Availability Assessment of Multi-State System by Hybrid Universal Generating Function and Probability Intervals

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Abstract: In this paper a non-repairable multi-state system with imprecise probabilities and performance rates are taken, representing imprecise probabilities by interval valued probabilities. These intervals are evaluated by computing bound of interval valued ordinary differential equation of the system. For imprecise performance rates random fuzzy variables are introduced. The proposed method is based on hybrid universal generating function (universal generating function representation of random fuzzy variable) and probability intervals to incorporate the uncertainty problem in availability assessment of the system. Finally, availability p-boxes of the system have been evaluated along with a numerical example.

Keywords: Multi-State System, Universal Generating Function, Random Fuzzy Variable, Probability intervals, P-boxes, Hybrid Universal Generating Function, System Availability Assessment

Notations

\( x_{j,k} \) \( j^{th} \) variable in \( k^{th} \) state
\( N \) number of components
\( N_{j} \) highest state of \( j^{th} \) component
\( [p_{i,k,l}] \) probability interval
\( \lambda_{i,k} \) failure rate of \( j^{th} \) component in \( i^{th} \) state from one state to lower state \( k^{th} \)
\( P_{i,j} \) probability of \( j^{th} \) component in \( i^{th} \) state
\( A \) availability of system
\( A_{\alpha} \) lower bound of availability p-box at \( \alpha \) level
\( A_{\alpha} \) upper bound of availability p-box at \( \alpha \) level
\( F \) cumulative distribution function
\( F_{\alpha} \) element of cumulative distribution function
\( [E_{\alpha}, F_{\alpha}] \) set of random fuzzy variables at \( \alpha \) cut level (where \( E_{\alpha} \) lower bound, \( F_{\alpha} \) upper bound)

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\[ \tilde{X}_j = [x_{ja}, \bar{x}_{ja}] \]  

fuzzy variable at \( \alpha \) cut level

\[ \tilde{X}^* = (\tilde{X}_0, \tilde{X}_1, \ldots, \tilde{X}_j) \]  

random fuzzy variable

\[ [\lambda_{ik}, \bar{\lambda}_{ik}] \]  

transition rate interval

\( w \)  

performance demand

\( b_{ja} = \tilde{X}_j - w \)  

system adequacy level at state \( j \)

\[ [p_j] = [p_j', \bar{p}_j] \]  

probability interval

1. Introduction

A system is a set of interacting and interdependent components which work together as a network. Basically, there are two types of systems, namely binary state system (BSS) and multi-state system (MSS). In a BSS it is assumed that the system and its components will be in two possible states i.e. working or failed, whereas in a multi-state system a system and its components works with finite number of performance rates or will be in more than two possible states. Reliability analysis of binary system is a foundation for mathematical treatment of reliability theory. Many problems related to binary state system reliability has been solved by several authors ([9, 17]). But BSS approach fails to describe the condition when system has more than two states. MSS models clearly describe the system state distribution and its gradual development. It is more than obvious to reliability engineers that MSS reliability models provide tremendous opportunity to researchers because of its flexibility. This is one of the reasons due to which now a days it is used more popularly in different industries such as transportation system, power generating system, computer generation system etc. There are two types of MSS’s namely repairable and non-repairable system. Non-repairable systems would be one shot type systems i.e. light bulb and pacemaker, transistor, rocket motor etc. Many researchers have done lots of work in the field of non-repairable multi-state systems [26] with different approaches. Upshot of the above points is that the MSS model more popular in comparison to BSS model. The BSS theory have replaced with theory of MSS for multi-state components by many researchers [7, 19]. As far as computation of MSS reliability are concerned there exist, different approaches to estimate like structure functions approach, stochastic process approach, universal generating function (UGF) approach and Monte Carlo simulation technique [14]. Out of these, UGF [10] is one of the widely used approaches to evaluate the reliability of the system. Levitin and Lisnianski have done their work in the field of earlier discussed MSS reliability analysis and optimization along with computation efficiency of UGF [2, 8, 9]. Many researchers have done work in the area of availability assessment and improvement of availability [13, 15] in various types of engineering systems with different approaches.

It is increasingly evident that some MSS have more complexity in their modeling and analysis process. These complexities may be due to the ill knowledge about the data or
other reasons which leads to uncertainty in the data. In reliability assessment uncertainty is an important and unavoidable factor which directly affects the result of design and assessment process. There are different types of uncertainties, two of them are: aleatory and epistemic uncertainties. The aleatory uncertainty arises due to the natural stochasticity or variability whereas the epistemic uncertainty arises due the lack of knowledge. There are many methods introduced in the past to improve reliability of the system under uncertainty and many models which are approached for availability/reliability assessment of the system [3, 24]. As of now authors [6, 12] developed uncertainty models where epistemic uncertainty is modeled by interval approach, belief function theory, possibility theory and fuzzy set theory. It is hard fact that the uncertainty may exist in many ways in the system. The main tools to represent uncertainties are parameter of probability distribution and imprecise probabilities. The problem of imprecise probabilities reduced by introducing interval valued UGF [5]. To solve the identified uncertainty problem the authors [20] have done their work on ill known probability and transition rates for MSS considering epistemic uncertainty. Further researchers [18, 21] have proposed different methods like joint uncertainty propagation to deal with both aleatory and epistemic uncertainties. Recently hybrid universal generating function (HUGF) approach for joint uncertainty (aleatory and epistemic) has been proposed in the article [25]. It is worth mentioning here that HUGF is a relation between traditional UGF and random fuzzy variable (RFV). HUGF is an extended form of UGF. Randomness and fuzziness are two important sources of uncertainty and they are complimentary to each other. The relationship between randomness and fuzziness is expressed by RFVs. The concept of RFV is first introduced by Zadeh. Later this concept is extended [1] and introduced the RFV as a tool to jointly express two types of uncertainties i.e. epistemic and aleatory. There after RFV have been extended for hybrid uncertainty propagation in the field of risk analysis [4, 11].

