International Journal of Performability Engineering, Vol. 1, No. 2, October 2005, pp. 179-189.

Printed in India

Stochastic Analysis of A System Containing N-Redundant Robots and M-Redundant Built-in Safety Units

B.S. DHILLON* and ZHIJIAN LI

Department of Mechanical Engineering
University of Ottawa
Ottawa, Ontario, K1N 6N5
Canada

(Received on October 13, 2004)

Abstract: This paper presents a mathematical model to perform availability analysis of a robot-safety system having n-redundant robots and m-redundant built-in safety units with common-cause failures. The system failure rates and the partially failed system repair rates are assumed constant, and the failed system repair time is assumed arbitrarily distributed. The supplementary variable method is used to develop generalized expressions for Laplace transforms of state probabilities and system availabilities.

Key Words: maintenance, robot, system, safety, availability, common-cause failures

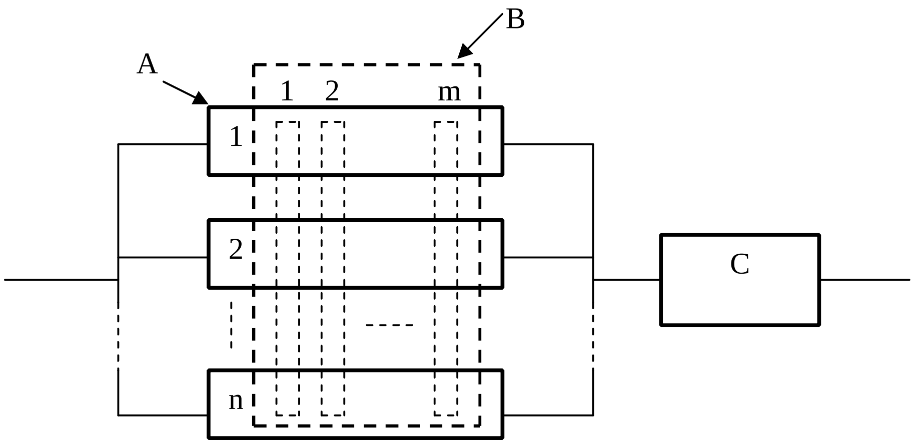
1. Introduction

In recent years, robots are increasingly used in various sectors of industry for purposes such as arc or spot welding, underwater exploration, outer space exploration, fire fighting, medical aid, and relieving humans from performing hazardous tasks. According to the International Federation of Robotics (IFR) the worldwide industrial robot population reached 350,000 in 1987 (Solem, 1987), and increased to 710,000 in 1997 (United Nations, 1998). A conservative forecast for industrial robots for 2005 is 965,000 (United Nations, 2002).

Robot safety is of utmost importance as Ramirez (1985) pointed out, “In short, safety is a concern because generally robots are extremely strong, fast, deaf, dumb, blind, automatic, and therefore dangerous”. Over the years a number of serious accidents and other safety-related problems involving robots have occurred (Nicolaisen, 1987; Nagamachi, 1988; Dhillon, 1991). There is absolutely no doubt that a robot has to be safe and reliable. An unreliable robot may lead to unsafe conditions, high maintenance costs, inconvenience, etc. Therefore, to perform robot reliability analysis, the coupling between reliability and safety must be carefully considered. Moreover, in robot reliability and availability analyses, the occurrence of common-cause failures is overlooked and only general failures are considered. A common-cause failure may be defined as any instance where multiple units or elements fail due to a single cause (Dhillon, 1983; Dhillon, 1999). Under such conditions, the end results may not present a true picture regarding the actual system reliability and availability. To improve a system’s reliability, the concept of redundancy is widely employed, and it can also be applied to robot-safety systems.

* Corresponding author’s email: dhillon@genie.uottawa.ca

Therefore, this paper presents availability analyses of a robot-safety system having n-redundant robots and m-redundant built-in safety units with common-cause failures. The block diagram of this robot-safety system is shown in Figure 1 and its corresponding state space diagram is given in Figure 2. The numerals and letters n and m in the boxes and ellipses of Figure 2 denote system states.



