Classical and Bayesian Inference on 3-Step Step-Stress Accelerated Life Test Plan for Weibull Model under Modified Progressive Type-I Censoring

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Abstract: In this paper, the optimum test plan and statistical inference of 3-step step-stress accelerated life tests (SSALT) under progressive Type-I censoring is studied. It is assumed that the lifetime of a test unit follow a Weibull distribution with mean lifetime of a unit is a log-quadratic function of stress level. The maximum likelihood and Bayesian method are used to obtain the point and interval estimates of the model parameters under progressive Type-I censoring. The Bayes estimates are obtained using Markov Chain Monte Carlo (MCMC) simulation based on Gibbs sampling. The optimum 3-step SSALT plan under progressive Type-I censoring is developed by minimizing asymptotic variance of the maximum likelihood estimators of log of mean life at the design stress. Finally, the numerical study is presented to illustrate the proposed study.

Keywords: Accelerated life test, Weibull distribution, Maximum likelihood estimation, Bayesian estimation, Progressive Type-I censoring.

1. Introduction

The testing of products under usual stress conditions usually take a long period of time, often in years or decades. Moreover, these experiments are often expensive, which makes it difficult or even impossible to obtain the failure information under usage conditions for such products. Therefore, accelerated life testing (ALT) is widely used to collect information about the lifetime of the products quickly and economically under more severe environment conditions. Censoring is frequently used in life-testing experiment and most commonly used censoring schemes are Type-I and Type-II censoring. But, these conventional censoring schemes have a disadvantage i.e., continuous monitoring up to termination of experiment. Recently, progressive censoring scheme (PC) has received considerable attention in the SSALT planning.

The main advantage of PC schemes is that it is possible to remove experimental units during the experiment, even if they do not fail. In literature of SSALT, the majority of the SSALT planning problems, commonly used variance optimality criteria to minimize the asymptotic variance of the MLEs of mean life or some percentile of the lifetime distribution at the normal operating conditions. Miller and Nelson [9] is the first propose the optimum simple SSALT plans by assuming exponential lifetimes and complete failure data. They also assumed that the mean life is a log-linear function of stress level and the cumulative exposure model is used for data analysis. Bai et al. [2] extended the results of Miller and Nelson [9] to the censoring case. Khamis and Higgins [7] proposed optimum 3-step step-stress plans for exponential distribution. Khamis [6] presented a generalized
optimum \( m \)-step SSALT design with \( k \) stress variables, assuming complete knowledge of the life-stress relationship with multiple stress variables. Gouno et al. [5] proposed an optimum \( m \)-step SSALT plans with equal test duration and investigated in detail the case of progressive Type-I censoring with a single stress variable, assuming exponential lifetime. Balakrishnan and Han [3] extended the Gouno et al. [5] model with a practical modification for small to moderate sample sizes. In all the above case, the authors assumed only log-linear relationship between life and stress level. For more recent studies regarding optimum SSALT plans, see, for example, Wu et al. [15], Ebrahem and Al-Masri [1] and Chandra et al. [4].

The Bayesian method has not been widely used to analyze the SSALT data. By applying the Bayesian approach to SSALT model, the statistical precision of the parameter inference can be improved. Van Dorp et al. [13] and Van Dorp and Mazzuchi [14] developed a general Bayesian inference model for SSALT on the basis of the exponentially distributed failure data. Lee and Pan [8] described the Bayesian inference model for simple SSALT when failure times at each stress are exponentially distributed with Type-II censoring. Sha and Pan [12] presented a Bayesian analysis for Weibull proportional hazard (PH) model used in simple SSALT. They point out that the PH model as compared to CE model provides more flexibility in fitting step-stress data. A MCMC algorithm with adaptive rejection sampling technique is used for posterior inference.