As mentioned earlier uncertainty plays a dominant role in the failure data as well as in parameter of probability distribution in reliability analysis. Our main objective in present study is to address the problem faced by the reliability engineers in availability/reliability assessment of the non-reparable MSS when they have following two concerns: (i) when non-repairable multi-stage system components has interval valued transition rate/failure rate (i.e. when they have uncertainty in transition rate), (ii) when components state have probability intervals corresponding to RFV performance level. Keeping above points in view plans are afoot to study ill known probabilities, transition rates and parameters of probability distributions (performance rates) of non-repairable system components and proposed a method for availability analysis of the system with the help of HUGF, probability intervals and p-box unlike done in past. It is worth mentioning that the HUGF representation of probability intervals is named as interval valued HUGF. In this study, probability intervals of differential equation of the proposed system have been obtained with the help of interval valued transition rate. Ordinary differential equations for probabilities computed with the help of well-defined stochastic process. Finally, a method has been proposed to extract availability p-boxes of the system.

The present study is arranged in the following manner. In section 2, some basic definitions related to the study are taken. In section 3, method for finding probability intervals, proposed method for assessment by HUGF and probability intervals along with extraction of p-boxes has been discussed. A numerical method is taken in section 4 to demonstrate the proposed approach and its comparison with interval valued UGF.
2. Preliminaries

2.1 Universal Generating Function

Universal generating function is conceptualized by Ushakov in 1986 and extended by G. Levitin and A. Lisnianski. UGF is a form of moment generating function in a polynomial form to represent the probability mass function of variables. It can very effectively be applied to find reliability of MSS’s. It is also known that some basic properties of moment generating function preserved by $z$-transformation. The $z$-transformation of random variable represents its probability mass function in polynomial form. Let independent variable $X$ has $N$ possible values $(x_1, x_2, ..., x_N)$ and $p_j$ be the probability corresponding to the random variable $x_j$, where $j=1, 2, 3, ..., N$ and $Z$ is a variable then UGF of discrete random variable $X = (x_1, x_2, ..., x_N)$ is represented by a polynomial:

$$u(z) = \sum_{j=1}^{N} p_j z^{X=x_j}$$

Let us suppose a MSS with $X_j$ component and every component has $k$ ($k=1, 2, ..., N_j$) possible state denoted by $x_{jk}$ so probability distribution of each variable $x_{jk}$ ($j^{th}$ variable at $k^{th}$ state) is represented by $p_{jk}$. If the variable has $N_j$ possible states and $Z$ is a variable then the UGF of random variable $X_j$ is defined by

$$u_{X_j}(z) = \sum_{k=1}^{N_j} p_{jk} z^{X_{jk}}$$  \hspace{1cm} (1)$$

Combination of $n$ $u$-functions is defined by composition operator $\otimes_f$ where the properties of the composition operator strictly depend on properties of the function $f(X_1, X_2, ..., X_n)$. Therefore, the composition is

$$\otimes_f \left( \sum_{k_j=1}^{N_j} p_{jk_j} z^{X_{jk_j}} \right) = \sum_{k_1=1}^{N_1} \sum_{k_2=1}^{N_2} ... \sum_{k_n=1}^{N_n} \left( \prod_{j=1}^{n} p_{jk_j} z^{f(x_{jk_1}, ..., x_{jk_n})} \right)$$  \hspace{1cm} (2)$$

$u$-function of a system with $n$ components is denoted by $U(z)$ and defined as

$$U(z) = \otimes_f \left( u_{X_1}(z), u_{X_2}(z), ..., u_{X_n}(z) \right).$$  \hspace{1cm} (3)$$

2.2 Random Fuzzy Variable

**Definition** [21]: Let $F$ denote the set of all cumulative distribution functions (CDFs) defined on the real number set $[0, 1]$ and each element $F \in F$ is an onto function $\mathcal{F} : \mathbb{R} \rightarrow [0, 1]$ such that $F(x_1) \geq F(x_2)$ whenever $x_1 \geq x_2$. A RFV is a set of closed intervals, each characterized by a pair of functions from $F$:

$$H : [0, 1] \rightarrow \mathcal{F} \times \mathcal{F} : \alpha \rightarrow [\mathcal{F}_\alpha \mathcal{F}_\alpha]$$  \hspace{1cm} (4)$$
such as for $\alpha_1, \alpha_2 \in [0, 1]$ such as for $\alpha_1, \alpha_2 \in [0, 1]$, $\bar{F}_{\alpha_1} \geq \bar{F}_{\alpha_2} \geq \bar{F}_{\alpha_2}$ whenever $\alpha_1 < \alpha_2$, where $\alpha_1$ and $\alpha_2$ represent fuzzy membership values of $x$. Here $[\bar{F}_{\alpha_2}, \bar{F}_{\alpha_2}]$ is random fuzzy variable.

### 2.3 P-Boxes

**Definition** [22]: Suppose $\bar{F}$ and $\bar{F}$ are two nondecreasing functions from real line $R$ into $[0, 1]$ and $\bar{F}(x) \leq \bar{F}(x)$ for all $x \in R$. Let $[\bar{F}, \bar{F}]$ denote a set of nondecreasing function $F$ from reals into $[0, 1]$ such that $\bar{F} \leq F \leq \bar{F}$. If $\bar{F}$ and $\bar{F}$ are defined as imprecisely known probability distribution then $[\bar{F}, \bar{F}]$ is called p-box of random variable.

The concept of p-box is same as FRV. A method is proposed ([23]) to fix the $\alpha$ level then built the lower and upper probability bounds $[\bar{F}_\alpha(A), \bar{F}_\alpha(A)]$ for an event $A$. There are two cases: one at $\alpha = 0$ and another at $\alpha = 1$ to represent the p-boxes. The p-boxes at $\alpha = 0$ is $[\bar{F}_0(A), \bar{F}_0(A)]$ and at $\alpha = 1$ is $[\bar{F}_1(A), \bar{F}_1(A)]$.

### 2.4 Hybrid UGF

The UGF representation of RFV is named as HUGF. This concept is proposed in [25]. HUGF is an extension of the UGF in which random variable is replaced by random fuzzy variable in probability distribution. HUGF is an efficient method to deal with problem of uncertainty in any complex system.

