Joint Lot-size and Preventive Maintenance Optimization for a Production System

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Abstract: This paper considers a production system producing multiple products alternately, and a production cycle is formed by a complete run of all products which go through the system in sequence. Two different preventive maintenance policies are studied. A framework is proposed to solve the optimal lot-size of each product together with the optimal preventive maintenance epoch which minimizes the system’s cost rate in long run.

Keywords: Lot sizing, production, inventory, preventive maintenance, delay time

1. Introduction

The classical economic production quantity (EPQ) model calculates the optimal lot size of the product by minimizing the sum of the inventory holding and set-up costs [1]. Some works have been devoted to extend this basic EPQ model for close fitting to real-world situations, such as the consideration of preventive maintenance (PM) in EPQ model [2-4]. These integrated PM and EPQ works only consider a single type of product going through the production system, but in reality, multiple products are often produced on the same production system in typical batch production situations, and there is no literature in this integrated area considering multiple products.

In this paper, we consider the problem of jointly determining the optimal lot sizes and PM policy for a production system that can produce multiple products alternately. The demand for each product is assumed to be steadied, and the total supply per year for each product is well balanced at the production planning stage based on the demand and the capacity of the production system. Our task is to determine the optimal lot size for each product in conjunction with the PM policy. We start with a simple case of three products with two different maintenance policies: either carrying out the PM after completing all products' lot sizes, or at each set-up time of the products. Then, the case of k-products is proposed for both PM policies.

To model the PM policy for the system, we use the delay-time concept in our model. The delay-time concept has been widely applied in maintenance modeling, regarding the failure process as a two-stage process: the period from new to the initial point of the defect, usually referred to as the normal stage or time-to-defect; and the period from the initial point to the component failure, referred to as the delay-time stage [5]. If the PM, which is considered to be perfect, is carried out during the delay-time, the defect can be identified and removed by repair or replacement. The delay-time models have been applied to single-component systems and complex systems with many components [5]. In this paper, we use the complex system delay-time model since typical production systems are equipped with many components.

This paper is organized as follows. The model assumptions and notations are given in Section 2. Section 3 introduces the cost models based on different PM policies. Numerical examples are presented in Section 4. Section 5 concludes the paper.

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2. Assumptions and Notation

2.1. Assumptions

(1) The demands of all products are steady and can be divided into smaller lot sizes that are produced according to a fixed sequence, which is preset.
(2) The defects of the system arrive independently according to a HPP.
(3) The delay-time of all defects is independent and identically distributed.
(4) The PM is carried out at some set-up points for less interruption to the production. The PM is perfect and renews the system.
(5) The downtime caused by the failure of the system, set-up and PM can be ignored.
(6) A minimal repair is always performed at a failure.

Assumption (1) is typical for batch production when the demand is steady. The sequence of the products to be produced can vary because of the delivery time, but we fix it for simplicity in this paper. It can be relaxed to allow dynamic scheduling, which is a different research topic from the one presented in this paper. Assumptions (2) and (3) have been used in previous delay-time models. The first part of assumption (4) can be observed in industry while the second part is for modeling simplification. Assumption (5) is an approximation, but, in reality, compared with the production time, the downtime caused by maintenance is small and can be ignored. Assumption (6) is commonly used in maintenance modeling, where due to the time constraint and the need to resume the production as soon as possible.

2.2. Notation

- $D_i$: Total demand of the $i$th product per year, $i = 1, 2, 3, \cdots, k$.
- $d_i$: The demand rate.
- $p_i$: The production rate.
- $Q_i$: The production lot size of the $i$th product.
- $n$: The number of production cycles per year.
- $T_i$: The production time of the $i$th product in one production cycle.
- $u$: The defect arrival point.
- $f(\cdot)$: The probability density function (pdf) of the delay-time.
- $F(\cdot)$: The cumulative density function (cdf) of the delay-time.
- $\lambda$: The rate of the occurrence of defects.
- $h_j$: The average inventory holding cost per unit product per unit time for the $j$th product.
- $C_i$: The average set-up cost for product $i$.
- $C_d$: The average cost of repairing a defect that is identified at a PM.
- $C_f$: The average cost for a failure, $C_f > C_d$.
- $C_p$: The average cost of an inspection.
- $E_c(\cdot)$: The expected cost of set-up $(s)$, holding $(h)$ and maintenance $(m)$ within one production cycle where $\cdot = s, h$ and $m$, respectively.
- $E_c(n)$: The expected total cost per year with $n$ as the decision variable.
3. The Models

3.1. Model 1: The PM is carried out at the end of a Production Cycle

We first consider three products and the situation that the PM is carried out at the end of a production cycle, as shown in Figure 1. Several types of cost will be calculated during one cycle: inventory holding cost, set-up cost and the maintenance costs. The expected total cost per year is modeled based on the expected total cost per cycle.

