

A Simulation-based Multi-level Redundancy Allocation for a Multi-level System

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Abstract: In this paper, we deal with a multi-level redundancy allocation problem for a multi-level system and system, modules and component levels in the system are simultaneously considered as candidates for redundancy. System availability and life cycle cost are used as the constraint and objective function, respectively, and are estimated by simulation. The level and degree of redundancy are determined that minimize the life cycle cost and satisfy the target system availability. An estimation of distribution algorithm is used to determine best solutions, and numerical examples are studied in order to compare module redundancy with component redundancy.

Keywords: *Multi-level redundancy; system availability; simulation; estimation of distribution algorithm*

1. Introduction

Modern large-scale systems are required to carry out various functions in the performance of multiple missions during their life cycle. Hence, these systems consist of many units in a complex structure, and should be evaluated quantitatively and accurately. System availability is usually used as a key measure to assess the performance of complex repairable systems, and is a function of system reliability and system maintainability. Therefore, system reliability and maintainability should be enhanced in order to develop complex repairable systems with higher system availability. System reliability is an important design measure, and it receives much attention in design studies. Designing a system with higher reliability tends to result in a system with more complex structure, leading to greater production and maintenance expense. Thus, the optimization problems of system reliability have been studied by many researchers. Kuo and Prasad [1] introduced reliability optimization problems with various optimization methods such as meta-heuristic algorithms, and branch and bound methods. In general, there are two commonly accepted means of increasing system reliability: increasing the reliability of components in the system (reliability allocation) and allocating redundant components to units in the system (redundancy allocation). The redundancy of units in the system is more commonly considered as it is difficult to increase component reliability. Many redundancy allocation optimization problems for various system structures, such as series, parallel, series-parallel, network, and k -out-of- n systems, have been studied [2]. To date, most studies of redundancy optimization have restricted redundancy to the component level in the system, as it is well-known principle for design engineers that redundancy at the component level is superior to redundancy at the system level. However, redundancy may be allocated to any level of the system if it will result in a reduction of design or life-cycle costs.

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Modern complex systems have multi-leveled structures and the entire system is at the top level (system level), while components are at the lowest level. Also, modules, consisting of a group of components are represented at different levels between system and component levels. Therefore, the reliability of modules at all levels depends on the reliability of lower modules or their components and thus the configuration of multi-level systems can be changed by redundancy of components or modules. In some cases, module redundancy can reduce the overall design costs and enhance efficiency, flexibility and reliability. Hence, multi-level redundancy in which system, module, and component levels are considered for redundancy can be applied to redundancy allocation optimization problems. Levitin [3] studied a multi-level redundancy allocation problem for series-parallel systems and proposed a genetic algorithm to determine optimal protection levels for a series-parallel system that minimize the protection cost and satisfy the target survivability. Yun and Kim [4] studied a multi-level redundancy allocation problem for a multi-level system and compare the proposed optimization methods (a MIP model, a heuristic method and a genetic algorithm). Yun *et al.* [5] proposed a simulated annealing algorithm to determine the optimal redundancy level and redundancy units which maximize the system reliability. Later, Yun *et al.* [6] considered the extended case in which all available units (system, module and component) can be selected simultaneously (multiple multi-level redundancy) and compared it with the multi-level redundancy case studied by Yun *et al.* [4]. Agarwal and Aggarwal [7] studied a redundancy allocation problem for a complex network system and compared the performance of the proposed heuristic method with the existing method proposed by Kim and Yum [8]. Yeh [9] proposed a two-stage discrete particle swarm optimization algorithm and compare the algorithm with the genetic algorithm proposed by Yun *et al.* [6]. Many studies have applied the conventional genetic algorithm to such problems, using a one dimensional representation but the design variables for the multi-level redundancy model are hierarchically structured. Hence, Kumar *et al.* [10] proposed a hierarchical genotype encoding method and compared it with the conventional genetic algorithm proposed by Yun and Kim [4]. Later, they applied the hierarchical genetic algorithm to the optimization problem of module redundancy allocation in series and series-parallel systems [11]. Using the hierarchical genetic algorithm, Kumar *et al.* [12] studied a multi-objective multi-level redundancy allocation problem for a multi-level system. Also, Wang *et al.* [13] and Hsieh [14] proposed a novel memetic algorithm and a bacterial inspired evolutionary algorithm (BiEA) based on hierarchical genotype representation for the multi-level redundancy. However, the length of each chromosome may be different because it depends on the redundant degree of its parent unit and the genetic operators become more complex. For that reason, He *et al.* [15] proposed a two dimensional encoding method to fix the length of a chromosome.