**Definition:** A random fuzzy variable (RFV) defined on a finite set of fuzzy numbers $\pi$, $|\pi| = j + 1$. UGF of random fuzzy variable denoted by $u_{X^*}(z)$ and expressed as

$$u_{X^*}(z) = \sum_{j=0}^{j} p_j z^{X^*_j} = \sum_{j=0}^{j} p_j z^{[X^{\alpha_j}, X^{\alpha_j}]}$$

where $X^*$ is a Random fuzzy number, $z^*$ is a fuzzy variable and $[X^{\alpha_j}, X^{\alpha_j}]$ is $\alpha$-cut bound of fuzzy number.

### 3. System Modeling

#### 3.1 Probability Intervals

In this study, probability interval has been obtained by using the results of ordinary differential equations of systems with interval valued parameters. Suppose $C_i$ be the component of non-repairable multi-state system and $P_{ik}$ is probability of being in state $g_i$, where $j=1, 2... n$ denote number of components of the system and $i, k = 1, 2... k_j$ denote the system states. Let $\lambda_{ik}$ be degradation (transition) rate from one state $i$ to another lower state $k$ which is in the interval $[\lambda_{ik}, \lambda_{ik}]$. The characteristics of
component $C_i$ of non-repairable multi-state system are described by ordinary differential equation with the help of stochastic process bearing probability as a variables and degradation (transition) rate as parameter. In the considered system derivatives of system components probabilities are evaluated as

$$f_i'(p, \lambda, t) = \frac{dp_i'(t)}{dt} = \sum_{k=i+1}^{k_j} \lambda_{ki}^j p_k'(t) - p_i'(t) \sum_{k=i}^{i-1} \lambda_{ik}^j$$

(6)

where $\sum_{k=i+1}^{i-1} = 0$ when $i=0,1,2,...n$. 

Equation (6) is describing the behavior of system states. A method [16] has been used to find the probability bounds in equation (6). Lower and upper probability bounds are evaluated by substituting value of transition rates $\lambda_{ki}^j, \lambda_{ik}^j$ in differential equation (6). The same can be expressed as follows:

If $\frac{\partial f_i'}{\partial p_i'}(p, \lambda, t) \geq 0, \forall j, i, k$, $\forall \lambda_{ki}^j \in [\lambda_{ki}^j, \lambda_{ki}^j], \lambda_{ik}^j \in [\lambda_{ik}^j, \lambda_{ik}^j] k \neq i, t \geq t_0$

then lower bound of probability can be obtained as follows:

(i) If $\frac{\partial f_i'}{\partial \lambda_{ki}^j}(p, \lambda, t) \geq 0$ for all $t>0$ then substitute $\lambda_{ki}^j = \underline{\lambda}_{ki}^j$ in equation (6)

(ii) If $\frac{\partial f_i'}{\partial \lambda_{ik}^j}(p, \lambda, t) \leq 0$ for all $t>0$ then substitute $\lambda_{ik}^j = \overline{\lambda}_{ik}^j$ in equation (6).

With the help of above substitutions, we get

$$\frac{dp_i'(t)}{dt} = \sum_{k=i+1}^{k_j} \lambda_{ki}^j p_k'(t) - p_i'(t) \sum_{k=i}^{i-1} \lambda_{ik}^j$$

(7)

Similarly, upper bound of probability can be obtained as

(iii) If $\frac{\partial f_i'}{\partial \lambda_{ki}^j}(p, \lambda, t) \geq 0$ for all $t>0$ then substitute $\lambda_{ki}^j = \overline{\lambda}_{ki}^j$

(iv) if $\frac{\partial f_i'}{\partial \lambda_{ik}^j}(p, \lambda, t) \leq 0$ for all $t>0$ then substitute $\lambda_{ik}^j = \underline{\lambda}_{ik}^j$

Applying (iii) and (iv) in differential equation (6), we have

$$\frac{dp_i'(t)}{dt} = \sum_{k=i+1}^{k_j} \lambda_{ki}^j p_k'(t) - p_i'(t) \sum_{k=i}^{i-1} \lambda_{ik}^j$$

(8)

The probability bounds can be obtained after solving equation (7) for lower bound and equation (8) for upper bound by using Laplace-Stieltjes transform.
3.2 Interval Valued HUGF and Composition Operator

A single probability distribution corresponding to a state can’t properly represent the incompleteness in information. However this can be represented by probability intervals. In this study, we have extended the HUGF approach by using probability interval for representation of uncertainty in the system’s probability distribution. Probability interval at the state \( j \) is denoted by \( [p_j, \bar{p}_j] \) where \([p_j, \bar{p}_j] \). HUGF with interval valued probability is defined as follows

\[
u_{x^*, z} = \sum_{j=0}^{J}[p_j]z^{x_j} = \sum_{j=0}^{J}[p_j, \bar{p}_j]z^{x_j} \quad \text{[equation (9)]}
\]

3.2.1 Series Arrangement

![Fig.1: Series System](image)

**Preposition 1.** Let a system contains \( N \) components, connected in series arrangement then interval valued HUGF of the series system \( U_{ss}(z) \) is given by

\[
U_{ss}(z) = \sum_{\eta_1=0}^{R_1} \sum_{\eta_2=0}^{R_2} \ldots \sum_{\eta_N=0}^{R_N} \left[ \prod_{k=1}^{N} p_{k}^{\eta_k} \prod_{k=1}^{N} \bar{p}_{k}^{\eta_k} \right] z_{\min(x_1^\eta_1, x_2^\eta_2, \ldots, x_N^\eta_N)} \quad \text{[equation (10)]}
\]

where \((R_1+1),(R_2+1),\ldots,(R_N+1)\) possible states and corresponding probability intervals \([p_{\eta_1}^{\eta_1}, \bar{p}_{\eta_1}^{\eta_1}], [p_{\eta_2}^{\eta_2}, \bar{p}_{\eta_2}^{\eta_2}], \ldots, [p_{\eta_N}^{\eta_N}, \bar{p}_{\eta_N}^{\eta_N}]\) are associated with system components \(X_1^\eta, X_2^\eta, \ldots, X_N^\eta\).