A: n-identical robots
B: m-identical safety units
C: common-cause failure

Figure 1: The Block Diagram of the Robot-Safety System with Common-Cause Failures

At time t=0, all n-redundant robots and m-redundant safety units start operating. The robot-safety system can fail due to the failure of all n robots or the occurrence of a common-cause failure. Nonetheless, the robot-safety system will function normally until at least one safety unit and one robot are operating normally. The system goes through [(m+1)n] distinct operating states. A common-cause failure can occur only if at least two units (including at least one robot) are functioning successfully. Once all m safety units fail, the robots may continue to operate until the failure of the nth robot. The degraded or fully failed robot-safety system may be repaired.

The following assumptions are associated with this model:

(i). The robot-safety system is composed of n identical robots and m-identical safety units.
(ii). The redundant robots and safety units are operating simultaneously.
(iii). All failures are statistically independent.
(iv). All failure rates and the partially failed system repair rates are constant.
(v). The repair of the safety unit has the priority over the repair of the robot when the overall system is in the partially failed operating state. The failed robot-safety system repair rates can be constant or non-constant.
(vi). A repaired robot or safety unit is as good as new.
(vii). The overall system fails only when all the active robots fail.

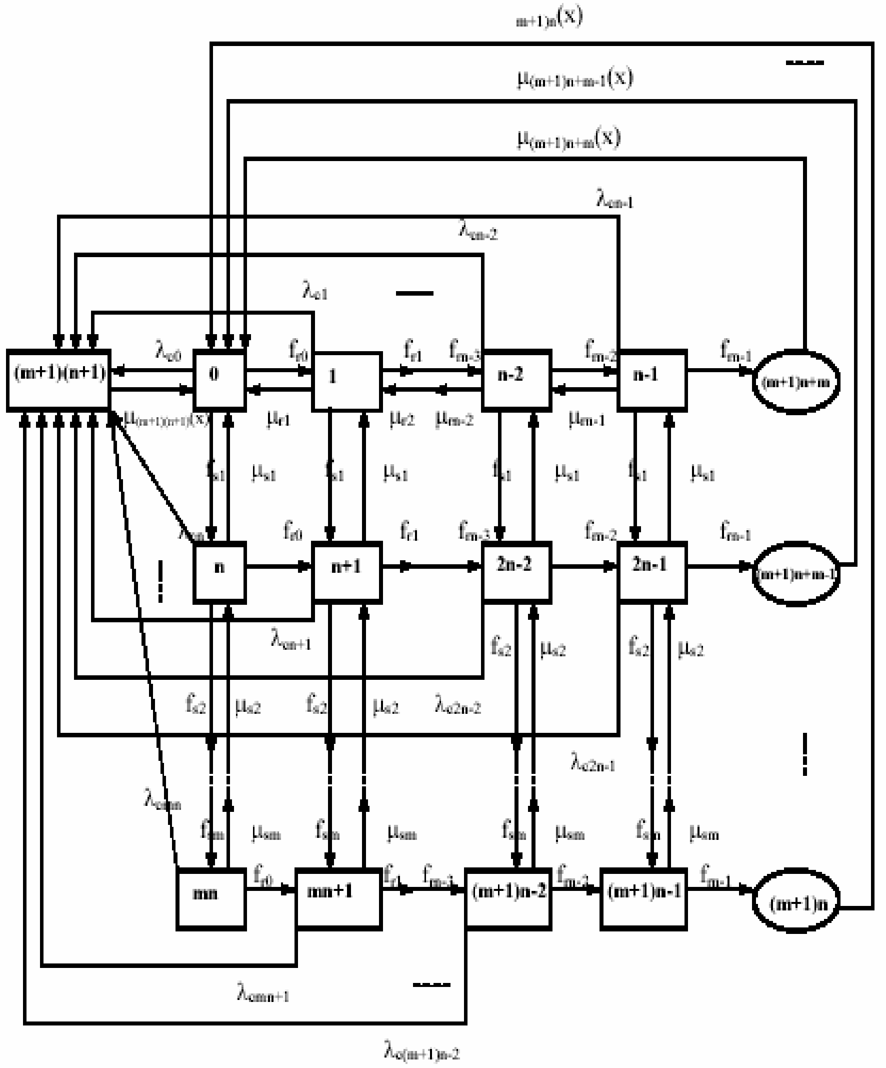


Figure 2: The state space diagram of the robot-safety system with common-cause failures. The numerals and letters n and m in squares, rectangles, and ellipses denote system states, and $f_{sk}=(m-k+1)\lambda_s$, for k=1, 2,…, m; $f_{rq}=(n-q)\lambda_r$, for q=0,1, 2,…, n-1.

2. Notation

The following symbols are associated with the model:

i — i^{th} state of the overall robot-safety system: for i=0 means all n robots and m safety units are in perfect working condition; for i=kn+q (where k=0, 1,…, m, and q=0, 1,…, n-1) means (n-q) robots and (m-k) safety units operating normally while q robots and k safety units have failed; for i= (m+1)n-1

	means only one robot operating normally while (n-1) robots and all m safety units have failed.