Maintainability is also an important design measure to enhance the availability of complex repairable systems and thus system availability can be improved by enhancing system maintainability, and this is achieved through reducing the duration required for maintenance procedures. Generally, complex repairable systems are repaired at higher module levels in the system because it requires less maintenance times but may incur a higher maintenance costs [16]. Therefore, when we design complex repairable systems, the maintenance unit should be determined optimally and balance between maintenance times and costs is maintained. Dandotiya *et al.* [17] proposed a mathematical model to determine the maintenance intervals of LRUs in the aircraft that satisfy the desired availability and minimize the life cycle cost. Chung [18] proposed a discrete-event

simulation model based on the object-oriented design method in order to estimate the demand of spare parts for LRUs in the multi-indenture system. Joo [19] proposed an algorithm in order to determine the overhaul intervals of the modules in the aircraft engine that minimize maintenance costs under limited availability of spare parts.

In this paper, we deal with a multi-level redundancy allocation problem for a multi-level system. The term ‘unit’ is used as a common name for system, module and component. The system availability and life cycle cost are considered as the constraint and objective function, and are estimated by simulation. The objective of this paper is to determine the optimal redundancy level and redundancy degree which will increase system reliability while minimizing life cycle cost and to satisfy the target system availability. This paper is organized as follows: First, we introduce the redundancy allocation model for a multi-level system in Section 2. A simulation-based optimization procedure with an estimation of distribution algorithm is proposed to find near-optimal solutions for the multi-level redundancy allocation in Section 3. Next, numerical examples are studied to compare the module redundancy with the component redundancy in Section 4, and Section 5 concludes the paper.

Notation

i	Index of units ($i = 1, 2, \dots, n$)
r	Index of resources ($r = 1, 2, \dots, n_r$)
x_i	Number of redundant ones of unit i
y_i	0 or 1 indicator variable to represent whether unit i is the redundancy level or not
v_k	k^{th} chromosome
b_r	Amount of available resource r
A_j	A set of ancestor units of unit j
$C_i(x_i)$	Design cost function of unit i with x_i redundant ones
$E[LC]$	Expected life cycle cost
RC_i	Replacement cost of unit i
A_T	Target system availability
A_{sys}	System availability
$E[NR_i]$	Expected number of unit i replaced during life cycle

2. Multi-level Redundancy Allocation Model for a Multi-level Redundancy

In this paper, we study a new redundancy allocation problem, in which the system availability and the life cycle cost are considered as new optimization criteria in multi-level redundancy problems. Usually, component redundancy is more effective than module redundancy in improving system reliability but module redundancy can be more cost effective than component redundancy in modularized systems. For example, a system is assumed to be a series system and is composed of modules A and B. Also, module A is composed of components A_1 and A_2 as shown in Fig. 1. The system reliability of the redundant components A_1 and A_2 is higher than that of the redundant module A but their assembly costs may be higher than that of the redundant module A because of larger maintenance times and higher skill. Therefore, we need to consider a multi-level

redundancy method in which system, module and component levels are simultaneously considered as the objects of redundancy.

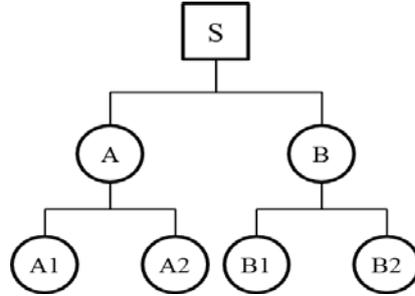


Figure 1: Example of a Three-level System

In the multi-level system, a direct line is defined as a set of units from the system level down to the component level (A_1 -A-S, A_2 -A-S, B_1 -B-S, and B_2 -B-S in Fig. 1). Most complex systems have modularized structures which are not easy to change and thus we assume that only one level in a direct line can be chosen for redundancy. For example, all units in the direct line A_1 -A-S are considered as the candidates of redundancy, but only one unit among A_1 , A, and S should be finally selected. Generally, maintenance at higher module levels in the system can reduce the maintenance times because modules are easier to be replaced than components but it incurs higher maintenance costs than maintenance at component level. Thus, the maintenance unit also needs to be determined optimally. Two following assumptions are made about maintenance:

- (1) The failed system is restored only by replacing the units which are determined for redundancy.
- (2) The failed units in the system are replaced upon system failure.