**Proof.** Consider a series system consisting of \( N \) components then interval valued HUGF of the system (Fig.1) is

\[
U_{ss}(z) = u_{x_1^\eta_1, x_2^\eta_2, \ldots, x_N^\eta_N} (z) = u_{x_1^\eta_1} (z) \otimes u_{x_2^\eta_2} (z) \otimes u_{x_N^\eta_N} (z)
\]

\[
= \sum_{\eta_1=0}^{R_1} [p_{\eta_1}^{1}, \bar{p}_{\eta_1}^{1}] z_{\min(x_1^\eta_1, x_2^\eta_2, \ldots, x_N^\eta_N)} \otimes \sum_{\eta_2=0}^{R_2} [p_{\eta_2}^{2}, \bar{p}_{\eta_2}^{2}] z_{\min(x_2^\eta_2, x_3^\eta_3, \ldots, x_N^\eta_N)} \otimes \ldots \otimes \sum_{\eta_N=0}^{R_N} [p_{\eta_N}^{N}, \bar{p}_{\eta_N}^{N}] z_{\min(x_N^\eta_N)}
\]

(from equation (9))
3.2.2 Parallel Arrangement

Fig.2: Parallel System

**Preposition 2.** Let a system contains $N$ components arranged in parallel then interval valued HUGF of the parallel system is given by

$$U_{ps}(z) = \sum_{r_1=0}^{R_1} \sum_{r_2=0}^{R_2} \ldots \sum_{r_N=0}^{R_N} \left[ \prod_{k=1}^{N} \frac{p_{r_k}^k}{\prod_{k=1}^{N} \bar{p}_{r_k}^k} ! \right] z^{\min(x_{r_1}, x_{r_2}, \ldots, x_{r_N})} \tag{11}$$

**Proof.** Let $X_1^*, X_2^*, \ldots, X_N^*$ are $N$ components of a system arranged in parallel (Fig.2) with $R_1 + 1, R_2 + 1, \ldots, R_N + 1$ possible states and corresponding probability intervals $[p_{r_1}^1, \bar{p}_{r_1}^1], [p_{r_2}^2, \bar{p}_{r_2}^2], \ldots, [p_{r_N}^N, \bar{p}_{r_N}^N]$ respectively, then the interval valued HUGF of the parallel system ($U_{ps}(z)$) is given by

$$U_{ps}(z) = u_{X_1^*}(z) \oplus u_{X_2^*}(z) \ldots \oplus u_{X_N^*}(z)$$

$$= \sum_{r_1=0}^{R_1} \left[ p_{r_1}^1 \bar{p}_{r_1}^1 \right] z^x_{r_1} \oplus \sum_{r_2=0}^{R_2} \left[ p_{r_2}^2 \bar{p}_{r_2}^2 \right] z^x_{r_2} \ldots \oplus \sum_{r_N=0}^{R_N} \left[ p_{r_N}^N \bar{p}_{r_N}^N \right] z^x_{r_N}$$

(from equation (9))

$$= \sum_{r_1=0}^{R_1} \sum_{r_2=0}^{R_2} \ldots \sum_{r_N=0}^{R_N} \left[ \prod_{k=1}^{N} \frac{p_{r_k}^k}{\prod_{k=1}^{N} \bar{p}_{r_k}^k} ! \right] z^{\min(x_{r_1}, x_{r_2}, \ldots, x_{r_N})} \tag{from equation (9)}$$
3.2.3 For Series-Parallel Arrangement

**Proposition 3.** If $S_1, S_2, \ldots, S_N$ numbers of subcomponents of components of a system are connected in series and $N$ components are arranged in parallel then interval valued HUGF of the series-parallel system is given by

$$U_{ps}(z) = \sum_{r_1=0}^{R_1} \sum_{r_2=0}^{R_2} \ldots \sum_{r_N=0}^{R_N} \prod_{k=1}^{S_k} \prod_{r=1}^{r_k} P^r_{k} \prod_{r=1}^{r_k} P^r_{k}^{r_k} \nonumber$$

where $(R_1+1), (R_2+1), \ldots, (R_N+1)$ possible states and corresponding probability intervals $[P^1_{1r}, P^1_{r}], [P^2_{2r}, P^2_{r}], \ldots, [P^N_{N}, P^N_{r}]$ are associated with system components $X_1^*, X_2^*, \ldots, X^*_N$.

**Proof.** Let a system components be arranged in series as well as parallel as shown in Fig.3 then interval valued HUGF of the series-parallel system ($U_{ps}(z)$) can be obtained by

$$U_{ps}(z) = u_{1r}(z) \oplus u_{2r}(z) \oplus u_{3r}(z) \oplus \ldots \oplus u_{r_N}(z)$$

where $(R_1+1), (R_2+1), \ldots, (R_N+1)$ possible states and corresponding probability intervals $[P^1_{1r}, P^1_{r}], [P^2_{2r}, P^2_{r}], \ldots, [P^N_{N}, P^N_{r}]$ are associated with system components $X_1^*, X_2^*, \ldots, X^*_N$.

![Fig.3: Series-Parallel System](image)

(from equations (10) and (11))
3.2.4 For Parallel-Series Arrangement

Preposition 4. Let in a system $N$ components be connected in series and every series component has $S_1$, $S_2$, ..., $S_N$ number of subcomponents respectively, arranged in parallel then interval valued HUGF of the parallel-series system is obtained as

$$U_{pys}(z) = \sum_{\eta=0}^{R_1} \sum_{\tau_1=0}^{R_2} ... \sum_{\tau_N=0}^{R_N} \prod_{k=1}^{S_1} \prod_{k=1}^{S_2} \prod_{k=1}^{S_N} \left[ p^{[\eta]}_{\tau_1} \cdot \tilde{p}^{[\eta]}_{\tau_1}, \min(\tilde{p}^{[\eta]}_{\tau_1}), \max(\tilde{p}^{[\eta]}_{\tau_1}) \right] \in$$

(3)

where $(R_1+1), (R_2+1), ..., (R_N+1)$ possible states and corresponding probability intervals $[p^{[\eta]}_{\tau_1}, \tilde{p}^{[\eta]}_{\tau_1}], [p^{[\eta]}_{\tau_2}, \tilde{p}^{[\eta]}_{\tau_2}], ..., [p^{[\eta]}_{\tau_N}, \tilde{p}^{[\eta]}_{\tau_N}]$ are associated with system components $X_1^*, X_2^*, ..., X_N^*$.