No researcher has paid any attention for analysis and test design of 3-step SSALT under quadratic life-stress relationship with modified progressive Type-I censoring schemes. However, it is motivated me to give some contribution in this direction. In this paper, a 3-step SSALT model for Weibull distribution under modified progressive Type-I censoring is considered. Also, it assumed that the relationship between life and stress is log-quadratic. The maximum likelihood (ML) method and Bayesian method are proposed for drawing inferences of all the related model parameters for the modified progressive Type-I censored data. The optimum plan under modified progressive Type-I censoring is developed with an optimization criteria based on minimizing asymptotic variance of the MLEs of the log of mean life at the design stress.

In the subsequent sections, model description and assumptions are discussed in section 2. Modified progressive Type-I censoring for small sample size is discussed in section 3. Section 4 described the maximum likelihood estimation and Fisher information matrix. The optimality criterion is given in section 5. A Bayesian method for parameter estimation is presented in section 6. A real life is presented to illustrate the proposed study in section 7. The conclusion based on proposed study is summarized in section 8.

2. Model Description and Assumptions

In Step-Stress model with progressive Type-I censoring scheme, the experiment is terminated at pre-specified time with some prefixing number of units censored at stress level \( x_i \). A total of \( N_i = n \) identical units are initially placed at low stress level \( x_1 \), and run until pre-specified stress change time \( t_1 \), the number of failed units \( n_1 \) is recorded and \( R_1 \) (prefix number) surviving units are randomly removed from the test. The test is continued on \( N_2 = n - n_1 - R_1 \) non-removed surviving units placed at the higher stress level \( x_2 \) and run until time \( t_2 \), when the stress is increased to \( x_3 \) and \( R_2 \) surviving units are randomly
withdrawn from the test, and so on. Finally, at time $\tau_3$ under stress $x_3$, all surviving items
$R_3 = n - \sum_{i=1}^{n} n_i - \sum_{i=4}^{n} R_i$ are withdrawn from the test, thereby terminating the life test. Note
that, when there is no intermediate censoring (viz., $R_i = R_2 = 0$), this situation corresponds to 3-level step-stress testing under Type-I right censoring as a special case.

Basic Assumptions

i. The life of a test unit at each stress level follows the Weibull distribution.

ii. At the stress level $x_i$, $i = 1, 2, 3$, the scale parameter $\theta_i$, is a log-quadratic function of stress, i.e., $\log(\theta_i) = \beta_0 + \beta_1 x_i + \beta_2 x_i^2$, $i = 1, 2, 3$.
   Where; $\beta_0$, $\beta_1$ and $\beta_2$ are unknown model parameters, and to be estimated from test data.

iii. For all stress level, the shape $\delta$ is common, constant and independent of time and stress.

iv. The lifetimes of test units are independent and identically distributed.

From the assumption (i), the cumulative distribution function (c.d.f.) of a test unit under 3-step step-stress ALT follows the K-M model:

$$F(t) = \begin{cases} 
1 - \exp(-A), & 0 \leq t < \tau_1 \\
1 - \exp(-B), & \tau_1 \leq t < \tau_2 \\
1 - \exp(-C), & \tau_2 \leq t < \infty 
\end{cases} \quad (1)$$

The probability density function (p.d.f.) of the random variable $T$ is obtained as follows:

$$f(t) = \begin{cases} 
\delta t^{\delta-1} \theta_1^{-\delta} \exp(-A), & 0 \leq t < \tau_1 \\
\delta t^{\delta-1} \theta_2^{-\delta} \exp(-B), & \tau_1 \leq t < \tau_2 \\
\delta t^{\delta-1} \theta_3^{-\delta} \exp(-C), & \tau_2 \leq t < \infty 
\end{cases} \quad (2)$$

Where, $A = \frac{t^\delta}{\theta_1^\delta}$, $B = \left(\frac{t^\delta - t_1^\delta}{\theta_2^\delta} + \frac{t_1^\delta}{\theta_1^\delta}\right)$ and $C = \left(\frac{t^\delta - t_2^\delta}{\theta_3^\delta} + \frac{t_2^\delta - t_1^\delta}{\theta_2^\delta} + \frac{t_1^\delta}{\theta_1^\delta}\right)$