j	j^{th} state of the failed robot-safety system: for j=(m+1)n means n robots and m safety units have failed; for j=(m+1)n+k (where k=0, 1,…, m) means n robots failed while k safety units are functioning; for j=(m+1)(n+1) means the robot-safety system failed due to a common-cause failure.
t	time
λ_s	Constant failure rate of a safety unit.
λ_r	Constant failure rate of a robot.
λ_{ci}	Constant common-cause failure rate of the robot-safety system in state i; for i = 0,1,2,…, (m+1)n-2.
μ_{sk}	Constant repair rate of the safety unit in state i=kn+q; for k = 1,2,…,m and q=0, 1,…, n-1.
μ_{ri}	Constant repair rate of the robot in state i; for i = 1,2,…,n-1.
Δx	Finite repair time interval.
$\mu_j(x)$	Time-dependent repair rate when the failed robot-safety system is in state j and has an elapsed repair time of x; for j=(m+1)n, (m+1)n+1,…, (m+1)(n+1).
$p_j(x,t)\Delta x$	The probability that at time t, the failed robot-safety system is in state j and the elapsed repair time lies in the interval [x,x+Δx]; for j=(m+1)n, (m+1)n+1,…, (m+1)(n+1).
pdf	Probability density function.
$z_j(x)$	pdf of repair time when the failed robot-safety system is in state j and has an elapsed time of x; for j=(m+1)n, (m+1)n+1,…, (m+1)(n+1).
$P_i(t)$	Probability that the robot-safety system is in state i at time t; for i = 0,1,…, (m+1)n-1.
$P_j(t)$	Probability that the robot-safety system is in state j at time t; for j=(m+1)n, (m+1)n+1,…, (m+1)(n+1).
s	Laplace transform variable.
$P_i(s)$	Laplace transform of the probability that the robot-safety system is in state i; for i = 0,1,…, (m+1)n-1.
$P_j(s)$	Laplace transform of the probability that the robot-safety system is in state j; for j=(m+1)n, (m+1)n+1,…, (m+1)(n+1).
$AV_{rs}(s)$	Laplace transform of the robot-safety system availability when the robot-safety system working with at least one safety unit.
$AV_r(s)$	Laplace transform of the robot-safety system availability when the robot-safety system working with or without the safety unit(s).
$AV_{rs}(t)$	Robot-safety system time-dependent availability when the robot-safety system working with at least one safety unit.
$AV_r(t)$	Robot-safety system time-dependent availability when the robot-safety system working with or without the safety unit(s).