Proof. If components of system arranged in parallel and series as depicted in Fig.4 then interval valued HUGF of the parallel-series system $(U_{pys}(z))$ is given by

$$U_{pys}(z) = u_{x_1}(z) \oplus u_{x_2}(z) \oplus ... \oplus u_{x_N}(z)$$

$$= (u_{x_1}(z) \oplus u_{x_2}(z) \oplus ... \oplus u_{x_N}(z)) \otimes (u_{x_1}(z) \oplus u_{x_2}(z) \oplus ... \oplus u_{x_N}(z)) \otimes ...$$

$$= \sum_{\eta=0}^{R_1} \sum_{\tau_1=0}^{R_2} ... \sum_{\tau_N=0}^{R_N} \prod_{k=1}^{S_1} \prod_{k=1}^{S_2} \prod_{k=1}^{S_N} \left[ p^{[\eta]}_{\tau_1} \cdot \tilde{p}^{[\eta]}_{\tau_1}, \min(\tilde{p}^{[\eta]}_{\tau_1}), \max(\tilde{p}^{[\eta]}_{\tau_1}) \right]$$

(from equations (9), (10) and (11))
\[ U_{q_j}(z) = \sum_{\eta_1=0}^{n_1} \ldots \sum_{\eta_m=0}^{n_m} \sum_{r_1=0}^{n_1} \ldots \sum_{r_m=0}^{n_m} [\prod_{k=1}^{m} p_{j_k}^i] \sum_{\eta_1=0}^{n_1} \ldots \sum_{\eta_m=0}^{n_m} \sum_{r_1=0}^{n_1} \ldots \sum_{r_m=0}^{n_m} \prod_{k=1}^{m} p_{j_k}^i \] 

\[ \text{max}(b_{j_1}, \ldots, b_{j_m}, \ldots, b_{j_1}, b_{j_2}, \ldots, b_{j_m}, \ldots, b_{j_1}, \ldots, b_{j_2}, \ldots, b_{j_m}) \] 

3.3 Extraction of System Availability P -Box from Interval Valued HUGF

In this study we are extending the method of article [25] by using probability intervals unlike crisp probabilities used by them.

To obtain the parameters for extracting p-boxes substitute the given $\alpha = 0,1$ into the fuzzy number expression. For evaluating the availability p-box at the given demand following method is proposed.

Let $b_{j_1} = [b_{j_1}, \bar{b}_{j_1}]$ and $\bar{X}_j = [\bar{x}_{j_1}, \bar{x}_{j_1}]$, where $b_{j_1} = \bar{X}_j - \omega$, $b_{j_1}$ is system adequacy level at state $j$ and $\omega$ is a performance demand of the system and $\bar{X}_j = [\bar{x}_{j_1}, \bar{x}_{j_1}]$ is $\alpha$-cut interval of fuzzy number.

If $\bar{b}_{j_1} \geq 0$ then $A_{av} = \sum_{b_{j_1} \geq 0} \frac{(p + \overline{p})(b_{j_1})}{2}$, where $j = 0, 1 \ldots$ \hspace{1cm} (14)

If $\bar{b}_{j_1} \leq 0$ then $A_{av} = \sum_{b_{j_1} \leq 0} \frac{(p + \overline{p})(b_{j_1})}{2}$, where $j = 0, 1 \ldots$ \hspace{1cm} (15)

Now with the help of equations (14) and (15) we get availability P-boxes $[A_{a_0}, \overline{A}_{a_0}]$ and $[A_{a_1}, \overline{A}_{a_1}]$ at $\alpha = 0, 1$.

If $\bar{b}_{j_1} < 0$ and $\bar{b}_{j_1} \geq 0$ then calculate $\alpha_L$ where $\bar{b}_{a_2} = 0$ and

$A_{av} = \int_0^{\alpha_L} \sum_{b_{j_1} \geq 0} \frac{(p + \overline{p})(b_{j_1})}{2} \, d\alpha$

For the computation of $A_{av}$, at particular state $j$ the following mutually exclusive conditions are identified:

i) $\bar{b}_{j_1} \geq 0$ for any $\alpha \in [0,1]$, then we have \[ \int_0^{\alpha} \frac{(p + \overline{p})(b_{j_1})}{2} \, d\alpha = \frac{(p + \overline{p})(b_{j_1})}{2} \]

because \( \frac{(p + \overline{p})(b_{j_1})}{2} \) is a constant for any $\alpha$.

ii) $\bar{b}_{j_1} < 0$ for every $\alpha \in [0,1]$

\[ \int_0^{\alpha} \frac{(p + \overline{p})(b_{j_1})}{2} \, d\alpha = 0 \]

iii) $\bar{b}_{j_1} < 0$ and $\bar{b}_{j_1} \geq 0$ then \[ \int_0^{\alpha} \frac{(p + \overline{p})(b_{j_1})}{2} \, d\alpha = (1 - \alpha_L) \times \frac{(p + \overline{p})(b_{j_1})}{2} \]

Similarly for $\overline{A}_{av}$ if $\bar{b}_{j_1} < 0$ and $\bar{b}_{j_1} \geq 0$ then calculate $\alpha_U$ where $\bar{b}_{a_U} = 0$ and

$\overline{A}_{av} = \int_0^{\alpha_U} \sum_{b_{j_1} \geq 0} \frac{(p + \overline{p})(b_{j_1})}{2} \, d\alpha$
For the computation of $A_{av}$, at particular state $j$ the following conditions are identified:

i) $\bar{b}_{j_{00}} \geq 0$ for any $\alpha_0 \in [0,1]$

$$\int_0^1 \frac{(p + \bar{p})(\bar{b}_{j_{00}})}{2} d\alpha = \frac{(p + \bar{p})(\bar{b}_{j_{00}})}{2}$$

where $\frac{(p + \bar{p})(\bar{b}_{j_{00}})}{2}$ is a constant for any $\alpha$.