3. Modified Progressive Censoring for Small Samples

The modified progressive censoring (PC) scheme is suggested by Balakrishnan and Han [3] for small sample size which ensures the availability of enough surviving items to censor on average at the end of each stress level. Under proposed modified PC, it is easy to decide on a fixed proportion of surviving units to be removed at the end of each stage, rather than to decide on a global proportion over initial sample size. Therefore, we define a vector of proportions to revise the model accordingly, as follows

$$\pi^* = (\pi_1^*, \pi_2^*, \ldots, \pi_{m-1}^*, 1) \quad (3)$$
of surviving units to be censored at the end of each stage, where \( 0 \leq \pi_i^* < 1 \) for \( i = 1, 2, 3 \). Then, the number of censored units at end of the \( i \)-th stage is the integer part of \((N_i - n_i)\pi_i^*\). For more detail one may refer to Balakrishnan and Han [3].

\[
R_i = \text{round}\left((N_i - n_i)\pi_i^*\right),
\]

(4)

4. Maximum Likelihood Estimation and Fisher Information Matrix

The likelihood functions under modified progressive Type-I censored for Weibull failure data \( t_{ij}, i = 1, 2, 3; j = 1, 2, \ldots, n_i \) is obtained as:

\[
L(\delta, \theta_1, \theta_2, \theta_3) = \prod_{i=1}^{3} \prod_{j=1}^{n_i} \delta^{t_{ij}/\theta_i} \exp\left(-\sum_{j=1}^{n_i} \frac{U_i}{\theta_i^2}\right)
\]

(5)

where \( U_i = \sum_{j=1}^{n_i} \left(x_i^\delta - \tau_{i-1}^\delta\right) \left(\frac{R_i}{\pi_i} - \tau_i^\delta - \tau_{i-1}^\delta\right) \) \( i = 1, 2, 3 \).

(6)

Note that \( U_i \) is the total time on test statistic for the \( i \)-th stage and \( R_i \) is as defined in equation (4). Now, using assumption (ii) and equation (5), the log-likelihood function can be written as

\[
\log L = \sum_{i=1}^{3} \left(n_i \log(\delta) - n_i \delta(\beta_0 + \beta_1 x_i + \beta_2 x_i^2) + (\delta - 1) \sum_{j=1}^{n_i} \log(t_{ij}) - D\right)
\]

(7)

where, \( D = U_i \exp\left(-\delta(\beta_0 + \beta_1 x_i + \beta_2 x_i^2)\right)\)

The MLEs \((\hat{\delta}, \hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2)\) can be obtained by partially derivatives of equation (7) with respect to respective parameters and equating to zero. It has been seen that there is no closed-form expression exist for maximum likelihood estimate, which makes necessary to approximate it via a numerical techniques such as Newton-Raphson approximation. However, statistical software such as R is used to implement the numerical technique.

Since the MLEs of the model parameters are not in closed-form, it is not possible to derive the exact confidence intervals (CI), so asymptotic CIs instead of exact CIs are used. The variance-covariance matrix can be obtained as the inverse of Fisher information matrix.

Then the two sided \( 100(1-\alpha)\% \) asymptotic confidence interval (CI) of the model parameter \( \hat{\delta} \) can be obtained from

\[
\hat{\delta} \pm Z_{\alpha/2} \sqrt{AV(\hat{\delta})}
\]

(8)

where, \( Z_{\alpha/2} \) is the \((1-\alpha/2)\th\) quantile of the standard normal distribution. Similarly, the two sided \( 100(1-\alpha)\% \) CIs for parameters \( \beta_0, \beta_1 \) and \( \beta_2 \) can be obtained.
5. **Optimality Criterion**

The optimal test plans under accelerated conditions that minimize or maximize the selected objective function which are purely based on the Fisher information matrix. The Fisher information is a way of measuring the amount of information that an observable random variable carries about an unknown parameter upon which the likelihood function depends. However, in order to obtain the asymptotic-variance of $\beta_0$, $\beta_1$ and $\beta_2$, the Fisher information matrix is needed, is defined as

$$F = \begin{bmatrix} I_{22} & I_{23} & I_{24} \\ I_{32} & I_{33} & I_{34} \\ I_{42} & I_{43} & I_{44} \end{bmatrix}$$ (9)