3. Analysis

Using the supplementary method (Gaver, 1963; Grag, 1963), the system of Equations associated with the Figure 2 model can be expressed as follows:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_{r1} P_1(t) + \mu_{s1} P_n(t) + \sum_{j=(m+1)n}^{(m+1)(n+1)} \int_0^\infty P_j(x,t)\mu_j(x)dx \qquad (1)$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = (n-i+1)\lambda_r P_{i-1}(t) + \mu_{ri+1} P_{i+1}(t) + \mu_{s1} P_{n+i}(t) \tag{2}$$

$$(\,for \quad i = 1,2,\ldots,n-2)$$

$$\frac{dP_{n-1}(t)}{dt} + a_{n-1} P_{n-1}(t) = 2\lambda_r P_{n-2}(t) + \mu_{s1} P_{2n-1}(t) \qquad (\,for \quad i = n-1) \tag{3}$$

$$\frac{dP_{kn}(t)}{dt} + a_{kn} P_{kn}(t) = (m-k+1)\lambda_s P_{kn-n}(t) + \mu_{sk+1} P_{kn+n}(t) \tag{4}$$

$$(\,for \quad k = 1,2,\ldots,m-1)$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = [(k+1)n-i+1]\lambda_r P_{i-1}(t) + (m-k+1)\lambda_s P_{i-n}(t) + \mu_{sk+1} P_{i+n}(t) \tag{5}$$

$$(\,for \quad \begin{matrix} i = kn+1, kn+2, \ldots, (k+1)n-2 \\ k = 1,2,\ldots,m-1 \end{matrix})$$

$$\frac{dP_{(k+1)n-1}(t)}{dt} + a_{(k+1)n-1} P_{(k+1)n-1}(t) = 2\lambda_r P_{(k+1)n-2}(t) + (m-k+1)\lambda_s P_{kn-1}(t) + \mu_{sk+1} P_{(k+2)n-1}(t) \tag{6}$$

$$(\,for \quad k = 1,2,\ldots,m-1)$$

$$\frac{dP_{mn}(t)}{dt} + a_{mn} P_{mn}(t) = \lambda_s P_{(m-1)n}(t) \qquad (\,for \quad i = mn) \tag{7}$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = [(m+1)n-i+1]\lambda_r P_{i-1}(t) + \lambda_s P_{i-n}(t) \tag{8}$$

$$(\,for \quad i = mn+1, mn+2, \ldots, (m+1)n-2)$$

$$\frac{dP_{(m+1)n-1}(t)}{dt} + a_{(m+1)n-1} P_{(m+1)n-1}(t) = 2\lambda_r P_{(m+1)n-2}(t) + \lambda_s P_{mn-1}(t) \tag{9}$$

$$(\,for \quad i = (m+1)n-1)$$

where

$$a_0 = n\lambda_r + \lambda_{c0} + m\lambda_s$$

$$a_i = (n-i)\lambda_r + \lambda_{ci} + m\lambda_s + \mu_{ri} \qquad (\,for \quad i = 1,2,\ldots,n-2)$$

$$a_{n-1} = \lambda_r + \lambda_{cn-1} + m\lambda_s + \mu_{rn-1} \qquad (\,for \quad i = n-1)$$

$$a_{kn} = n\lambda_r + \lambda_{ckn} + (m-k)\lambda_s + \mu_{sk} \qquad (\,for \quad k = 1,2,\ldots,m-1)$$

$$a_i = [(k+1)n-i]\lambda_r + \lambda_{ci} + (m-k)\lambda_s + \mu_{sk}$$

$$(for \quad \begin{matrix} i = kn+1, kn+2,..., (k+1)n-2 \\ k = 1,2,..., m-1 \end{matrix})$$

$$a_{(k+1)n-1} = \lambda_r + \lambda_{c(k+1)n-1} + (m-k)\lambda_s + \mu_{sk} \qquad (for \quad k = 1,2,..., m-1)$$

$$a_{mn} = n\lambda_r + \lambda_{cmn} + \mu_{sm} \qquad (for \quad i = mn)$$

$$a_i = [(m+1)n - i]\lambda_r + \lambda_{ci} + \mu_{sm}$$

$$(for \quad i = mn+1, mn+2,..., (m+1)n-2)$$

$$a_{(m+1)n-1} = \lambda_r + \mu_{sm} \qquad (for \quad i = (m+1)n-1)$$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} + \mu_j(x)P_j(x,t) = 0 \tag{10}$$