ii) $\bar{b}_{j_{0}} < 0$ for every $\alpha \in [0,1]$ then we have

$$\int_0^1 \frac{(p + \bar{p})(\bar{b}_{j_{0}})}{2} d\alpha = 0$$

iii) $\bar{b}_{j_{22}} < 0$ and $\bar{b}_{j_{10}} \geq 0$ then

$$\int_0^1 \frac{(p + \bar{p})(\bar{b}_{j_{22}})}{2} d\alpha = \alpha \times \frac{(p + \bar{p})(\bar{b}_{j_{10}})}{2}$$

4. Illustrative Example

Consider a non-repairable series-parallel system with three components as shown in Fig 5:

Let $C_1^*$, $C_2^*$ and $C_3^*$ be three components of a non-repairable series-parallel system. Let $C_1^*$ and $C_2^*$ each have three states and component $C_3^*$ has four states. Let the degradation rates of component $C_1^*$ are:

i) From state third to second state is $\lambda_{32} = [1.10^{-4},3.10^{-4}]$.

ii) Second state to first is $\lambda_{21} = [3.10^{-4},5.10^{-4}]$ and from third higher state to first state is $\lambda_{31} = [2.10^{-4},4.10^{-4}]$.

Similarly for component $C_2^*$ degradation rates from higher state to lower are

i) From third to second state is $\lambda_{32} = [2.10^{-4},3.10^{-4}]$

ii) Similarly from second to first and third to first states are $\lambda_{21} = [10^{-4},2.10^{-4}]$, $\lambda_{31} = [4.10^{-4},5.10^{-4}]$ respectively.

By applying equations (7) and (8), we get following equations for the considered system

$$\frac{dp_1(t)}{dt} = -(\tau_{32} + \tau_{31}) p_1(t)$$

(18)

$$\frac{dp_2(t)}{dt} = \lambda_{32} p_1(t) - \lambda_{21} p_2(t)$$

(19)

$$\frac{dp_3(t)}{dt} = \lambda_{31} p_2(t) + \lambda_{23} p_3(t)$$

(20)
Initial condition: \( p_j'(0) = 1, p_j''(0) = 0, p_j'''(0) = 0 \) where \( j = 1, 2 \).

Lower bound of the probabilities of components at a time \( t \) can be obtained with the help of equations (10)-(20)

\[
\frac{dp_j'(t)}{dt} = -(\Delta_{32} + \Delta_{31}) p_j'(t).
\]

\[
\frac{dp_j''(t)}{dt} = \lambda_{23} p_j''(t) - \Delta_{21} p_j''(t)
\]

\[
\frac{dp_j'''(t)}{dt} = \lambda_{31} p_j'''(t) + \lambda_{31} p_j''(t)
\]

Similarly we will get upper bounds of probabilities of components.

HUGF with probability interval of the system components are defined by equation (9) as follows

\[
u_{C_1}(z) = [p_1^1, p_1^2] z^{[1+\alpha,5-a]} + [p_2^1, p_2^2] z^{[2+\alpha,4-a]} + [p_3^1, p_3^2] z^{[3+\alpha,2-a]}
\]

\[
u_{C_2}(z) = [p_1^3, p_1^4] z^{[6+\alpha,8-a]} + [p_2^3, p_2^4] z^{[5+\alpha,7-a]} + [p_3^3, p_3^4] z^{[4+\alpha,6-a]} + [p_4^3, p_4^4] z^{[3+\alpha,5-a]}
\]

Now interval valued HUGF of the non-repairable system is computed with the help of equations (10) & (11) and expressed as:
Interval valued HUGF of non-reparable series-parallel system is evaluated with the help of equation (24) as

\[
U_{\text{sys}*}(z) = U_{c_1}(z) \otimes U_{c_2}(z) \otimes U_{c_3}(z)
\]

\[
= \left[ p_1 e_1^2 p_1, p_1 e_1^3 \right] z^{-\alpha(3+\alpha-a)[3+\alpha-a]} + \left[ p_1 e_1^2 p_1, p_1 e_1^3 \right] z^{-\alpha(3+\alpha-a)[3+\alpha-a]} + \left[ p_1 e_1^2 p_1, p_1 e_1^3 \right] z^{-\alpha(3+\alpha-a)[3+\alpha-a]} + \left[ p_1 e_1^2 p_1, p_1 e_1^3 \right] z^{-\alpha(3+\alpha-a)[3+\alpha-a]}
\]

\[
+ \left[ p_1 e_1^2 p_1, p_1 e_1^3 \right] z^{-\alpha(3+\alpha-a)[3+\alpha-a]} + \left[ p_1 e_1^2 p_1, p_1 e_1^3 \right] z^{-\alpha(3+\alpha-a)[3+\alpha-a]}
\]

\[
U_{\text{sys}*}(z) = (U_{c_1}(z) \otimes U_{c_2}(z) \otimes U_{c_3}(z))
\]

\[
= \left[ p_1 e_1^2 p_1, p_1 e_1^3 \right] z^{-\alpha(3+\alpha-a)[3+\alpha-a]} + \left[ p_1 e_1^2 p_1, p_1 e_1^3 \right] z^{-\alpha(3+\alpha-a)[3+\alpha-a]}
\]

\[
+ \left[ p_1 e_1^2 p_1, p_1 e_1^3 \right] z^{-\alpha(3+\alpha-a)[3+\alpha-a]} + \left[ p_1 e_1^2 p_1, p_1 e_1^3 \right] z^{-\alpha(3+\alpha-a)[3+\alpha-a]}
\]

\[
U_{\text{sys}*}(z) = (U_{c_1}(z) \otimes U_{c_2}(z) \otimes U_{c_3}(z))
\]

\[
= \left[ p_1 e_1^2 p_1, p_1 e_1^3 \right] z^{-\alpha(3+\alpha-a)[3+\alpha-a]} + \left[ p_1 e_1^2 p_1, p_1 e_1^3 \right] z^{-\alpha(3+\alpha-a)[3+\alpha-a]}
\]

\[
+ \left[ p_1 e_1^2 p_1, p_1 e_1^3 \right] z^{-\alpha(3+\alpha-a)[3+\alpha-a]} + \left[ p_1 e_1^2 p_1, p_1 e_1^3 \right] z^{-\alpha(3+\alpha-a)[3+\alpha-a]}
\]

\[
(24)
\]