To obtain the expectations of the elements of $F$, the following properties is used:

**Properties:**

1. Given $n_1,...,n_{i-1}$, the random variable $n_i$ has a binomial distribution with parameters $(N_i, F_i(\tau))$, where

$$F_i(\tau) = \frac{F(\tau_i) - F(\tau_{i-1})}{1 - F(\tau_{i-1})}$$ (10)

   is the probability that a unit fail in the interval $(\tau_{i-1}, \tau_i]$ with $\tau_0 = 0$, and $F(\tau_i)$ is as given in (1).

2. For each $i = 1, 2, 3$, the random variables $\left( \tilde{\tau}_{i,j} - \tilde{\tau}_{i-1,j} \right)$, $j = 1, 2,...,n_i$ constitute a random sample from a truncated Weibull distribution on $(\tau_{i-1}, \tau_i]$, where $\tau_0 = 0$, with the p.d.f

   $$f_{i \tau}(z) = \frac{f_{i}(z)}{F(\tau_i) - F(\tau_{i-1})} \text{ for } \tau_{i-1} \leq z \leq \tau_i .$$

Using property (1) and the property of conditional expectation, we get

$$E(n_i) = E(N_i)F_i(\tau) .$$

Let us compute the expectation of $N_i$ and $R_i$, $i = 1, 2, 3$. Beginning with $E(N_1) = n$ and $N_{i+1} = N_i - n_i - R_i$, we obtain, by induction,

$$E(N_i) = \prod_{j=1}^{i-1} S_j(\tau)(1 - \pi_j)$$ (11)

$$E(R_i) = E(N_i)(1 - F_i(\tau))\pi_i$$ (12)

Hence, the expected value of $U_i$ given in equation (6) can be obtained as

$$E(U_i) = \exp\left(\beta_0 + \beta_1 x_i + \beta_2 x_i^2\right)E(N_i)F_i(\tau), i = 1, 2, 3.$$ (13)
and then the Fisher information matrix $F_1$ is

$$F_1 = n \sum_{i=1}^{3} A_i(r) \sum_{j=1}^{3} A_j(r)x_j \sum_{i=1}^{3} A_i(r)x_i^2 \sum_{i=1}^{3} A_i(r)x_i^3 \sum_{i=1}^{3} A_i(r)x_i^4$$

(14)

where $A_i(r) = F_i(r) \prod_{j=1}^{2} S_j(r)(1 - \pi_j^*)$ with $F_i(r)$ is as given in (10).

Hence, the optimum test plan to minimize the AV of the MLEs of the log of mean life at normal condition can be obtained as follows:

$$\text{AV}(\hat{\theta}_0) = \text{AV}(\hat{\beta}_0 + \hat{\beta}_1 x_0 + \hat{\beta}_2 x_0^2)$$

$$= \begin{vmatrix} t^2 & x_0^2 \end{vmatrix} F_1^{-1} \begin{vmatrix} t & x_0^2 \end{vmatrix}$$

(15)

where $F_1^{-1}(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2)$ is the asymptotic variance-covariance matrix, which is obtained from the inverse of Fisher information matrix given in equation (14).

6. Bayesian Inference under Progressive Type-I censoring

In order to reduce the testing time and cost of the experiment, small sample sizes data are often used in life-tests. In such situation, the Bayesian approach with prior knowledge (is collected from historical data, experience with similar items, design specifications, and experts’ opinions) on the unknown model parameters is very useful. In general, priors are regularly chosen according to ones subjective knowledge and beliefs.