$$(for \quad j = (m+1)n, (m+1)n+1,..., (m+1)(n+1))$$

The associated boundary conditions are as follows:

$$P_j(0,t) = \lambda_r P_{[m+1-j+(m+1)n]n-1}(t) \tag{11}$$

$$(for \quad j = (m+1)n, (m+1)n+1,..., (m+1)n+m)$$

$$P_{(m+1)(n+1)}(0,t) = \sum_{i=0}^{(m+1)n-2} \lambda_{ci} P_i(t) \tag{12}$$

At time t=0, $P_0(0)=1$, and all other initial state probabilities are equal to zero.

Unfortunately, it is very difficult to obtain general formulas for robot-safety system reliability and availability using Equations (1)-(12). However, for special values of n and m, Equations (1)-(12) can be solved.

Setting $\mu_{ri}=0$ (for i = 1,2,…,n-1) in Figure 2, which means robots are irreparable at the operable state of the robot-safety system, generalized expressions are developed. Thus, Equations (1)-(3) become:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_{s1} P_n(t) + \sum_{j=(m+1)n}^{(m+1)(n+1)} \int_0^\infty P_j(x,t)\mu_j(x)dx \tag{13}$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = (n-i+1)\lambda_r P_{i-1}(t) + \mu_{s1} P_{n+i}(t) \tag{14}$$

$$(for \quad i = 1,2,..., n-2)$$

$$\frac{dP_{n-1}(t)}{dt} + a_{n-1} P_{n-1}(t) = 2\lambda_r P_{n-2}(t) + \mu_{s1} P_{2n-1}(t) \qquad (for \quad i = n-1) \tag{15}$$

where

$$a_0 = n\lambda_r + \lambda_{c0} + m\lambda_s$$

$$a_i = (n-i)\lambda_r + \lambda_{ci} + m\lambda_s \qquad (for \quad i = 1,2,..., n-2)$$

$$a_{n-1} = \lambda_r + \lambda_{cn-1} + m\lambda_s \qquad (for \quad i = n-1)$$

Equations (4)-(12) remain same.

4. State Probabilities and System Availability *(i.e.,* $\mu ri = 0$*, for i = 1 ... n-1)*

Using Laplace Transform technique and the initial conditions in Equations (4) – (15), we get

$$(s+a_0)P_0(s) = 1 + \mu_{s1}P_n(s) + \sum_{j=(m+1)n}^{(m+1)(n+1)} \int_0^\infty P_j(x,s)\mu_j(x)dx \tag{16}$$

$$(s+a_i)P_i(s) = (n-i+1)\lambda_r P_{i-1}(s) + \mu_{s1}P_{n+i}(s) \tag{17}$$
$$(for \quad i = 1,2,...,n-2)$$

$$(s+a_{n-1})P_{n-1}(s) = 2\lambda_r P_{n-2}(s) + \mu_{s1}P_{2n-1}(s) \qquad (for \quad i = n-1) \tag{18}$$

$$(s+a_{kn})P_{kn}(s) = (m-k+1)\lambda_s P_{kn-n}(s) + \mu_{sk+1}P_{kn+n}(s) \tag{19}$$
$$(for \quad k = 1,2,...,m-1)$$

$$\begin{aligned}(s+a_i)P_i(s) = {} & [(k+1)n-i+1]\lambda_r P_{i-1}(s) + (m-k+1)\lambda_s P_{i-n}(s) \\ & + \mu_{sk+1}P_{i+n}(s)\end{aligned} \tag{20}$$
$$\left(for \quad \begin{matrix} i = kn+1, kn+2,...,(k+1)n-2 \\ k = 1,2,...,m-1 \end{matrix} \right)$$

