Optimum Time-Censored Step-Stress PALTSP with Competing Causes of Failure Using Tampered Failure Rate Model

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Abstract: In this paper accelerated life testing is incorporated in life test sampling plans to induce early failures of high reliability items. Life test under accelerated environmental conditions may be fully accelerated or partially accelerated. In fully accelerated life testing all the test units are run at accelerated condition, while in partially accelerated life testing they are run at both normal and accelerated conditions. Many products have more than one cause of failure. Optimum time-censored step-stress partially accelerated life test sampling plan (PALTSP) with competing causes of failure has been designed using tampered failure rate model and variable repetitive group sampling plan. The optimum plan consists in finding out optimum stress change point and optimum sample size by minimizing the average sample number of a lot such that producer’s and consumer’s interests are safeguarded. Bilevel programming approach is used for the purpose. The method developed has been explained using an example.

Keywords: Reliability, quality control, step-stress, repetitive group sampling plan, bi-level programming.

1. Introduction

The Acceptance Sampling Plan is concerned with either rejection or acceptance of a collection of items (or a lot) for the purpose of quality assurance. Many modern high reliability products are designed to operate without failure for a very long period of time. Life testing for these products under normal operating conditions is time consuming, and therefore accelerated life testing is incorporated in life test sampling plans to induce early failures. Life test under accelerated environmental conditions may be fully accelerated or partially accelerated. In fully accelerated life testing all the test units are run at accelerated condition, while in partially accelerated life testing they are run at both normal and accelerated conditions. Stress under accelerated condition can be applied using constant-stress, step-stress, progressive-stress, cyclic-stress, random-stress, or combinations of such loadings. Many products have more than one cause of failure. Examples include:

1. Ball bearing assemblies, in which a ball or the race can fail.
2. A semiconductor device can fail at a junction or at a lead.
3. Any solder joint on a circuit board can fail.
4. In motors, the Turn, Phase or Ground insulation can fail, etc.

When there are competing causes of failure, the life of product depends on the characteristics of the different failure modes.

Accelerated tests that yield a mix of failure modes have for long attracted the attention of experimenters. They have endeavored to find a valid way to extrapolate such data to estimate product life distribution at a design stress as it is clearly wrong to use data on one failure mode to estimate the life distribution of another failure mode. Several papers have been written on analyzing ALT when failures can occur from any one of p statistically independent causes. ([4], [6], [8], [9], [10]). Authors of [2] have formulated variable sampling plans for life test in the presence of single/multiple failure modes under Type I
progressive censoring. However, there does not seem to exist any work in the literature that couples the acceptance sampling plan with life testing under partially accelerated environment and competing causes of failure.

The repetitive group sampling (RGS) plan was first proposed by [11] for the inspection of attributes’ quality characteristics in which a sample is drawn, the number of defectives counted, then according to a fixed criterion, the lot is either accepted, rejected, or the sample is completely disregarded and a new one drawn. This is continued until the fixed criterion tells us to either reject or accept the lot without any need to keep a track of how many times sampling has been carried out. According to Sherman, the RGS plan gives the minimal sample size, the desired protection, and an intermediate value in sample size efficiency between the single sampling and sequential sampling plan. [11] has pointed out that the RGS plans lead to discarding the data and therefore, throwing information away, however, this is compensated with minimum inspection in terms of average sample number. An RGS plan for variable inspection has been studied by [1] for a normal distribution. A variable repetitive group sampling plan under failure-censored reliability tests for Weibull distribution by [7].

In this paper, we have formulated time-censored step-stress partially accelerated life test variable repetitive group sampling (RGS) plan with competing causes of failure using tampered failure rate model ([3], [5]), and Weibull life distribution. The optimum plan proposed consists in finding out optimum stress change point and optimum sample size by minimizing the average sample number of a lot such that producer’s and consumer’s interests are safeguarded. Bilevel programming approach is used for the purpose. The method developed has been explained using an example.

**Notation**

\[ n_{1j} \] number of units that fail before \( \tau \) due to the failure cause \( j \), \( j = 1, 2 \)

\[ n_{2j} \] number of units that fail after \( \tau \) due to the failure cause \( j \), \( j = 1, 2 \).

\[ n_c \] number of units censored

**Assumptions**

1. Each unit has two statistically independent potential failure times corresponding to two causes of failure.
2. Failure time of a unit is the smallest of its two potential failure times.
3. The lifetime of an item tested under each failure cause at use as well as at accelerated condition follows a Weibull distribution.
4. For each failure cause, a tampered failure rate model is assumed.