However, if one has adequate information about the parameter(s) one should use informative prior(s); otherwise it is preferable to use non informative prior(s). We assume that there is no information available on the unknown model parameters, and therefore non-informative priors are chosen in this problem. In Bayesian approach, after collecting sample data the prior distribution is updated according to Bayes’ theorem

$$\pi(\delta, \beta_0, \beta_1, \beta_2 | t) = \frac{L(\delta, \beta_0, \beta_1, \beta_2 | t)g(\delta, \beta_0, \beta_1, \beta_2)}{f(t)}$$

(16)

where $g(\delta, \beta_0, \beta_1, \beta_2)$ is the joint prior distribution, $\pi(\delta, \beta_0, \beta_1, \beta_2 | t)$ is the joint posterior distribution, and

$$f(t) = \int \int \int L(\delta, \beta_0, \beta_1, \beta_2 | t)g(\delta, \beta_0, \beta_1, \beta_2)d\delta d\beta_0 d\beta_1 d\beta_2.$$  

(17)

According to Sinha [11], the Jeffrey’s rule for choosing the non-informative prior (NIP) density functions for the independent random variables $\delta, \beta_0, \beta_1,$ and $\beta_2$ are considered as uniform distribution as follows:
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\[ g(\beta_l) \propto k_l - a_l \leq \beta_l \leq a_l, l = 0, 1, 2, \]  
\[ g(\delta) \propto k_3, -a_3 < \delta < a_3. \]

and for shape parameter \( \delta \) is given as

\[ g(\delta) \propto k_3, -a_3 < \delta < a_3. \]

Therefore, the joint prior density function of the random parameters is obtained as

\[ g(\delta, \beta_l) \propto k_l k_3, -a_l < \beta_l < a_l, -a_3 < \delta < a_3 \]

In order to obtain Bayes estimator, Bayes theorem yields the joint posterior distribution of \( \delta, \beta_0, \beta_1, \beta_2 \) is obtained by combining the likelihood function of \( \delta, \beta_0, \beta_1, \beta_2 \) given in (5) and the prior \( g(\delta, \beta_l) \) in (20), given as

\[ \pi(\delta, \beta_l | t) \propto L(\delta, \beta_l | t) \times g(\delta, \beta_l) \]

Note that the above defined joint posterior density function cannot be derived in closed form. Therefore, we used the Markov Chain Monte Carlo (MCMC) simulation based on Gibbs sampling. WinBUGS software, a specialized package for implementing MCMC simulation and Gibbs sampling is used.

7. Numerical Illustrations

In this section, numerical study is performed to illustrate the proposed model by considering examples based on real data.

7.1 Example: Real Life Data

In order to validate the proposed SSALT Weibull model, we consider the real life data set on degradation experiments of the light emitting diodes (LED) from Zhao and Elsayed [16]. For convenience, we consider only last three accelerated temperature stress (i.e., 413, 433 and 448 in degree Kelvin). According to the Arrhenius model of reliability testing with temperature, the natural stress variable is the reciprocal of the temperature in degree Kelvin, i.e., \( T_i = 1/T_i \). We use the Kolmogorov-Smirnov (K-S) test to find the best fitted distribution among three distributions (Weibull, Gamma and Lognormal) and we got that the data set follows the Weibull distribution.

In the progressive Type-I censoring, a fixed proportion of surviving units are censored at the end of each stress levels. We considered modified progressive Type-I censoring scheme with average censoring proportions \( \pi_0 = 0.10 \) and \( 0.20 \). According to Balakrishnan and Han [3], we can obtain censoring proportion over survival item at the end of each stress level, i.e.,

\[ \pi^*_i = \frac{n \pi_0}{E(N_i)(1 - F_i(\tau))}, i = 1, 2. \]

7.1.1 MLEs of the Model Parameters

The MLEs, standard deviation (SD) based on Hessian matrix and 95% confidence intervals of the proposed model parameters under modified progressive Type-I censoring
with the average censoring proportions 10% and 20%, respectively are reported in the Table 1 through R programming.