### Table 1: Parameters of System

<table>
<thead>
<tr>
<th>Component</th>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
<th>Performance Rate</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1^*$</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>[2+\alpha,4-a]</td>
<td>[.99302, .99700]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[3+\alpha,5-a]</td>
<td>[.00099, .00299]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[a,2-a]</td>
<td>[.00199, .00400]</td>
</tr>
<tr>
<td>$C_2^*$</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>[9+\alpha,11-a]</td>
<td>[.99104, .99401]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[8+\alpha,10-a]</td>
<td>[.00198, .00298]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[7+\alpha,9-a]</td>
<td>[.00398, .00598]</td>
</tr>
<tr>
<td>$C_3^*$</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>[6+\alpha,8-a]</td>
<td>.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[5+\alpha,7-a]</td>
<td>.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[4+\alpha,6-a]</td>
<td>.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[1+\alpha,3-a]</td>
<td>.1</td>
</tr>
</tbody>
</table>

Interval valued HUGF of non-reparable series-parallel system is evaluated with the help of equation (24) as
If load demand is 5.75 then HUGF with interval probability of a non-repairable series-parallel system is given by:

\[
U_{P_{ja}}(z) = [0.00019, 0.00059] [3.25 + 2a, 7.25 - 2a] + [0.19839, 0.2018] [2.25 + 2a, 6.25 - 2a] + [0.39630, 0.40087] [1.25 + 2a, 5.25 - 2a] + [0.29739, 0.30078] [2.25 + 2a, 4.25 - 2a] + [0.00059, 0.00120] [-7.75 + 2a, 3.25 - 2a] + [0.00058, 0.00150] [-1.75 + 2a, 2.25 - 2a] + [0.00090, 0.00999] [-2.75 + 2a, 1.25 - 2a] + [0.00019, 0.00040] [-4.75 + 2a, -2.25 - 2a] 
\]

Table 2: Elements for Construction of P-Boxes

<table>
<thead>
<tr>
<th>Terms</th>
<th>$b_{ja}$</th>
<th>$b_{ja}$</th>
<th>$b_{ja}$</th>
<th>$b_{ja}$</th>
<th>$\alpha_L$</th>
<th>$\alpha_U$</th>
<th>Probability of $p$-Boxes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-4.75</td>
<td>-2.75</td>
<td>-2.5</td>
<td>-2.25</td>
<td>.00029</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-2.75</td>
<td>-2.75</td>
<td>1.25</td>
<td>-2.25</td>
<td>.625</td>
<td>.09949</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-1.75</td>
<td>.25</td>
<td>2.25</td>
<td>-2.25</td>
<td>.00104</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>.75</td>
<td>1.25</td>
<td>3.25</td>
<td>1.25</td>
<td>.375</td>
<td>.00089</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>.25</td>
<td>2.25</td>
<td>4.25</td>
<td>2.25</td>
<td>.29904</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.25</td>
<td>3.25</td>
<td>5.25</td>
<td>3.25</td>
<td>.39858</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>2.25</td>
<td>4.25</td>
<td>6.25</td>
<td>4.25</td>
<td>.19978</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>3.25</td>
<td>5.25</td>
<td>7.25</td>
<td>5.25</td>
<td>.00039</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now by using equations (14)-(17) the upper and lower bounds of system availability p-boxes can be computed as follows

\[
A = 0.29904 + 0.39858 + 0.19978 + 0.00039 = 0.89779 \\
A_1 = 0.00049 + 0.00104 + 0.00089 + 0.29904 + 0.39858 + 0.19978 + 0.00039 = 0.99921 \\
A_2 = 0.00104 + 0.00089 + 0.29904 + 0.39858 + 0.19978 + 0.00039 = 0.89972 \\
A_3 = 0.00104 + 0.00089 + 0.29904 + 0.39858 + 0.19978 + 0.00039 = 0.89972 \\
A_{av} = 0.125 \times 0.00104 + 0.625 \times 0.00089 + 0.29904 + 0.39858 + 0.19978 + 0.00039 = 0.8947625 \\
A_{av} = 0.625 \times 0.00949 + 0.00104 + 0.00089 + 0.29904 + 0.39858 + 0.19978 + 0.00039 = 0.96190125 \\
A_0 = [0.89779, 0.99921], A_{av} = [0.8947625, 0.96190125], A_1 = 0.89972
\]

**Particular case**

If performance rate of component is $g_j$ and probability interval is $[p_j, \bar{p}_j]$ corresponding performance rate then interval valued UGF is:

\[
U(z) = \sum_{j=0}^{J} [p_j] z^{\delta_j} = \sum_{j=0}^{J} [p_j, \bar{p}_j] z^{\delta_j} \tag{25}
\]
where performances \( g_j \) are crisp valued and probabilities \( [p_j, \bar{p}_j] \) are imprecise.

