$$\frac{\partial P(G)}{\partial \tilde{P}(x)} = P(G_1) - P(G_2)$$

Note that $P(G)$ and $\partial P(G)/\partial \tilde{P}(x)$ are the unreliability and reliability importance of component x with respect to the current sub-MBDD, respectively. When x is the root node of the GPMS MBDD, $P(G)$ gives the Q_M, i.e., the unreliability of the GPMS ignoring the mission uncovered failures, and $\partial P(G)/\partial \tilde{P}$ gives and $\nabla_{\tilde{P}}(Q_M)$. $\tilde{P}(x)$ is the modified state occupation probability of component x, which can be obtained from Step 2) of the separable approach depicted in Section 4.2.2.

- Exit condition:
  - ⇒ If $G=0$, then $P(G)=0$, $\partial P(G)/\partial \tilde{P}=0$
  - ⇒ If $G=1$, then $P(G)=1$, $\partial P(G)/\partial \tilde{P}=0$.

End-of-Recursive-Algorithm

Lastly, we can fulfill the reliability importance analysis for each component in each phase of the GPMS using (8), (9), and (11). Using these Birnbaum's measures, the analyst can ascertain the importance of each component in each individual phase. However, an analyst also needs a single measure of importance over the entire mission for each component. For this reason, we evaluate a combined importance measure (CIM) as a weighted sum of component importance at each phase as in [2]. Specifically, let S_{A_i} be the importance of component A in phase i, that is, $S_{A_i} = \partial U_{GPMS}/\partial q_{a_i}$, and W_i be the weight associated with phase i. The weight is evaluated as $W_i=U_i/U_{GPMS}$, where U_i represents the unreliability of phase i of the GPMS. Then, the CIM of A over the entire mission is: $S_A = \sum_{i=1}^{m}(W_i \cdot S_{A_i})$. The normalized format is: $S_A = \sum_{i=1}^{m}(W_i \cdot S_{A_i}) / \sum_{i=1}^{m} W_i$.

6. Example Analysis Results

Table 1 details the input parameters except the coverage factor p used in the analysis of the example GPMS. All the failure rates (λ) are given in 10^{-6}/hour, and the coverage factor r is 0 for all components. Six different sets of the coverage factor p (Table 2) are designed for illustrating the effects of MIPC on the reliability importance analysis. Note that for simplicity we assume the value of p is the same for all components within a phase in this example; our methodology is applicable to different values of p for different components.

We analyze the example GPMS (Figure 1) using the separable MBDD-based approach described in Sections 4 and 5, and tabulate the results in Tables 3-7. Specifically, Table 3 provides the unreliability of each phase and the resulted weight of each phase for the example system. The values of U_{GPMS} corresponding to the six sets of p used in the weight calculation are collected in Table 6 of [4]. The weight values show that phase 2 is the most unreliable, phase 3 is the most reliable. Tables 4-7 list the reliability importance measure for each component in each phase as well as the combined measure for the entire mission for each set of *p*. The divergence in the importance measure across phases reflects the difference in the failure requirements for each phase. The divergence for different sets of *p* reflects the effects of two-level MIPC on the reliability importance analysis.

Table 1: Input Parameters for the Example GPMS

Basic Event	Phase 1 (*72* hours)		Phase 2 (*96* hours)		Phase 3 (*48* hours)	
	Failure prob. /rate λ	c	Failure prob. /rate λ	c	Failure prob. /rate λ	c
A	1 (λ)	0.9	0.0001	0.8	1 (λ)	0.9
B	0.0015	0.7	1.5 (λ)	0.7	0.0001	0.7
C	0.001	0.9	0.002	0.9	0.0001	0.8

Table 2: The Phase Imperfect Coverage Factor *p*

	Coverage Factor *p*		
	Phase 1	Phase 2	Phase 3
set I	1	1	1
set II	0.8	1	1
set III	0.8	0.6	0.8
set IV	0.3	0.6	0.8
set V	0.3	0.2	0.1
set VI	0	0	0

Table 3: Unreliability and Weight of Each Phase

		Phase 1 (72 hours)	Phase 2 (96 hours)	Phase 3 (48 hours)
Unreliability U_i		5.5822e-4	2.1437e-3	3.4803e-5
$W_i = U_i/U_{GPMS}$	Set I	0.6797	2.6104	0.0424
	Set II	0.6797	2.6104	0.0424
	Set III	0.6708	2.5761	0.0418
	Set IV	0.6708	2.5761	0.0418
	Set V	0.6413	2.4629	0.0400
	Set VI	0.6374	2.4476	0.0397