### Table 1: MLEs, SD and 95% Confidence Interval of the Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>MLE</th>
<th>SD</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta )</td>
<td>6.7181</td>
<td>2.2157</td>
<td>(2.3753, 11.0609)</td>
</tr>
<tr>
<td>( \beta_0 )</td>
<td>-45.1509</td>
<td>4.4793</td>
<td>(-53.9304, -36.3714)</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>44.2302</td>
<td>3.2804</td>
<td>(37.8006, 50.6598)</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>-9.4642</td>
<td>0.6123</td>
<td>(-10.6643, -8.2642)</td>
</tr>
<tr>
<td>( \pi_0 = 0.10 )</td>
<td>6.4164</td>
<td>2.7418</td>
<td>(1.0426, 11.7903)</td>
</tr>
<tr>
<td>( \pi_0 = 0.20 )</td>
<td>-41.3695</td>
<td>3.2347</td>
<td>(-46.7433, -35.9956)</td>
</tr>
<tr>
<td>( \beta_0 )</td>
<td>40.6633</td>
<td>2.0565</td>
<td>(35.2895, 46.0372)</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>-8.6335</td>
<td>0.4671</td>
<td>(-14.0073, -3.2596)</td>
</tr>
</tbody>
</table>

In Table 1, it is observed that the model parameters \( (\beta_0, \beta_1 \) and \( \beta_2 \) have smaller 95% interval width for \( \pi_0 = 0.20 \) in compare to \( \pi_0 = 0.10 \), but for shape parameter it is smaller for \( \pi_0 = 0.10 \) in compare to \( \pi_0 = 0.20 \).

### 7.1.2 Optimum Plan

The optimum stress changing times \( \tau_1^* \) and \( \tau_2^* \) is obtained by minimizing the asymptotic variance of the MLE of log of mean life at normal stress. R programming and Microsoft Excel are used to solve the problem. Figure 1 shows the plot of \( \text{AV}(\log \hat{\theta}_0) \) verses stress changing times \( \tau_1 \) and \( \tau_2 \). The optimal stress change times are: \( \tau_1^* = 484 \) and \( \tau_2^* = 630 \) with respect to \( \pi_0 = 0.10 \) and \( \tau_1^* = 452 \) and \( \tau_2^* = 590 \) with respect to \( \pi_0 = 0.20 \).

![Figure 1: AV(log\(\hat{\theta}_0\)) verses stress change times \(\tau_1\) and \(\tau_2\).](image)

### 7.1.3 Bayesian Point and Interval Estimates

Under Bayesian setup, we assumed two MCMC chains with different initial values \( \delta = 2, \beta_0 = -35, \beta_1 = 30, \beta_2 = -6 \text{ and } \delta = 4, \beta_0 = -45, \beta_1 = 40, \beta_2 = -10 \) were run simultaneously in one simulation. Each chain continues for 80000 iterations. Gelman-
Rubin convergence statistic of the parameters with average censoring proportions (ACP) proportions $\pi_0 = 0.10$ and $0.20$ shows that the simulation is believed to have converged as shown from Figure 2 and 3, respectively. The summary for the sampling results concerning the unknown parameters $(\delta, \beta_0, \beta_1, \beta_2)$ is displayed in Table 2.

<table>
<thead>
<tr>
<th>node</th>
<th>Mean</th>
<th>SD</th>
<th>MC error</th>
<th>Median</th>
<th>Credible Intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>3.3110</td>
<td>0.5443</td>
<td>0.0246</td>
<td>3.2760</td>
<td>(2.3980, 4.5060)</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>-42.2800</td>
<td>4.5630</td>
<td>0.2148</td>
<td>-42.0200</td>
<td>(-49.680, -35.310)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>42.5800</td>
<td>3.5660</td>
<td>0.1753</td>
<td>42.4900</td>
<td>(36.000, 49.560)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-6.8410</td>
<td>1.1690</td>
<td>0.0571</td>
<td>-6.7530</td>
<td>(-9.2560, -5.0770)</td>
</tr>
</tbody>
</table>

It is observed that the model parameters $\delta, \beta_0$ and $\beta_2$ have smaller 95% interval width for $\pi_0 = 0.10$ in compare to $\pi_0 = 0.20$. But for parameter $\beta_1$ it is smaller for $\pi_0 = 0.20$. 