Let \( \alpha=0.5 \) and components of a series-parallel system (Fig 5) be \( C_1^*, C_2^* \) and \( C_3^* \) with performances \( (3, 4, 1), (10, 9, 8), (7, 6, 5, 2) \) respectively. Assume that the probabilities of corresponding to performances are \( ([.99302, .99700], [.00099, .00299], [.00199, .00490]), ([.99104, .99401], [.00198, .00298], [.00398, .00598]) \) and \( (2, 4, 3, 1) \). Further, using equation (25) interval valued UGF of the system components can be expressed as

**Table 3: Performance of Components w.r.t. \( \alpha \)**

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>Performance of ( C_1^* )</th>
<th>Performance of ( C_2^* )</th>
<th>Performance of ( C_3^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>[2.4] [3.5] [0.2]</td>
<td>[9.1] [9.10] [7.9]</td>
<td>[6.8] [5.7] [4.6] [1.3]</td>
</tr>
<tr>
<td>0.1</td>
<td>[2.1.3] [3.1] [4.9] [0.1]</td>
<td>[9.1] [9.10] [7.9]</td>
<td>[6.7, 1.9] [5.1, 6.9] [4.1, 5.9] [1.1, 2.9]</td>
</tr>
<tr>
<td>0.2</td>
<td>[2.23] [3.2] [4.8] [0.2]</td>
<td>[9.2] [9.20] [7.8]</td>
<td>[6.2, 7.8] [5.2, 6.8] [4.2, 5.8] [1.2, 2.8]</td>
</tr>
<tr>
<td>0.3</td>
<td>[3.33] [3.3] [3.4] [0.3]</td>
<td>[9.3] [9.30] [7.3]</td>
<td>[6.3, 7.3] [5.3, 6.7] [4.3, 5.7] [1.3, 2.7]</td>
</tr>
<tr>
<td>0.4</td>
<td>[4.34] [4.3] [4.7] [0.4]</td>
<td>[9.4] [9.40] [7.4]</td>
<td>[6.4, 7.4] [5.4, 6.4] [4.4, 5.4] [1.4, 2.4]</td>
</tr>
<tr>
<td>0.5</td>
<td>[5.35] [5.3] [5.4] [0.5]</td>
<td>[9.5] [9.50] [7.5]</td>
<td>[6.5, 7.5] [5.5, 6.5] [4.5, 5.5] [1.5, 2.5]</td>
</tr>
<tr>
<td>0.6</td>
<td>[6.36] [6.3] [6.4] [0.6]</td>
<td>[9.6] [9.60] [7.6]</td>
<td>[6.6, 7.6] [5.6, 6.6] [4.6, 5.6] [1.6, 2.6]</td>
</tr>
<tr>
<td>0.7</td>
<td>[7.37] [7.3] [7.4] [0.7]</td>
<td>[9.7] [9.70] [7.7]</td>
<td>[6.7, 7.7] [5.7, 6.7] [4.7, 5.7] [1.7, 2.7]</td>
</tr>
<tr>
<td>0.8</td>
<td>[8.38] [8.3] [8.2] [0.8]</td>
<td>[9.8] [9.80] [7.8]</td>
<td>[6.8, 7.8] [5.8, 6.8] [4.8, 5.8] [1.8, 2.8]</td>
</tr>
<tr>
<td>0.9</td>
<td>[9.89] [9.8] [9.1] [0.9]</td>
<td>[9.9] [9.90] [7.9]</td>
<td>[6.9, 7.9] [5.9, 6.9] [4.9, 5.9] [1.9, 2.9]</td>
</tr>
<tr>
<td>1.0</td>
<td>3 4 1 10 9 8 7 6 5 2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ u_{c_1}(z) = [p_1, \bar{p}_1]z + [p_1, \bar{p}_1]z^2 + [p_1, \bar{p}_1]z^3 \]

\[ u_{c_2}(z) = [p_2, \bar{p}_2]z + [p_2, \bar{p}_2]z^2 + [p_2, \bar{p}_2]z^3 \]

\[ u_{c_3}(z) = [p_3, \bar{p}_3]z + [p_3, \bar{p}_3]z^2 + [p_3, \bar{p}_3]z^3 \]

The interval valued UGF of the considered system is evaluated as

\[ \begin{align*}
U_{sys}(z) &= (u_{c_1}(z) \otimes u_{c_2}(z) \otimes u_{c_3}(z))
\end{align*} \]
With the help of equation (26) interval valued UGF of given system (Fig. 5) on load demand 5.75 can be evaluated as follows

\[
U_{sys}(z) = [0.00019, 0.00059]z^{5.25} + [0.19839, 0.20118]z^{4.25} + [0.39630, 0.40087]z^{3.25} +
\]

\[
[0.29739, 0.30078]z^{2.25} + [0.00059, 0.0120]z^{1.25} + [0.00058, 0.0150]z^{0.25}
\]

\[
+ [0.09900, 0.09999]z^{-0.75} + [0.00019, 0.00040]z^{-2.75}
\]

(27)

For the considered values, availability of system obtained to be [0.89344, 0.90612] by equation (27). Since there are infinite many points in every interval so we have to calculate for every one of them for the value of \( \alpha \in [0,1] \) which results into larger computations for availability analysis of any system having uncertainty in their performance. Hence it is not possible to compare the present study with the previously developed methods as such because it is an extension of all of earlier approaches. However in order to compare the proposed method to interval valued UGF we have calculated the performances intervals at \( \alpha = 0.5 \). Gradual development of UGF’s is shown in Table 4.

### Table 4: Different UGFs Corresponding to Different Environmental Conditions

| UGF | | If performance and corresponding probabilities are crisp value. | If system component state have imprecise probability. |
|-----|-------------------------------------|--------------------------------------------------|
| Interval Valued UGF | | | |
| HUGF | | | If system state performance have uncertain data. |
| HUGF with probability intervals | | If system state performance has uncertainty and corresponding probability is imprecise |

### 5. Conclusion

One of the major problem in the system reliability/availability analysis is to tackle uncertainty arises in the system modeling, data etc. as it is an important factor in reliability assessment. In this work, we have proposed an approach for uncertainty representation and utilization in availability assessment of the non-repairable MSS. Foundation of this method is HUGF approach and probability intervals unlike done in the past. In this study, availability/reliability p-boxes of the non-repairable MSS evaluated from interval valued HUGF. The proposed approach is efficient for assessment of availability/reliability of the MSS with imprecise probabilities and performance rates in non-repairable MSS. This method is simple and easy to calculate in more generalized uncertain environmental condition in comparison to other approaches as described in Table 4.

### Reference


[4]. C. Baudrit, D. Dubois, and D. Guyonnet. *Joint Propagation of Probabilistic and


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