**Test Procedure**

1. All ‘\( n \)’ items are first tested at use condition.
2. If any item out of ‘\( n \)’ items does not fail at use condition by pre-specified time \( \tau \), then it is put on accelerated condition, and run until censoring time \( \eta \).
3. The test is continued until all test items fail, or a prescribed censoring time \( \eta \) whichever occurs earlier, and the test conditions remain the same.
4. Failure times and failure causes of test units are jointly observed continuously.

**Model**

The cdf of Weibull distribution with shape parameter \( \delta \) and scale parameter \( \theta \) is given by:
\[ G'(t) = 1 - \exp \left\{-t^\delta / \theta_1 \right\}, \quad 0 \leq t < \infty. \] \hfill (1)

Thus, the cdf of lifetime \( T_j \) due to failure cause \( j = 1, 2 \) under normal operating condition is 
\[ H_j(t) = 1 - \exp \left\{-t^\delta / \theta_j \right\}, \quad 0 \leq t < \infty, \] 
which is the smallest of its two potential failure times, \( i.e. \), \( T = \min\{T_1, T_2\} \) with cdf given by:
\[ H(t) = 1 - (1 - H_1(t))(1 - H_2(t)) = 1 - \exp \left\{-(\theta_1^\delta + \theta_2^\delta) t^\delta \right\} \] \hfill (2)

It is assumed that there is a lower specification, \( L \), so that an item having the lifetime, \( T \), less than \( L \) is regarded as non-conforming. Using (1), the fraction nonconforming (or unreliability at time \( L \)) is obtained as:
\[ p = P[T < L] = 1 - \exp \left\{-L^\delta / \theta_1 \right\}, \quad 0 \leq t < \infty. \] \hfill (3)

(3) can be re-written as:
\[ p = 1 - \exp \{-w\}, \] \hfill (4)

where \( w = L^\delta / \theta_1 = -\log(1-p) \). A part is considered as “good” if the unreliability is low. Also, the mean life is given by:
\[ \mu = \int_0^\infty \exp \left\{-t^\delta / \theta_1 \right\} dt = (1/\delta) \theta_1^\delta / \delta. \] \hfill (5)

Under partially accelerated environmental condition, using the tampered failure rate model and Weibull distributed life assumptions, the cdf of \( W_j \) due to failure cause \( j = 1, 2 \) is given by:
\[ G_j(t) = G_j(t; \theta_j) = \begin{cases} 1 - \exp \left\{-t^\delta / \theta_j \right\}, & \text{if } 0 \leq t < \tau \\ 1 - \exp \left\{-t^\delta / \theta_j \right\} \exp \left\{-A(t^\delta - t^\delta) / \theta_j \right\}, & \text{if } \tau \leq t < \infty, \end{cases} \]
for \( j = 1, 2 \), and the corresponding pdf of \( W_j \) is given by
\[ g_j(t) = g_j(t; \theta_j) = \begin{cases} (\delta t^{\delta-1} / \theta_j) \exp \left\{-t^\delta / \theta_j \right\}, & \text{if } 0 < t < \tau \\ 6A \delta t^{\delta-1} / \theta_j \exp \left\{-t^\delta / \theta_j \right\} \exp \left\{-A(t^\delta - t^\delta) / \theta_j \right\}, & \text{if } \tau \leq t < \infty, \end{cases} \]
for \( j = 1, 2 \) and acceleration factor, \( A > 1 \). Since we will observe only the smaller of \( W_1 \) and \( W_2 \), let \( W = \min\{W_1, W_2\} \) denote the overall failure time of a test unit under PALT then, its cdf and pdf are obtained to be:
\[ F(t) = F(t; \theta_1, \theta_2) = 1 - (1 - G_1(t))(1 - G_2(t)) \]
\[ = \begin{cases} 1 - \exp \left\{-((\theta_1^\delta + \theta_2^\delta) t^\delta \right\}, & \text{if } 0 \leq t < \tau \\ 1 - \exp \left\{-((\theta_1^\delta + \theta_2^\delta) t^\delta \right\} \exp \left\{A((\theta_1^\delta + \theta_2^\delta) t^\delta - \tau^\delta) / \theta_1 \right\}, & \text{if } \tau \leq t < \infty. \end{cases} \] \hfill (6)