Figure 2: Gelman-Rubin Statistic of $\beta_0, \beta_1, \beta_2$ and $\delta$ for $\pi_0 = 0.10$.

Figure 3: Gelman-Rubin Statistic of $\beta_0, \beta_1, \beta_2$ and $\delta$ for $\pi_0 = 0.20$.
### 7.1.4 Comparative Study

A comparative study is performed between the proposed 3-step SSALT plan with Type-I censoring in terms of the optimum plan, the results are given in Table 3.

#### Table 3: Comparative Study with Type-I Censoring

<table>
<thead>
<tr>
<th>SSALT Models</th>
<th>$\pi_0 = 0.10$</th>
<th>$\pi_0 = 0.20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under progressive Type-I censoring</td>
<td>$\tau_1^* = 484$ and $\tau_2^* = 630$</td>
<td>$\tau_1^* = 452$ and $\tau_2^* = 590$</td>
</tr>
<tr>
<td>Under Type-I censoring</td>
<td>$\tau_1^* = 516$ and $\tau_2^* = 670$</td>
<td></td>
</tr>
</tbody>
</table>

Results presented in Table 3 show that the optimum stress changing times ($\tau_1^*$ and $\tau_2^*$) under modified progressive Type-I censoring is reduces as compared to Type-I censoring. Hence, the proposed optimum test plan performs better than the plan under Type-I censoring.

### 7.1.5 Sensitivity Analysis

To observe the effect of changes in the initially estimated model parameters $\delta$, $\beta_0$, $\beta_1$, and $\beta_2$ on the optimum value of stress change times ($\tau_1$ and $\tau_2$), sensitivity analysis is performed and the results are displayed in Table 4.

The sensitivity analysis result in the Table 4 shows that the parameters have small effect particularly $\beta_3$ and $\delta$ on the optimum test plan. Therefore, the proposed optimum plan is robust, and the pre-estimates have a small effect on optimum values.

#### Table 4: Sensitivity analysis for 3-step SSALT plan

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Deviation (%)</th>
<th>$\pi_0 = 0.10$</th>
<th>$\pi_0 = 0.20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>-5%</td>
<td>484</td>
<td>444</td>
</tr>
<tr>
<td></td>
<td>+5%</td>
<td>484</td>
<td>452</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-5%</td>
<td>476</td>
<td>436</td>
</tr>
<tr>
<td></td>
<td>+5%</td>
<td>500</td>
<td>460</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>-5%</td>
<td>476</td>
<td>436</td>
</tr>
<tr>
<td></td>
<td>+5%</td>
<td>500</td>
<td>460</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>-5%</td>
<td>492</td>
<td>452</td>
</tr>
<tr>
<td></td>
<td>+5%</td>
<td>484</td>
<td>444</td>
</tr>
</tbody>
</table>

### 8. Conclusions

This paper presents the 3-step step-stress accelerated life tests with Weibull distribution under modified progressive Type-I censored data. The mean life that is a log-quadratic function of stress levels is assumed. We have obtained the point and interval estimates using maximum likelihood as well as Bayesian Methods. Also, optimum test plan for SSALT model is developed using ML procedure. In this study, we have considered...
modified progressive Type-I censoring with average censoring proportions (ACP) \( \pi_0 = 0.10 \) and 0.20.

Finally, the procedures developed have been illustrated using real life data. For Bayesian approach, MCMC technique with WinBUGS software is used to obtain the posterior point and interval estimates of model parameters and the simulation used in posterior analysis is believed to have converged as shown in Figures 2 and 3 for given data set. A comparative between modified progressive Type-I censoring and Type-I censoring in terms of optimum test plan and it is observed that the proposed optimum plan perform better than the plan under Type-I censoring (see Table 4). From the sensitivity analysis it is seen that the incorrect pre-estimates of model parameters havea small effect on the optimum value of stress changing times and therefore, proposed optimum plan is robust for given data set. It is also found that the optimum test duration from one step to another step under modified progressive Type-I censoring is reduced as the amount of ACP increase.

References


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