$$\begin{aligned}(s+a_{(k+1)n-1})P_{(k+1)n-1}(s) = {} & 2\lambda_r P_{(k+1)n-2}(s) + (m-k+1)\lambda_s P_{kn-1}(s) \\ & + \mu_{sk+1}P_{(k+2)n-1}(s)\end{aligned} \tag{21}$$
$$(for \quad k = 1,2,...,m-1)$$

$$(s+a_{mn})P_{mn}(s) = \lambda_s P_{(m-1)n}(s) \qquad (for \quad i = mn) \tag{22}$$

$$(s+a_i)P_i(s) = [(m+1)n-i+1]\lambda_r P_{i-1}(s) + \lambda_s P_{i-n}(s) \tag{23}$$
$$(for \quad i = mn+1, mn+2,...,(m+1)n-2)$$

$$(s+a_{(m+1)n-1})P_{(m+1)n-1}(s) = 2\lambda_r P_{(m+1)n-2}(s) + \lambda_s P_{mn-1}(s) \tag{24}$$

$$sP_j(x,s) + \frac{\partial P_j(x,s)}{\partial x} + \mu_j(x)P_j(x,s) = 0 \tag{25}$$
$$(for \quad j = (m+1)n, (m+1)n+1,...,(m+1)(n+1))$$

and the boundary conditions:

$$P_j(0,s) = \lambda_r P_{[m+1-j+(m+1)n]n-1}(s) \tag{26}$$
$$(for \quad j = (m+1)n, (m+1)n+1,...,(m+1)n+m)$$

$$P_{(m+1)(n+1)}(0,s) = \sum_{i=0}^{(m+1)n-2} \lambda_{ci} P_i(s) \tag{27}$$

Solving differential Equation (25), we get the following expression:

$$P_j(x,s) = P_j(0,s)e^{-sx}\exp[-\int_0^x \mu_j(\delta)d\delta] \tag{28}$$

$$(for \quad j=(m+1)n,(m+1)n+1,...,(m+1)(n+1))$$

Since,

$$P_j(s) = \int_0^\infty P_j(x,s)dx \qquad (for \quad j=(m+1)n,(m+1)n+1,...,(m+1)(n+1)) \tag{29}$$

and together with Equation (28), we get

$$P_j(s) = P_j(0,s)\frac{1-Z_j(s)}{s} \tag{30}$$

$$(for \quad j=(m+1)n,(m+1)n+1,...,(m+1)(n+1))$$

where

$$\frac{1-Z_j(s)}{s} = P_j(0,s)\int_0^\infty e^{-sx}\exp[-\int_0^x \mu_j(\delta)d\delta]dx \tag{31}$$

$$(for \quad j=(m+1)n,(m+1)n+1,...,(m+1)(n+1))$$

or

$$Z_j(s) = \int_0^\infty e^{-sx} z_j(x)dx \tag{32}$$

$$(for \quad j=(m+1)n,(m+1)n+1,...,(m+1)(n+1))$$

$$z_j(x) = \exp[-\int_0^x \mu_j(\delta)d\delta]\mu_j(x)$$

where $z_j(x)$ is the failed robot-safety system repair time probability density function.

Let i=kn+q (for k=0, 1,…, m, and q=0, 1,…, n-1) and it denotes the i^{th} state of the robot-safety system, where k and q are the number of failed safety units and failed robots, respectively.

Simplifying Equations (17) – (18), (20) – (21), and (23) – (24), we get

$$\begin{aligned} P_{kn+q}(s) = & \sum_{g=k}^{m} \frac{P_{gn+q-1}(s)}{\mu_{sk}[(n-q+1)\lambda_r]^{g-k}} \prod_{l=k}^{g} K_{r[q]l} \\ & + \sum_{h=0}^{k-1} (\prod_{f=h+1}^{k} U_{r[q]f})(\sum_{g=h}^{m} \frac{P_{gn+q-1}(s)}{\mu_{sh}[(n-q+1)\lambda_r]^{g-h}} \prod_{l=h}^{g} K_{r[q]l}) \end{aligned} \tag{33}$$