For an example, if module A in Fig. 1 is assumed to be the redundancy level and only component A_1 fails, the entire system fails, and module A is replaced even though component A_2 is functioning. Under the above assumptions, we aim to determine the redundancy levels, and redundancy degrees which minimize the life cycle cost and satisfy the target system availability. Thus, the multi-level redundancy allocation problem for a multi-level system is formulated as follows:

$$\text{Min } E[LC] = \sum_{i=1}^n y_i (E[NR_i] \times RC_i) \quad (1)$$

$$\text{subject to } A_{\text{sys}} \geq A_T \quad (2)$$

$$\sum_{i=1}^n y_i C_i(x_i) \leq b \quad (3)$$

$$y_j + \sum_{k \in A_j} y_k = 1 \quad (4)$$

$$y_i = 0 \text{ or } 1, \quad \forall i \quad (5)$$

$$x_i \in \text{non-negative integer}, \quad \forall i \quad (6)$$

Two prime variables, x_i and y_i , are used. x_i denotes the number of redundant units of unit i , and y_i indicates whether unit i is actually considered or not for redundancy. Constraint (2) guarantees to satisfy the target system availability and (3) represents the

cost function used in this paper, $C_i(x_i) = c_i x_i + \lambda^{x_i}$ which is the sum of the price of unit i , $c_i x_i$, and the additive cost, λ^{x_i} . Constraint (4) ensures the selection of only one level among system, module and component level in a direct line.

3. An Optimization Method

In this paper, we consider a multi-unit system with series structure and assume that the failure times of the components follow an exponential distribution. It is assumed that the failed components are only replaced when the system fails. In addition, if modules are redundant levels and have at least one failed components, all the components in the module are replaced. Therefore, it is difficult to obtain the system availability and life cycle cost analytically and thus, simulation is used to estimate the two terms. Also, appropriate optimization methods are needed to find near-optimal solutions and an estimation of distribution algorithm is proposed for this purpose. In the estimation of distribution algorithm, the probabilistic model of the promising solutions in the search space is built and used to guide the search for optimal solutions. A gene in a chromosome is defined as an ordered couple of an allocated number, x_{ki} , and an indicator variable, y_{ki} ; $v_{ki} = (x_{ki}, y_{ki})$ where the subscript k is index of chromosome that the gene belongs to and the subscript i denotes units. Therefore, a chromosome is represented as:

$$v_k = [(x_{k1}, y_{k1}), (x_{k1}, y_{k1}) \cdots (x_{kn}, y_{kn})]$$

The initial solutions are randomly generated with a uniform distribution, but the direct line should be considered in order to satisfy the constraint (Equation 4) when y is selected randomly. After generating initial solutions of which the number is the same as the population size by the random number generating method, the life cycle cost and system availability for each initial solution are estimated by simulation. We select the half of the solutions with the lowest life cycle cost among the solutions whose system availability satisfies the target system availability. Next, we obtain the joint probability distribution of design variables of the selected solutions and generate new solutions by the joint probability distribution as follows.

- Step 1: Select a direct line randomly among the direct lines in which the values of y of all units are '0'.
- Step 2: Select a unit in the selected direct line and sample the value of y based on the probability distribution for y variable of the selected unit.
 - 2.1: If the value of y of the unit is '0', return to Step 2. Otherwise, go to Step 2.2.
 - 2.2: Assign a redundant number to the selected unit randomly within the current available design cost.
 - 2.3: Change the values of y and x to '0' for ancestor units of the selected unit.
 - 2.4: Update the current available design cost.
 - 2.5: If the selected unit is the component, go to Step 3. Otherwise, go to Step 4.
- Step 3: Change the value of y to '1' for the sibling units of the selected unit.
 - 3.1: Change the values of y and x to '0' for ancestor units of each sibling units.
 - 3.2: Assign the redundant numbers to the sibling units randomly within the current design cost.
 - 3.3: Update the current available design cost and go to Step 4.
- Step 4: If each direct line has only a unit whose value of y is '1', stop the procedure. Otherwise, go to Step 1.

After generating new solutions (as many as the population), the system availability and life cycle cost are estimated by simulation. The solutions which satisfy the target system availability and have minimal life cycle cost are selected by as many as the half of the population. The estimation of distribution algorithm runs until the termination condition is satisfied [20].

4. Numerical Examples

A series system with input data shown in Table I is considered, and the system availability with the original system structure (without redundant units) is 0.70. The design cost limit is set to 250 and the system life cycle time is given 50,000. The parameters for the estimation of distribution algorithm are as follows: population size (100), generation size (100) and selection rate (0.5). The number of simulation replications is 50.