\[ f(t) = f(t; \theta_1, \theta_2) = \begin{cases} (\delta t^{\delta-1} / \theta_1^\delta) \exp \left\{-((\theta_1^\delta + \theta_2^\delta) t^\delta \right\}, & \text{if } 0 < t < \tau \\ (6A \delta t^{\delta-1} / \theta_1^\delta) \exp \left\{-((\theta_1^\delta + \theta_2^\delta) t^\delta \right\} \exp \left\{A((\theta_1^\delta + \theta_2^\delta) t^\delta - \tau^\delta) / \theta_1 \right\}, & \text{if } \tau \leq t < \infty. \end{cases} \]
respectively, where \( \theta_1^{-1} = \theta_2^{-1} + \theta_3^{-1} \). Furthermore let \( i \) denote the indicator for the cause of failure. Then, under our assumptions, the joint pdf of \((W, C)\) is given by

\[
f_{W,C}(t, j) = \begin{cases} 
\delta \theta_j^{-1} \exp\{-t^\delta / \theta_j\}, & \text{if } 0 < t < \tau \\
A \delta \theta_j^{-1} \exp\{-t^\delta / \theta_j\} \exp\{-A(t^\delta / \theta_j)\}, & \text{if } \tau \leq t < \infty,
\end{cases}
\]

for \( j, j' = 1, 2 \) and \( j' \neq j \).

The relative risk imposed on a test unit before \( \tau \) due to failure cause \( j \) by

\[
\pi_{ij} = \Pr[C = j \mid W = \theta_j + \theta_j] = \Pr[C = j \mid W = \theta_j], \quad i = 1, 2.
\]

Similarly, the relative risk after \( \tau \) due to the cause \( j \) is denoted by

\[
\pi_{ij} = \Pr[C = j \mid W = \theta_j + \theta_j] = \Pr[C = j \mid W = \theta_j], \quad i = 1, 2.
\]

They are simply the proportion of failure rates in the given time frame. It follows from (8)-(9) that \( W \) and \( C \) are independent given the time frame in which a failure has occurred.

3. Lot Acceptance Procedure

3.1 Single Sampling Plan (SSP)

1. Draw a random sample of size \( n \) from a lot.
2. Calculate the quantity \( \hat{\nu} = \hat{\mu} \), where \( \hat{\mu} \) is the estimate of mean life time, \( \mu \), given in (5). Accept the lot if \( \nu \geq \nu_0 \) and reject the lot if \( \nu < \nu_0 \).

3.2 Variables Repetitive Group Sampling Plan (VRGSP)

The proposed variables repetitive group sampling plan (VRGSP) for the Weibull distribution is as follows:

1. Take a random sample of size \( n \) from a lot.
2. Calculate the quantity \( \hat{\nu} = \hat{\mu} \), where \( \hat{\mu} \) is the estimate of mean life time, \( \mu \), given in (5). Accept the lot if \( \nu \geq \nu_0 \) and reject the lot if \( \nu < \nu_0 \).
3. If \( \nu_0 \leq \nu < \nu_0 \), then repeat the above steps (1)-(2) through re-sampling.

4. Log Likelihood

Under the assumption of the tampered failure rate model, the likelihood function of \( \theta \) based on the Type-I censored sample, using (1) and (2), is

\[
L(\theta) = \prod_{i=1}^{n_1} f(t_i, c_i) \prod_{j=1}^{n_2} f(T_j, c_j) (1 - F(T_j))^k_j,
\]

where

\[
L(\theta) = \prod_{i=1}^{n_1} [\delta \theta_i^{-1} e^{-t_i^\delta / \theta_i}] \times \prod_{i=1}^{n_2} [\delta \theta_i^{-1} e^{-t_i^\delta / \theta_i}] \times \prod_{j=1}^{m} [A \delta \theta_j^{-1} e^{-t_j^\delta / \theta_j} e^{-A(t_j^\delta / \theta_j)}] \times \prod_{j=1}^{m} [A \delta \theta_j^{-1} e^{-t_j^\delta / \theta_j} e^{-A(t_j^\delta / \theta_j)}] \
\]