![Figure 1: PM is carried out at the end of the production cycle. '△' PM point.](image)

As \( D_i \) \((i = 1, 2, 3)\) is the annual demand for product \( i \), the demand of product \( i \) in each production cycle is \( D_i / n \) and demand rate \( d_i \) is \( D_i / (n(T + T + T)) \). Furthermore, the maximum inventory level for product \( i \) can be obtained as \( \left( p_i - D_i / n(T + T + T) \right) T_i \). The inventory holding cost of product \( i \) per production cycle is \( \left( \frac{1}{2} \sum_{j=1}^{3} T_j \right) \sum_{j=1}^{3} p_j - D_i / n \sum_{j=1}^{3} T_j T_j \). As the system needs to be set up three times during one cycle, the expected set-up cost is \( C_{Es} \).

The PM is carried out only once per cycle; the inspection cost is \( C_p \).

From assumption (2), the defects arrive according to an HPP with the occurrence rate \( \lambda \). Based on the delay time concept, the expected number of the failures within one cycle is \( \int_0^{T_2 + T_3} \int_0^{T_1 + T_2 + T_3} \lambda f(t - u) du dt = \int_0^{T_1 + T_2 + T_3} \lambda F(t) dt \), and the expected number of the defects identified at PM is \( \int_0^{T_1 + T_2 + T_3} \lambda \left[1 - F(t)\right] dt \) \[6\]. The failure and the defects repairing cost can be expressed as \( C_f \int_0^{T_1 + T_2 + T_3} \lambda F(t) dt \) and \( C_d \int_0^{T_1 + T_2 + T_3} \lambda \left[1 - F(t)\right] dt \) respectively. This gives \( E_c(m) = C_p + C_f \int_0^{T_1 + T_2 + T_3} \lambda F(t) dt + C_d \int_0^{T_1 + T_2 + T_3} \lambda \left[1 - F(t)\right] dt \).

It is obvious that \( T_i = D_i / np_i \) is the production time of product \( i \) in one cycle, since each product needs to meet the demand. The total expected cost per year is given by

\[
E_c(n) = n[E_c(s) + E_c(h) + E_c(m)] = n \left\{ \frac{1}{2} \left[ \sum_{i=1}^{3} D_i / np_i \right] \left[ \sum_{j=1}^{3} p_j - D_i / n \left( \sum_{i=1}^{3} D_i / np_i \right) \right] \left[ \sum_{i=1}^{3} C_i \right] \right\} + C_f \int_0^{T_1 + T_2 + T_3} \lambda F(t) dt + C_d \int_0^{T_1 + T_2 + T_3} \lambda \left[1 - F(t)\right] dt + C_p.
\]
Equation (1) can be extended to the case of an arbitrary number of products, $k$. The expected total cost per year is given by

$$E_{c}(n) = n\left[ E_{c}(s) + E_{c}(h) + E_{c}(m) \right]$$

$$= n \left[ \frac{1}{2} \left( \sum_{i=1}^{k} D_{i}/np_{i} \right) \sum_{j=1}^{k} \left( p_{j} - D_{j} \left( \sum_{i=1}^{k} D_{i}/np_{i} \right) \right) \left( D_{j}/np_{j} \right) h_{j} + \sum_{i=1}^{k} C_{i} \right]$$

$$+ C_{f} \int_{0}^{D_{f}/mp_{i}} \lambda F(t)dt + C_{r} \int_{0}^{D_{f}/mp_{i}} \lambda \left[ 1 - F(t) \right]dt + C_{p}$$

(2)

### 3.2. Model 2: The PM is carried out at each Set-up Point

We now consider that the PM is carried out at each set-up point, as shown in Figure 2.