Table 1: Input Data of Units in the System

Parent	Unit	Failure rate	Set-up time	MTTR	Price	Additive cost	Replacement cost
-	1	0.0258	3	2	60	1	61
1	11	0.0090	5	3	27	2	29
	12	0.0168	4	4	33	2	35
11	111	0.0045	7	4	15	3	18
	112	0.0045	6	5	12	2	14
12	121	0.0044	5	6	9	2	11
	122	0.0074	6	5	15	2	17
	123	0.0050	6	4	9	2	11
111	1111	0.0015	10	7	5	4	9
	1112	0.0010	10	5	6	4	10
	1113	0.0020	10	6	4	3	7
112	1121	0.0025	9	8	7	3	10
	1122	0.0020	9	7	5	3	8
121	1211	0.0028	7	8	4	2	6
	1212	0.0016	7	10	5	3	8
122	1221	0.0019	8	6	4	3	7
	1222	0.0022	8	7	5	3	8
	1223	0.0033	8	9	6	4	10
123	1231	0.0020	9	6	4	3	7
	1232	0.0030	9	6	5	3	8

For numerical examples, we first consider different target system availability as 0.80, 0.85 and 0.90. We then increase the failure rates of the components in the system by 15% and 30%. Lastly, MTTR of all units in the system is increased by 15% and 30%. From the numerical results, we compare module redundancy (MR) with component redundancy (CR). To satisfy high target system availability, more redundancy units must be allocated and also the modules at high levels must be replaced. Thus, the life cycle cost and the total number of redundant units increase as the target system availability increases as shown in Fig. 2 and Table 2. On average, module redundancy results in a 12.5% lower life cycle cost than component redundancy under the same target system availability.

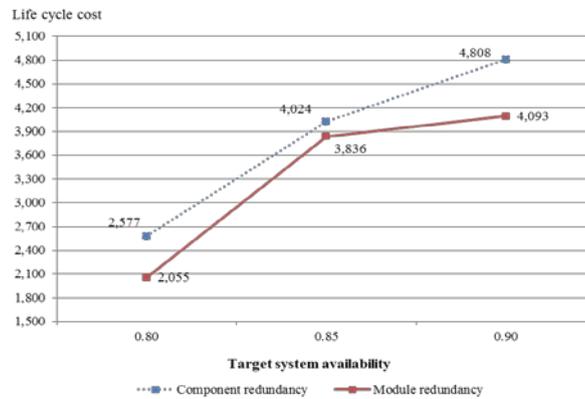


Figure 2: Life Cycle Cost for Different Target System Availability

Table 2: Optimal Redundancy Allocation of Units for Different Target System Availability

Parent	Unit	0.80		0.85		0.90	
		CR	MR	CR	MR	CR	MR
-	1	-	0	-	0	-	0
1	11	-	0	-	0	-	0
	12	-	0	-	0	-	0
11	111	-	0	-	0	-	1
	112	-	0	-	0	-	0
12	121	-	0	-	2	-	1
	122	-	0	-	0	-	0
	123	-	2	-	0	-	2
111	1111	0	0	0	0	0	0
	1112	0	0	0	0	1	0
	1113	2	0	1	0	1	0
112	1121	2	1	0	3	0	1
	1122	0	0	0	2	2	3
121	1211	5	0	4	0	5	0
	1212	2	2	3	0	0	0
122	1221	2	3	0	3	0	2
	1222	0	2	3	2	3	2
	1223	0	0	0	0	2	0
123	1231	0	0	0	0	2	0
	1232	0	0	3	3	3	0

In the second case, the target system availability is 0.85 and the failure rates of the components in the system are increased by 15% and 30%. As the reliability of the components decreases, their failure frequency increases and thus more redundant units must be allocated and the module at high levels must be replaced in order to satisfy the target system availability. Therefore, the life cycle cost and the total number of redundant units increase as the failure rates of the components increase as shown in Fig. 3 and Table 3. On average, module redundancy results in a 4.3% lower life cycle cost than component redundancy.

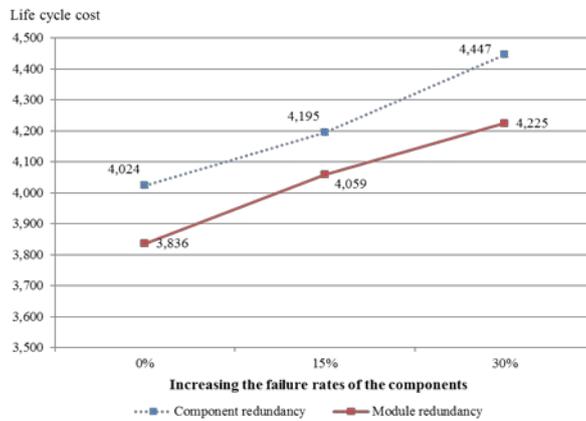


Figure 3: Life Cycle Cost for increasing the Failure Rates of the Components

Table 3: Optimal Redundancy Allocation of Units for increasing the Failure Rates of the components