where \( n_1 = n_{11} + n_{12} \), \( n_2 = n_{21} + n_{22} \), and \( n = n_1 + n_2 \). Thus, the log likelihood \( L \) of a Type I censored observation is
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\[
\log L = \left(n_{11} + n_{12} + n_{21} + n_{22}\right) \log \delta - \left(n_{11} + n_{12}\right) \log \theta_1 - \left(n_{12} + n_{22}\right) \log \theta_2 + \left(n_{12} + n_{22}\right) \log A \\
+ \left(\delta - 1\right) \left(\sum_{i=1}^{n_{11}} \log t_i + \sum_{i=1}^{n_{12}} \log t_i + \sum_{i=1}^{n_{21}} \log t_i - \sum_{i=1}^{n_{11}} \frac{t_i^\delta}{\theta_1} - \sum_{i=1}^{n_{12}} \frac{t_i^\delta}{\theta_1} + \sum_{i=1}^{n_{21}} \frac{t_i^\delta}{\theta_1} - \sum_{i=1}^{n_{22}} \frac{t_i^\delta}{\theta_1} - \sum_{i=1}^{n_{12}} A(t_i^\delta - \tau^\delta) / \theta_1 \right) \\
- \sum_{i=1}^{n_{22}} A(t_i^\delta - \tau^\delta) / \theta_1 - \left(n_{12} + n_{22}\right) (\tau^\delta / \theta_1) - n_{12} A(T^\delta - \tau^\delta) / \theta_1.
\]

The model proposed above can be easily extended and generalized to accommodate multiple stress levels and multiple competing failure causes.

5. Fisher Information Matrix

For a single observation, the first partial derivatives of \( \log L \) with respect to \( \theta_1, \theta_2 \) and \( A \) are given in the Appendix. These expressions when set equal to zero, yield likelihood equations. The parameters values that are the solution to these equations are the maximum likelihood estimates. The MLEs do not exist if \( n_{11} = n_{21} = 0 \), or \( n_{12} = n_{22} = 0 \), or \( n_{22} = n_{21} = 0 \). Thus, the MLEs exist if there is atleast one failure due to each failure cause.

The elements of the Fisher information matrix \( F \) obtained by taking the negative expectations of the second partial and mixed partial derivatives of \( \log L \) with respect to \( \theta_1, \theta_2 \) and \( A \) are also given in the Appendix.

6. Operating Characteristic (OC) Curve

The proposed variable RGS plan (VRGSP) is characterized by the parameters, namely, \( n, k_a, k_r \). It reduces to a single sampling plan (SSP) if \( k_a = k_r \). Based on the asymptotic distribution theory, \( \sqrt{n} \left( v - \mu \right) / \sqrt{\text{Var}(v)} \sim \text{N}(0,1) \) as \( n \to \infty \), therefore, the lot acceptance probability based on a single sample (group) when the lot quality is \( p \) will be

\[
P_a(p) = \text{Pr}[v \geq L | k_a] = 1 - \Phi \left( \sqrt{n} \left( k_a - \Gamma \left(1/\delta \right) / \delta w^{1/\delta} \right) / \sqrt{\text{Var}(v)} / L^2 \right),
\]

where \( \Phi(\cdot) \) is the cdf of a standard normal variate, and \( w = L^2 / \theta_1 = -\log(1-p) \) (see (4)). On the other hand, the lot rejection probability is given by

\[
P_r(p) = \text{Pr}[v < L | k_a] = \Phi \left( \sqrt{n} \left( k_a - \Gamma \left(1/\delta \right) / \delta w^{1/\delta} \right) / \sqrt{\text{Var}(v)} / L^2 \right).
\]