![Figure 2: PM is carried out at each Set-up Point.](image)

Supposing there are $k$ products. The expected inventory holding cost and set-up cost in this situation are the same as that in Section 3.1. The cost of PM is $kC_{p}$. The expected number of failures during one cycle is $\sum_{i=1}^{D_{f}/mp_{i}} \lambda F(t)dt$, so the expected failure repairing cost can be expressed as $C_{f} \sum_{i=1}^{D_{f}/mp_{i}} \lambda F(t)dt$. The expected defects repairing cost is $C_{d} \sum_{i=1}^{D_{f}/mp_{i}} \lambda \left[ 1 - F(t) \right]dt$. The total expected cost per year is given by

$$E_{c}(n) = n \left[ \frac{1}{2} \left( \sum_{i=1}^{k} D_{i}/np_{i} \right) \sum_{j=1}^{k} \left( p_{j} - D_{j} \left( \sum_{i=1}^{k} D_{i}/np_{i} \right) \right) \left( D_{j}/np_{j} \right) h_{j} \right]$$

$$+ C_{f} \sum_{i=1}^{D_{f}/mp_{i}} \lambda F(t)dt + C_{r} \sum_{i=1}^{D_{f}/mp_{i}} \lambda \left[ 1 - F(t) \right]dt + kC_{p}$$

(3)

### 3.3. Property of the Models

**Lemma 1:** Equation (2) or (3) is a strictly convex function of $n$, so there exists a unique optimal $n$ for minimizing $E_{c}(n)$.

**Proof:** The proof is the same for both equations so we only use Equation (3) to show the proof. Let $\int_{0}^{D_{f}/mp_{i}} \lambda F(t)dt = \int_{0}^{D_{f}/mp_{i}} \lambda F(t)dt = P(T_{i})$. The second derivative of Equation (3) with respect to $n$ can be derived as

$$\frac{\partial^{2} E_{c}(n)}{\partial n^{2}} = \frac{1}{n} \sum_{i=1}^{k} \left( \frac{D_{i}/np_{i}}{p_{i}} \sum_{j=1}^{k} D_{j} h_{j} \right) + \left( C_{f} - C_{d} \right) \sum_{i=1}^{D_{f}/mp_{i}} \left( \frac{P(T_{i})}{p_{i}} \right) \frac{1}{n^{2}}$$

(4)
From Equation (4), to prove \( \frac{\partial^2 E_c(n)}{\partial n^2} > 0 \), we just need to prove \( \frac{\partial^2 P(T_i)}{\partial T_i^2} > 0 \).

From \( P(T_i) = \int_0^{\infty} \lambda F(t)dt \), we have \( \frac{\partial P(T_i)}{\partial T_i} = \lambda F(T_i) \) and \( \frac{\partial^2 P(T_i)}{\partial T_i^2} = \lambda f(T_i) > 0 \), so \( \frac{\partial^2 E_c(n)}{\partial n^2} > 0 \) is always true when \( n > 0 \). This proves that Equation (3) is a strictly convex function of \( n \) and has only one minimum value with \( n > 0 \).

### 4. Numerical Examples

The pdf of the delay-time is assumed to follow an exponential distribution with parameter \( \alpha \). The exponential distribution as the delay-time has been used in previous delay-time-based case studies [6], and was chosen based on the best fit to the actual data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( D_i )</th>
<th>( p_i )</th>
<th>( h_i )</th>
<th>( C_i )</th>
<th>( C_f )</th>
<th>( C_d )</th>
<th>( C_p )</th>
<th>( \lambda )</th>
<th>( \alpha )</th>
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<tbody>
<tr>
<td>Product 1</td>
<td>6000</td>
<td>60</td>
<td>0.005</td>
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<td></td>
<td></td>
<td></td>
<td>0.0416</td>
<td>0.0833</td>
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<tr>
<td>Product 2</td>
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<td>80</td>
<td>0.007</td>
<td>25</td>
<td>600</td>
<td>100</td>
<td>10</td>
<td>0.0416</td>
<td></td>
</tr>
<tr>
<td>Product 3</td>
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<td>40</td>
<td>0.006</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Figure 3 shows the results in terms of the expected total cost per year of Equation (1) with \( n \) from 1 to 100. It is clear that \( n = 31 \) is the best, which gives \( E_c(31) = 7626.2 \).

Correspondingly, the optimal lot sizes of the three products are \( Q_1 = D_1/n = 6000/31 \approx 194 \), \( Q_2 \approx 286 \) and \( Q_3 \approx 286 \), respectively.

Comparing the results of these two models, it is clear that the minimum expected total cost calculated by Equation (1) is larger than that of Equation (3). So, having the PM carried out at each set-up point is a better choice in this example.
5. Conclusion

In this paper, we study the problem of a production system that can produce multiple products, which is more realistic than the previous works about EPQ studies where only one product was considered. The study has, of course, certain limitations, as some assumptions have to be proposed to simplify the modeling process. This leads to the need for further research: 1) general situations that PM can occur at some other set-up points; 2) considering the downtime caused by failure and PM.

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References


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