Parent	Unit	0%		15%		30%	
		CR	MR	CR	MR	CR	MR
-	1	-	0	-	0	-	0
1	11	-	0	-	0	-	0
	12	-	0	-	0	-	0
1	111	-	0	-	1	-	2
	112	-	0	-	1	-	2
12	121	-	2	-	2	-	2
	122	-	0	-	0	-	0
	123	-	0	-	3	-	3
111	1111	0	0	0	0	0	0
	1112	0	0	1	0	1	0
	1113	1	0	1	0	2	0
112	1121	0	3	2	0	0	0
	1122	0	2	0	0	0	0
121	1211	4	0	6	0	6	0
	1212	3	0	0	0	2	0
122	1221	0	3	0	3	0	2
	1222	3	2	0	2	0	0
	1223	0	0	0	0	0	0
123	1231	0	0	3	0	3	0
	1232	3	3	2	0	3	0

In the last case, the target system availability is 0.85 and MTTR of all units in the system is increased by 15% and 30%. By increasing the maintenance times of units, the mean down time (MDT) increases, and thus, the system availability decreases. Hence, in order to satisfy high system availability, we need to reduce the failure frequency of the system by allocating more redundant units and the maintenance times by replacing modules at high levels. As a result, the life cycle cost and the total number of redundant units increase as MTTR of all units increases as shown in Fig. 4 and Table 4. On average, module redundancy results in a 4.5% lower average life cycle cost than component redundancy.

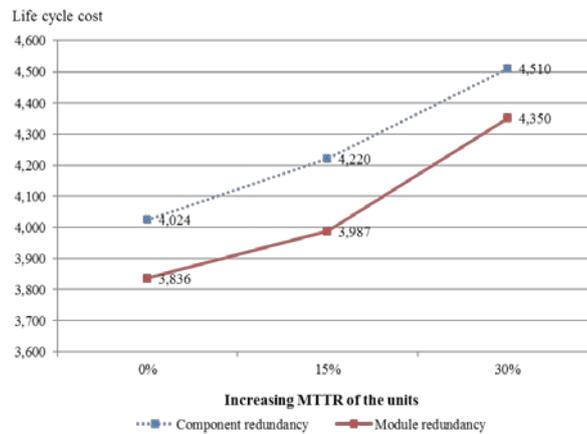


Figure 4: Life Cycle Cost for increasing MTTR of the Units

Table 4: Optimal Redundancy Allocation of Units for increasing MTTR of the Units

Parent	Unit	0%		15%		30%	
		CR	MR	CR	MR	CR	MR
-	1	-	0		0		0
1	11	-	0		0		0
	12	-	0		0		0
11	111	-	0		1		2
	112	-	0		0		2
12	121	-	2		2		3
	122	-	0		0		2
	123	-	0		2		3
111	1111	0	0	1	0	1	0
	1112	0	0	0	0	1	0
	1113	1	0	2	0	2	0
112	1121	0	3	0	3	1	0
	1122	0	2	0	2	1	0
121	1211	4	0	3	0	3	0
	1212	3	0	3	0	3	0
122	1221	0	3	1	3	2	0
	1222	3	2	3	2	3	0
	1223	0	0	1	0	1	0
123	1231	0	0	1	0	1	0
	1232	3	3	3	0	3	0

5. Conclusions

In this paper, we considered a multi-level redundancy allocation in which system, module and component levels are simultaneously considered as candidates for redundancy. System availability and life cycle cost were considered as the constraint and objective function and were estimated by simulation. The objective of this paper is to determine the redundancy level and redundancy degree that minimize the life cycle cost, including the replacement cost of the redundant units, and satisfy the target system availability. In order to generate alternatives, an estimation of distribution algorithm was proposed, and

numerical examples were studied to compare the module redundancy with the component redundancy. In the numerical examples, higher module levels were selected for redundancy in order to satisfy the target system availability. More redundant units were assigned in component redundancy than in module redundancy, because the target system availability can only be satisfied through increasing system reliability. Based on numerical results, it was known that module redundancy is more effective than component redundancy in some cases. In this paper, we assumed that the redundancy level to allocate redundant units and the maintenance level to restore the failed system are the same. For further studies, we will study an extended optimization problem to determine the maintenance level and redundancy level separately.

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