The OC curve is obtained by plotting \( L(p) \approx L_a \) against fraction nonconforming \( p \), where the lot acceptance probability based on a single sample (group) when the lot quality is \( p \) is

\[
L_{\text{SSP}}(p) = \text{Pr}[v \geq L | k | p] = 1 - \Phi \left( \sqrt{n} \left( k - \Gamma \left(1/\delta \right) / \delta w^{1/\delta} \right) / \sqrt{\text{Var}(v)} / L^2 \right),
\]

and the lot acceptance probability based on a repetitive group sample when the lot quality is \( p \) is given by

\[
L_{\text{VRGSP}}(p) = \frac{P_a(p)}{P_r(p) + P_a(p)} = \frac{1 - \Phi \left( \sqrt{n} \left( k_a - \Gamma \left(1/\delta \right) / \delta w^{1/\delta} \right) / \sqrt{\text{Var}(v)} / L^2 \right)}{1 - \Phi \left( \sqrt{n} \left( k_a - \Gamma \left(1/\delta \right) / \delta w^{1/\delta} \right) / \sqrt{\text{Var}(v)} / L^2 \right) + \Phi \left( \sqrt{n} \left( k_a - \Gamma \left(1/\delta \right) / \delta w^{1/\delta} \right) / \sqrt{\text{Var}(v)} / L^2 \right)}.
\]
7. Formulation of an Optimization Problem

Bilevel programming is a mathematical optimization problem that contains an optimization problem in the constraints. The bilevel programs facilitate the formulation of several practical problems that involve a hierarchical decision making process like engineering design, analysis of competitive economies, transport system planning, signal optimization, network design, strategic offensive and defensive force problems, government regulation, management, etc.

In this paper, in the first stage, optimum stress change point is obtained by using D-optimality criterion which consist in minimizing $1/F$; followed by the second stage in which optimal sample size has been obtained by minimizing the average sample number such that producer’s and consumer’s interest are safeguarded. In this section, bilevel programming has been devised for SSP and VRGSP.

7.1 Bilevel Programming-Variable Repetitive Group Sampling Plan

$$\min_{n} \frac{n}{F}$$

s.t. $0 \leq \tau \leq \eta$.

where $(\tau)$ solves

$$\min_{n} \text{ASN}_{VRGSP} = \text{ASN}(p_1) + \text{ASN}(p_2)$$

$$= \frac{1}{1 - \Phi} \left( \frac{\sqrt{n} (k_1 - \Gamma(1/\delta) / \delta w_1^{1/\delta})}{\sqrt{\text{Var}(v)/L^2}} \right) + \Phi \left( \frac{\sqrt{n} (k_1 - \Gamma(1/\delta) / \delta w_1^{1/\delta})}{\sqrt{\text{Var}(v)/L^2}} \right)$$

$$+ \frac{1}{1 - \Phi} \left( \frac{\sqrt{n} (k_2 - \Gamma(1/\delta) / \delta w_2^{1/\delta})}{\sqrt{\text{Var}(v)/L^2}} \right) + \Phi \left( \frac{\sqrt{n} (k_2 - \Gamma(1/\delta) / \delta w_2^{1/\delta})}{\sqrt{\text{Var}(v)/L^2}} \right)$$

$$s.t. A_1(w_1) = \frac{1 - \Phi} {1 - \Phi} \left( \frac{\sqrt{n} (k_1 - \Gamma(1/\delta) / \delta w_1^{1/\delta})}{\sqrt{\text{Var}(v)/L^2}} \right) + \Phi \left( \frac{\sqrt{n} (k_1 - \Gamma(1/\delta) / \delta w_1^{1/\delta})}{\sqrt{\text{Var}(v)/L^2}} \right) \geq 1 - \bar{\alpha},$$

$$A_2(w_2) = \frac{1 - \Phi} {1 - \Phi} \left( \frac{\sqrt{n} (k_2 - \Gamma(1/\delta) / \delta w_2^{1/\delta})}{\sqrt{\text{Var}(v)/L^2}} \right) + \Phi \left( \frac{\sqrt{n} (k_2 - \Gamma(1/\delta) / \delta w_2^{1/\delta})}{\sqrt{\text{Var}(v)/L^2}} \right) \leq \bar{\beta},$$

$$n > 2, k_1 > k_2, k_2 > 0, k_1 > 0, k_1 > 0,$$

Let $\Psi_3 = \{(\tau) : 0 \leq \tau \leq \eta\}$, $\Psi_4 = \{(\text{ASN}_{VRGSP}) : A_1(w_1) \geq 1 - \bar{\alpha}, A_2(w_2) \leq \bar{\beta}, n > 2, k_1 > k_2, k_2 > 0, k_1 > 0\}$, and $(\tau^*) = \text{Argmin}(n / |F| : (\tau) \in \Psi_3)$. Then the optimization problem reduces to $\min_{n} \{(\text{ASN}_{VRGSP}) : n \in \Omega_2, (\tau) = (\tau^*)\}$. 
7.2 Bilevel Programming-Single Sampling Plan