$$(for \quad i=kn+q, \quad \begin{matrix} k=0,1,2,...,m \\ q=1,2,3,...,n-1 \end{matrix})$$

where

$$K_{r[q]m} = \frac{(n-q+1)\lambda_r \mu_{sm}}{s+a_{mn+q}} \qquad (for \quad q=1,2,...,n-1)$$

$$U_{r[q]m} = \frac{\lambda_s}{s + a_{mn+q}} \qquad (for \quad q = 1,2,...,n-1)$$

$$K_{r[q]k} = \frac{(n-q+1)\lambda_r \mu_{sk}}{s + a_{kn+q} - \mu_{sk+1} U_{r[q]k+1}} \qquad (for \quad \begin{matrix} k = 1,2,...,m-1 \\ q = 1,2,...,n-1 \end{matrix})$$

$$U_{r[q]k} = \frac{(m-k+1)\lambda_s}{s + a_{kn+q} - \mu_{sk+1} U_{r[q]k+1}} \qquad (for \quad \begin{matrix} k = 1,2,...,m-1 \\ q = 1,2,...,n-1 \end{matrix})$$

$$\mu_{s0} = 1$$

$$K_{r[q]0} = \frac{(n-q+1)\lambda_r \mu_{s0}}{s + a_q - \mu_{s1} U_{r[q]1}} \qquad (for \quad q = 1,2,...,n-1)$$

Similarly, simplifying Equations (19) and (22), we obtain

$$P_{kn}(s) = \frac{K_{sk}}{\mu_{sk}} P_{(k-1)n}(s) = \prod_{q=1}^{k} \frac{K_{sq}}{\mu_{sq}} P_0(s) \qquad (for \quad \begin{matrix} k = 1,2,...,m \\ q = 0 \end{matrix}) \tag{34}$$

where

$$K_{sm} = \frac{\lambda_s \mu_{sm}}{s + a_{mn}} \qquad (for \quad k = m)$$

$$K_{sk} = \frac{(m-k+1)\lambda_s \mu_{sk}}{s + a_{kn} - K_{sk+1}} \qquad (for \quad k = 1,2,...,m-1)$$

From Equations (33) and (34), the Laplace transform of the ith state probability can be expressed as

$$P_i(s) = Y_i P_0(s) \qquad (for \quad i = kn + q, \quad \begin{matrix} k = 0,1,2,...,m \\ q = 0,1,2,...,n-1 \end{matrix}) \tag{35}$$

where Y_i is the function of the Laplace transform variable, s.

$$Y_0 = 1$$

$$Y_{kn} = \prod_{q=1}^{k} \frac{K_{sq}}{\mu_{sq}} \qquad (for \quad \begin{matrix} k = 1,2,...,m \\ q = 0 \end{matrix}) \tag{36}$$

$$\begin{aligned} Y_{kn+q} = & \sum_{g=k}^{m} \frac{Y_{gn+q-1}}{\mu_{sk} [(n-q+1)\lambda_r]^{g-k}} \prod_{l=k}^{g} K_{r[q]l} \\ & + \sum_{h=0}^{k-1} \Big(\prod_{f=h+1}^{k} U_{r[q]f} \Big) \Big(\sum_{g=h}^{m} \frac{Y_{gn+q-1}}{\mu_{sh} [(n-q+1)\lambda_r]^{g-h}} \prod_{l=h}^{g} K_{r[q]l} \Big) \\ & (for \quad i = kn + q, \quad \begin{matrix} k = 0,1,2,...,m \\ q = 1,2,3,...,n-1 \end{matrix}) \end{aligned} \tag{37}$$

The Laplace transforms of the probabilities of all the system states add up to 1/s, i.e.,

$$\sum_{i=0}^{(m+1)n-1} P_i(s) + \sum_{j=(m+1)n}^{(m+1)(n+1)} P_j(s) = \frac{1}{s} \tag{38}$$