Table 4: Reliability Importance Analysis Results for Sets I and II

Basic Event	Phase 1	Phase 2	Phase 3	Whole mission
A	0.10174	0.20075	1.4187e-3	0.5933
B	0.30064	0.2996	4.4816e-7	0.9864
C	0.10076	9.9852e-2	2.2939e-4	0.3292

Note that for sets I and II (or sets III and IV), they have the same *p* values in both phase 2 and phase 3, but different *p* values in phase 1. Because phase 1 is directly connected to the entire mission failure via a phase-OR gate (Figure 1), the phase uncovered failure and mission uncovered failure occurring in this phase will have the same effects on the entire mission failure. Therefore, the value of p in phase 1 has no effect on the entire mission unreliability, and thus the reliability importance measures. Hence, we obtain the same reliability importance values for sets I and II as shown in Table 4 (or sets III and IV as shown in Table 5).

Table 5: Reliability Importance Analysis Results for Sets III and IV

Basic Event	Phase 1	Phase 2	Phase 3	Whole mission
A	0.10174	0.20074	2.1322e-2	0.5863
B	0.30063	0.29959	5.9882e-2	0.9759
C	0.10075	9.9847e-2	4.0043e-2	0.3265

Table 6: Reliability Importance Analysis Results for Set V

Basic Event	Phase 1	Phase 2	Phase 3	Whole mission
A	0.10173	0.20073	9.0982e-2	0.5633
B	0.30059	0.29955	0.26946	0.9413
C	0.10073	9.9827e-2	0.17938	0.3176

Table 7: Reliability Importance Analysis Results for Set VI

Basic Event	Phase 1	Phase 2	Phase 3	Whole mission
A	0.10173	0.20073	0.10093	0.5602
B	0.30058	0.29955	0.2994	0.9366
C	0.10073	9.9825e-2	0.19929	0.3164

7. Conclusions

We presented a MBDD-based combinatorial approach to the reliability importance analysis of components in a generalized phased-mission system subject to CPR and MIPC. The approach separates mission uncovered failures from the combinatorics of the solution, thereby reducing the overall computational complexity. In addition, similar to the efficient approach for computing Birnbaum's measure of components in single-phase systems [7], our approach allows the reliability of the GPMS and the reliability importance measures to be concurrently evaluated in a single-pass traversal of the GPMS MBDD.

Acknowledgments

Part of this paper was presented at the 48th Annual Reliability and Maintainability Symposium [4]. We appreciate the feedback given to us on this topic by reviewers and conference attendees.

References

[1] Meshkat, L., L. Xing, S. Donohue, and Y. Ou, *An Overview of the Phase-Modular Fault Tree Approach To Phased-Mission System Analysis*, Proceedings of The First Internal

Conference on Space Mission Challenges for Information Technology, Pasadena, CA, pp. 393-398, July 2003.

[2] Xing, L., and J. B. Dugan, *Analysis of Generalized Phased Mission System Reliability, Performance and Sensitivity*, IEEE Transactions on Reliability, Vol. 51, No. 2, pp. 199-211, June 2002.

[3] Dugan, J. B., and S. A. Doyle, *New Results in Fault-Tree Analysis*. Tutorial Notes of the Annual Reliability and Maintainability Symposium, pp.568-573, January 1996.

[4] Xing, L., and J. B. Dugan, *Generalized Imperfect Coverage Phased-Mission Analysis*, Proceedings of The Annual Reliability and Maintainability Symposium, pp.112-119, January 2002.

[5] Birnbaum, Z. W., *On the Importance of Different Components in a Multi-component System*, Multivariate Analysis II, Edited by P.R. Krishnaiah, Academic Press, Vol. 11, pp. 581-592, 1969.

[6] Rausand, M., and A. Hoyland, *System Reliability Theory: Models and Statistical Methods (2^{nd} Ed.)*, Wiley Series in Probability and Mathematical Statistics, John Wiley & Sons, 2003.

[7] Chang, Y. R., S. V. Amari, S. Y. Kuo, *Computing System Failure Frequencies and Reliability Importance Measures Using OBDD*, IEEE Transactions on Computers, Vol. 53, No. 1, pp 54-68, January 2004.