The optimal design problem can be formulated as a nonlinear optimization problem: The optimal design problem can be formulated as:

\[
\begin{align*}
\text{Min } n / F & \\
\text{s.t. } 0 \leq \tau \leq \eta, & \\
\text{where } (\tau) \text{ solves } & \\
\end{align*}
\]

\[
\begin{align*}
\text{Min } & \text{ASN}_{\text{SSP}} = \text{ASN}(w_1) + \text{ASN}(w_2) = 2n \\
\text{s.t. } & A_1(w_1) = 1 - \Phi \left( \frac{\sqrt{n} \left( k - \Gamma \left( 1/\delta \right) / \delta w_1^{1/\delta} \right)}{\text{Var}(v)} \right) \geq 1 - \bar{\alpha}, \\
& A_4(w_2) = 1 - \Phi \left( \frac{\sqrt{n} \left( k - \Gamma \left( 1/\delta \right) / \delta w_2^{1/\delta} \right)}{\text{Var}(v)} \right) \leq \bar{\beta}, n \geq 1, n \in Z^+, \\
\end{align*}
\]

Let \( \Psi_1 = \{ (\tau) : 0 \leq \tau \leq \eta \}, \quad \Psi_2 = \{ (\text{ASN}_{\text{SSP}}) : A_1(w_1) \geq 1 - \bar{\alpha}, A_4(w_2) \geq 1, n \geq 1, n \in Z^+ \}, \)

\[\text{and } (\tau^*) = \text{Argmin}\{ n / F : (\tau) \in \Psi_1 \}. \]

Then the optimization problem reduces to:

\[\text{Min } \{ (\text{ASN}_{\text{SSP}}) : n \in \Omega, (\tau) = (\tau^*) \}.\]

8. Illustrative Example

Consider the hypothetical dataset: \( \theta_1 = 25, \theta_2 = 35, A = 1.1, \delta = 1.5 \) & \( \eta = 2.5. \) The optimal value of \( \tau \) obtained using (21) is \( \tau^* = 1.485. \) The estimates of relative risks are \( \hat{\eta}_{11} = 0.583, \hat{\eta}_{12} = 0.416. \) This shows that there is a 58% chance for a test unit to fail by the failure cause 1 and 42% chance for a test unit to fail by the failure cause 2.

The optimal values of sample size and acceptability constant w.r.t. producer and consumer’s non-confirming proportions are \( n_{\text{VRGSP}}^* = 16, \quad k_{\text{VRGSP}}^* = 141.646 \) and \( k_{\text{SSP}}^* = 77.113 \) under VRGSP and \( n_{\text{SSP}}^* = 25, \quad k_{\text{SSP}}^* = 183.627 \) under single sampling plan, respectively.

Figure 1 depicts OC curve under VRGSPs and SSPs by plotting \( L(p) \) against different values of \( p \) using the two points \( (p_1, 1 - \bar{\alpha}) \) and \( (p_2, \bar{\beta}) \) as \( (0.01, 0.99) \) and \( (0.10, 0.05). \)
Table 1 presents optimum VRGSP and SSP for various values of $p_1$ and $p_2$ when $(1-\alpha, \beta) = (0.99, 0.10)$. It can be seen that the ASN$_{VRGSP}$ is smaller than the ASN$_{SSP}$. It is also observed that for given $p_2$ as $p_1$ decreases, $n^*$ decreases.