Solving Equations (33)-(38), we get

$$P_0(s) = [s(1 + \sum_{i=1}^{(m+1)n-1} Y_i + \sum_{j=(m+1)n}^{(m+1)(n+1)} a_j \frac{1 - Z_j(s)}{s})]^{-1} = \frac{1}{H} \tag{39}$$

$$P_i(s) = \frac{Y_i}{H} \qquad (for \quad i = kn + q, \quad \begin{matrix} k = 0,1,2,...,m \\ q = 0,1,2,...,n-1 \end{matrix}) \tag{40}$$

$$P_j(s) = \frac{a_j[1 - Z_j(s)]}{sH} \tag{41}$$

$$(for \quad j = (m+1)n, (m+1)n+1, ..., (m+1)(n+1))$$

where

$$a_j = \lambda_r Y_{[m+1-j+(m+1)n]n-1}$$

$$(for \quad j = (m+1)n, (m+1)n+1, ..., (m+1)n+m)$$

$$a_{(m+1)(n+1)} = \lambda_{c0} + \sum_{i=1}^{(m+1)n-2} \lambda_{ci} Y_i$$

$$H = s(1 + \sum_{i=1}^{(m+1)n-1} Y_i + \sum_{j=(m+1)n}^{(m+1)(n+1)} a_j \frac{1 - Z_j(s)}{s})$$

Thus, the Laplace transform of the robot-safety system availability with at least one working safety unit is

$$AV_{rs}(s) = \sum_{i=0}^{mn-1} P_i(s) = \frac{1 + \sum_{i=1}^{mn-1} Y_i}{H} \tag{42}$$

Similarly, the Laplace transform of the robot-safety system availability with or without working safety units is given by

$$AV_r(s) = \sum_{i=0}^{(m+1)n-1} P_i(s) = \frac{1 + \sum_{i=1}^{(m+1)n-1} Y_i}{H} \tag{43}$$

Substituting the Laplace transform of $z_j(x)$ for different repair time distributions into Equations (42) and (43), and taking the inverse Laplace transforms of the resulting equations, we can get the time-dependent system availabilities, $AV_{rs}(t)$ and $AV_r(t)$.

References

[1]. Dhillon, B S. (1983), *Reliability Engineering in Systems Design and Operation*, Van Nostrand Reinhold, New York.

[2]. Dhillon, B S. (1991), *Robot Reliability and Safety*, Springer-Verlag, New York.

[3]. Dhillon, B S. (1999), *Design Reliability: Fundamentals and Applications*, CRC Press, Boca Raton, Florida.

[4]. Gaver, D.P. (1963), "Time to Failure and Availability of Paralleled Systems with Repair", *IEEE Transactions on Reliability,* Vol. 12, pp.30-8.

[5]. Grag, R.C. (1963), "Dependability of a Complex System Having Two Types of Components", *IEEE Transactions on Reliability,* Vol. 12, pp.11-15.

[6]. Nagamachi, M. (1988), "Ten Fatal Accidents due to Robots in Japan", in Karwowski, H.R. and Parsaei, M.R.(Eds), *Ergonomics of Hybird Automated Systems I,* Elsevier, Amsterdam, pp.391-396.

[7]. Nicolaisen, P. (1987), "Safety Problems Related to Robots", *Robotics,* Vol. 3, pp.205-211.

[8]. Ramirez, C.A. (1985), "Safety of Robot", in Nof, S.Y. (Ed.), *Handbook of Industrial Robots,* Wiley, New York, NY, pp. 131-48.

[9]. Solem, E. (1987), "Social and Economic Factors of Robotics Systems", *Proceedings of the 1987 Military Robotics Association*, Victoria, B.C., Canada, pp.83-94.

[10]. United Nations Economic Commission for Europe (1998), *World Robotics 1998-Statistics, Market Analysis, Forecasts, Case Studies and Profitability of Robot Investment,* United Nations, New York.

[11]. United Nations Economic Commission for Europe (2002), *World Robotics 2002-Statistics, Market Analysis, Forecasts, Case Studies and Profitability of Robot Investment,* United Nations, New York.