[8] Chang, Y. R., S. V. Amari, S. Y. Kuo, *OBDD-Based Evaluation of Reliability and Importance Measures for Multistate Systems Subject to Imperfect Fault Coverage*, IEEE Transactions on Dependable and Secure Computing, Vol. 2, No. 4, pp 336-347, October-December 2005.

[9] Andrews, J. D., *Identifying the Major Contributions to Risk in Phased Missions*, Proceedings of The Annual Reliability and Maintainability Symposium, pp. 624-629, January 2006.

[10] Andrews, J. D., and S. Beeson, *Birnbaum's Measure of Component Importance for Noncoherent Systems*, IEEE Transactions on Reliability, Vol. 52, No. 2, pp.213-219, June 2003.

[11] Dugan, J. B., K. S. Trivedi, M. K. Smotherman, and R. M. Geist, *The Hybrid Automated Reliability Predictor*, AIAA Journal of Guidance, Control and Dynamics, Vol. 9, No. 3, pp. 319-331, May-June 1986.

[12] Dugan, J. B., and K. S. Trivedi, *Coverage Modeling for Dependability Analysis of Fault-Tolerant Systems*, IEEE Transactions on Computers, Vol. 38, No. 6, pp. 775-787, June 1989.

[13] Dugan, J. B., *Automated Analysis of Phased-Mission Reliability*, IEEE Transactions on Reliability, Vol. 40, No. 1, pp. 45-52, 55, April 1991.

[14] Esary, J. D., and H. Ziehms, *Reliability Analysis of Phased Missions*, in Reliability and Fault Tree Analysis: Theoretical and Applied Aspects of System Reliability and Safety Assessment, Edited by R. E. Barlow, J. B. Fussell, and N. D. Singpurwalla, pp. 213-236, SIAM, 1975.

[15] Mura, I., and A. Bondavalli, *Hierarchical Modeling & Evaluation of Phased-Mission Systems*, IEEE Transactions on Reliability, Vol. 48, No. 4, pp. 360-368, December 1999.

[16] Somani, A. K., and K. S. Trivedi, *Phased-Mission System Analysis Using Boolean Algebraic Methods*, Proceedings of the ACM Sigmetrics Conference on Measurement and Modeling of Computer Systems, pp.98-107, May 1994.

[17] Zang, X, H. Sun, and K. S. Trivedi, *A BDD-based Algorithm for Reliability Analysis of Phased-Mission Systems*, IEEE Transactions on Reliability, Vol. 48, No. 1, pp. 50-60, March 1999.

[18] Zang, X., H. Sun, and K. S. Trivedi, *Dependability Analysis of Distributed Computer Systems with Imperfect Coverage*, Proceedings of The 29th Annual International Symposium on Fault-Tolerant Computing, pp. 330-337, June 1999.

[19] Bryant, R. E., *Graph-based Algorithms for Boolean Function Manipulation*, IEEE Transactions on Computers, Vol. c-35, No. 8, pp. 677-691, August 1986.

[20] Bouissou, M., F. Bruyere, and A. Rauzy, *BDD Based Fault-Tree Processing: A Comparison of Variable Ordering Heuristics*, Proceedings of European Safety and Reliability Association Conference (ESREL'97), June 1997.

[21] Amari, S. V., J. B. Dugan, and R. B. Misra, *A Separable Method for Incorporating Imperfect Coverage in Combinatorial Model*, IEEE Transactions on Reliability, Vol. 48, No. 3, pp. 267-274, September 1999.

[22] Xing, L., *Dependability Modeling and Analysis of Hierarchical Computer-based Systems*, Ph.D. dissertation, Electrical and Computer Engineering, University of Virginia, May 2002.

Liudong Xing received her B.E. degree in Computer Science from Zhengzhou University, China, in 1996, and was a Research Assistant in the Chinese Academy of Sciences from 1996 to 1998. She was awarded the M.S. and Ph.D. degree in Electrical Engineering from the University of Virginia in 2000 and 2002, respectively. Since 2002, Dr. Xing has been an Assistant Professor with the Electrical and Computer Engineering Department, University of Massachusetts, Dartmouth. Dr. Xing was a Program co-Chair for the Second IEEE International Symposium on Dependable, Autonomic and Secure Computing (DASC'06). She was the associate Guest Editor for the Journal of Computer Science on a special issue of "Reliability and Autonomic Management". She also serves on the program committees of many international workshops and conferences. Her current research interests include dependable computing and networking, reliability engineering, fault-intrusion tolerant computing, risk analysis, and wireless sensor networks. She is a member of IEEE.