Table 1: Effects of Various Values of $(p_1, 1-\alpha)$ and $(p_2, \beta)$ in the Optimal ALTSP

<table>
<thead>
<tr>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$n_{\text{VRGSP}}$</th>
<th>$k_{\text{VRGSP}}$</th>
<th>$k_{\text{SSP}}$</th>
<th>ASN$_{VRGSP}$</th>
<th>ASN$_{SSP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.500</td>
<td>0.200</td>
<td>85</td>
<td>1.512</td>
<td>1.238</td>
<td>207</td>
<td>116</td>
</tr>
<tr>
<td>0.100</td>
<td>0.050</td>
<td>35</td>
<td>1.647</td>
<td>1.554</td>
<td>73</td>
<td>37</td>
</tr>
<tr>
<td>0.010</td>
<td>0.005</td>
<td>25</td>
<td>1.731</td>
<td>1.731</td>
<td>50</td>
<td>25</td>
</tr>
</tbody>
</table>

9. Sensitivity Analyses

To formulate an optimum test plan, one needs information about the values of $\theta_1$, $\theta_2$, $A$, which are usually unknown. Therefore, they have to be approximated from experience, similar data, or preliminary test. Incorrect choice of pre-estimates gives a non-optimal test plan and results in poor estimates of the parameters of life distribution at design stress. We have investigated the effect of pre-estimates for some selected values of $\theta_1$, $\theta_2$, $A$ in terms of relative stress change point, $\tau^*$ are presented in Table 2, using $\%\Delta \tau^* = (|\tau^* - \tau^0|/\tau^*)\times100$, where $\tau^*$ is the optimal stress change point for the plans obtained with the correctly specified values, and $\tau^0$ is the stress change point for the plans obtained with misspecified values. The sensitivity analysis indicates that because these values have a very small effect on the optimal value $\tau^*$, they are not sensitive. Therefore, the optimum plans proposed are robust, and initial estimates have a small effect on optimal values.

Table 2: Effects of Incorrect Pre-estimates of $\theta_1$, $\theta_2$, $A$ in the Optimal ALTSP

<table>
<thead>
<tr>
<th>% change</th>
<th>$\Delta \tau^*$ due to ±1% to ±5% change in $\theta_1$, $\theta_2$, $A$</th>
<th>$\Delta \tau^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>1.089 1.487 0.127 24.750 1.484 0.026 34.650 1.485 0.019</td>
<td></td>
</tr>
<tr>
<td>-1%</td>
<td>1.078 1.489 0.256 24.500 1.484 0.040 33.950 1.484 0.057</td>
<td></td>
</tr>
<tr>
<td>2%</td>
<td>1.122 1.481 0.127 25.500 1.486 0.050 35.700 1.485 0.036</td>
<td></td>
</tr>
<tr>
<td>-2%</td>
<td>1.078 1.489 0.256 24.500 1.484 0.040 33.950 1.484 0.057</td>
<td></td>
</tr>
<tr>
<td>3%</td>
<td>1.133 1.479 0.369 25.750 1.486 0.075 36.050 1.486 0.054</td>
<td></td>
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<tr>
<td>-3%</td>
<td>1.090 1.484 0.239 24.500 1.484 0.030 33.590 1.484 0.057</td>
<td></td>
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<tr>
<td>4%</td>
<td>1.144 1.478 0.488 26.000 1.486 0.099 36.400 1.486 0.071</td>
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<tr>
<td>-4%</td>
<td>1.056 1.493 0.521 24.000 1.483 0.136 33.250 1.483 0.097</td>
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<tr>
<td>5%</td>
<td>1.155 1.476 0.605 26.250 1.487 0.123 36.750 1.486 0.088</td>
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<tr>
<td>-5%</td>
<td>1.045 1.495 0.657 23.750 1.483 0.136 33.250 1.483 0.097</td>
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</tr>
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</table>
10. Conclusion

This paper deals with the design of optimum time-censored step-stress PALTSP with competing causes of failure using tampered failure rate model. The variable repetitive group sampling plan has been used with Weibull life distribution. The optimal plan consist in finding optimal stress change point, sample size and acceptability constants. The VRGSP is compared with SSP, and it is found that the optimum sample size is reduced by using the VRGSP as compared with the SSP.

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References


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Appendix

\[
\frac{\partial \log L}{\partial \theta_1} = -(n_i + n_{i\ast}) / \theta_1 + (1 / \theta_1^2) \left( \sum_{j=1}^{n_i} \sum_{k=1}^{n_{i\ast}} \left[ \frac{t_i^j + t_i^k + A \sum_{l=1}^{n_i} (t_i^l - \bar{t}) + A \sum_{l=1}^{n_{i\ast}} (t_i^l - \bar{t})}{\sqrt{n_i}} \right] + (n_i + n_{i\ast}) \theta_1^2 / \theta_1^2 + n_i A(T - \bar{t}) / \theta_1^2 \right),
\]

\[
\frac{\partial \log L}{\partial \theta_2} = -(n_i + n_{i\ast}) / \theta_2 + (1 / \theta_2^2) \left( \sum_{j=1}^{n_i} \sum_{k=1}^{n_{i\ast}} \left[ \frac{t_i^j + t_i^k + A \sum_{l=1}^{n_i} (t_i^l - \bar{t}) + A \sum_{l=1}^{n_{i\ast}} (t_i^l - \bar{t})}{\sqrt{n_i}} \right] + (n_i + n_{i\ast}) \theta_1^2 / \theta_1^2 + n_i A(T - \bar{t}) / \theta_1^2 \right),
\]

\[
\frac{\partial \log L}{\partial A} = n_i / A - (1 / \theta_1) \left( \sum_{j=1}^{n_i} \sum_{k=1}^{n_{i\ast}} (t_i^j - \bar{t}) + (t_i^k - \bar{t}) + n_i A(T - \bar{t}) \right).
\]

\[
E[-\delta^2 \log L / \partial \theta_j^2] = -(E[n_{i\ast}] + E[n_i]) / \theta_j^2 + (2 / \theta_j^4) (E[n_{i\ast}] + E[n_i]) \bar{t}^2 + A_{i\ast}(\bar{t}) + A_i(\bar{t})
\]

\[
+ (2 / \theta_j^2) (A_{i\ast}(\bar{t}) + A_i(\bar{t}) + E[n_i] (\bar{t}^2 - \bar{t}^2)); j = 1, 2,
\]

\[
E[-\delta^2 \log L / \partial \theta \partial A] = -(1 / \theta_1) (A_{i\ast}(\bar{t}) + A_i(\bar{t}) + E[n_i] (\bar{t}^2 - \bar{t}^2)); j = 1, 2,
\]

\[
E[-\delta^2 \log L / \partial A^2] = E[-\delta^2 \log L / \partial \theta \partial A],
\]

\[
E[-\delta^2 \log L / \partial A^2] = E[n_{i\ast}] / \lambda^2,
\]

where

\[
E[n_{i\ast}] = n(\lambda_i^2/) / \theta_i^2(1 - e^{-\lambda_i^2}),
\]

\[
E[n_i] = n(1 - \lambda_i^2 / \theta_i^2)(1 - e^{-\lambda_i^2}),
\]

\[
E[n_{i\ast}] = n(\lambda_i^2 / \theta_i^2) e^{-\lambda_i^2}(1 - e^{-\lambda_i^2}),
\]

\[
E[n_{i\ast}] = n(1 - \theta_i^2) e^{-\lambda_i^2}(1 - e^{-\lambda_i^2}),
\]

\[
A_{i\ast}(\bar{t}) = E[\sum_{j=1}^{n_{i\ast}} w_{ij\ast}] = n(1 - \theta_i^2 / \theta_i^2)(1 - e^{-\lambda_i^2}(\bar{t}^2 - \bar{t}^2)),
\]

\[
A_i(\bar{t}) = E[\sum_{j=1}^{n_i} w_{ij}] = n(1 - \theta_i^2 / \theta_i^2)(1 - e^{-\lambda_i^2}(\bar{t}^2 - \bar{t}^2)),
\]

\[
A_{i\ast}(\bar{t}) = E[\sum_{j=1}^{n_{i\ast}} (w_{ij\ast}^i - \bar{v})] = n(1 - \theta_i^2 / \theta_i^2)(1 - e^{-\lambda_i^2}(\bar{t}^2 - \bar{t}^2)),
\]

\[
A_i(\bar{t}) = E[\sum_{j=1}^{n_i} (w_{ij}^i - \bar{v})] = n(1 - \theta_i^2 / \theta_i^2)(1 - e^{-\lambda_i^2}(\bar{t}^2 - \bar{t}^2)),
\]

and

\[
A_{i\ast}(\bar{t}) = E[\sum_{j=1}^{n_{i\ast}} (w_{ij\ast}^i - \bar{v})] = n(1 - \theta_i^2 / \theta_i^2)(1 - e^{-\lambda_i^2}(\bar{t}^2 - \bar{t}^2)).
\]