Condition Monitoring of Shinkansen Tracks based on
Inverse Analysis

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Abstract: This paper demonstrates the possibility to estimate the track irregularities of
Shinkansen tracks using car-body motions only. In an inverse problem to estimate track
irregularity from car-body motions, a Kalman Filter was applied to solve the inverse
problem. This technique is utilized to estimate the track irregularity in longitudinal level.
It can be concluded that the track irregularity estimation in longitudinal level is possible
with acceptable accuracy for practical use with method.

Keywords: Railways, condition monitoring, track irregularity, inverse analysis, Kalman
filter

1. Introduction

The track irregularity deteriorates ride comfort and running safety, track condition
monitoring for maintenance is one of the most important tasks for railway companies. In
general, track irregularity is measured several times a month by a specially designed track
geometry car (TGC). However, this measurement method is limited by expensive cost of
equipment.

Waston et al. [1, 2] demonstrated the track irregularity monitoring using bogie-
mounted sensors. Alfi et al. [3] proposed a frequency domain technique for estimating
long wavelength track irregularity from on-board measurement. Naganuma et al. [4]
demonstrated that a dynamic programing filter with regularization can be used to estimate
vertical track irregularity from vertical car-body acceleration.

In 2009, the track condition monitoring system called RAIDARSS (Real time
Acceleration Inspection Device with Automatic Recording System for Shinkansen) [5]
started operation in the Tokaido Shinkansen line. Inertial measurement devices are
mounted on six N700 series Shinkansen train sets and they measure the track several
times a day by the in-service vehicle. The in-service vehicle measurement complements
the track geometry car measurement. However, RAIDARSS needs high frequent
maintenance because accelerometers are mounted on axle-boxes of vehicles. If the track
irregularity can be estimated from car-body acceleration of in-service vehicle, it will
enable more frequent track condition monitoring by a portable device [6].

This study proposes the track irregularity estimation techniques from car-body
motion only. The inverse analysis technique is applied to estimate the track irregularity.
This technique is frequently utilized to determine an unknown input signal (track
irregularity) from a known output signal (car-body acceleration).

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2. Construction and Validity of Vehicle Model

2.1 Vehicle Model

Figure 1 shows a railway vehicle model in vertical direction. Where $z_c$ is a car-body displacement, $z_{t1}$ and $z_{t2}$ are front and rear bogie displacement, $\theta_c$ is a car-body pitch angle, $\theta_{t1}$ and $\theta_{t2}$ are front and rear bogie pitch angle. Inputs, $r_{1a}$, $r_{1b}$, $r_{2a}$, $r_{2b}$, denote the track irregularities. The equation of motion for 6 DOF railway vehicle running on a straight track can be written as

$$M\ddot{z}(t) + C\dot{z}(t) + Kz(t) = Dr(t) + Er(t), \quad (1)$$

where, $Z(t) = [z_c, l_1 \theta_c, z_{t1}, l_2 \theta_{t1}, z_{t2}, l_3 \theta_{t2}]$, $r(t) = [r_{1a}, r_{1b}, r_{2a}, r_{2b}]$. Coefficient matrices of the equation (1) are shown in the appendix.

Discretizing the equation (1) using numerical integration yields the following state equation and observation equation:

$$x_n = Fx_{n-1} + Gu_n + w_n, \quad (2)$$

$$y_n = Hx_n + v_n, \quad (3)$$

For the discretization (Data interval $h$), we used the Newmark $\beta$ method ($\gamma = 1/2$, $\beta = 1/6$) which is the linear acceleration method. where $x_n$ is the state vector, $u_n$ is the input vector, $y_n$ is the output vector, $w_n$ and $v_n$ are process noise and measurement noise. $F$ is the state transition matrix, $G$ is the input matrix, $H$ is the observation matrix. The state transition matrix and the input matrix can be expressed by $F = A^kB$ and $G = A^kC$ using following $A$, $B$ and $C$:

$$A = \begin{bmatrix} I & 0 & -\beta h^2 \\ 0 & I & -0.5h \\ K & C & M \end{bmatrix}, \quad B = \begin{bmatrix} I & h & (0.5 - \beta)h^2 \\ 0 & I & 0.5h \\ 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

![Figure 1: Linear Vehicle Model](image)

2.2 Frequency Response Function and System Identification

When considered as a dynamic system to the running vehicle, it is considered that the input is the track irregularity, and the output is the vehicle motions in vertical direction.
This input and output relationship can be expressed by the frequency response functions if it is approximated to the linear system.

To confirm a validity of the simple model and parameter identification, frequency analysis was carried out. Figure 2 shows the measured track irregularity by the track geometry car, and figure 3 shows the measured car-body acceleration by the in-service car. Figure 4 shows the coherence between track irregularity and car-body acceleration. This relationship shows strong correlation, thus it can be assumed the dynamics as linear system. Figure 5 shows the frequency response functions of gain characteristic and phase characteristic (dotted line: estimated, solid line: model). There is good correspondence between the estimated from actual measurement and the calculated value. These time and frequency analysis demonstrate that the model and parameters are appropriate.
3. Track Irregularity Estimation Technique

3.1 Calculation of the Car-body Acceleration in Velocity Changes

The car-body acceleration has been calculated using the vehicle model in different vehicle velocity. Figure 6 shows the track irregularity and Figure 7 shows the corresponding vehicle velocity. The calculated car-body acceleration is shown on Figure 8. Stepwise vehicle velocity change is assumed in the calculation. In this study, a Gaussian noise was added to the calculated car-body acceleration, and it was used as the measurement data. The added Gaussian noise can be expressed by \( N(0, \sigma^2) \) with the standard deviation of 0.01 m/s².

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**Figure 5:** Frequency Response

**Figure 6:** Track Irregularity

**Figure 7:** Vehicle Velocity

**Figure 8:** Calculated Car-body Acceleration using Vehicle Model
3.2 Impulse Response Generation Algorithm

In this study, we use an impulse response for the inverse analysis. Figure 9 shows the vehicle characteristic at the velocity of 180km/h, 210km/h, 240km/h and 270km/h. It should be noted that the vehicle characteristic is affected by the vehicle velocity as shown in figure 9.

In this section, we propose the method to create the impulse response of a different velocity. The full vehicle impulse response (include 4-axes) is created by shifting the response of each axle according to the velocity and sampling frequency. This approach can reproduce the impulse response of a different velocity with only four responses. Figure 10 shows the concept of impulse response generation algorithm.

![Figure 9: Changes in Vehicle Characteristics (Vertical Direction)](image)

![Figure 10: Construction of Impulse Response with Velocity Change](image)

3.3 Track Irregularity Estimation Approach

This section analyses estimation technique of track geometry from measured signal on the car-body. This operation is generally unstable and the calculation result greatly receives the influence of the measurement noise because it is categorized as “inverse problems” that trace the causality of stable physical phenomena in the opposite direction. Therefore, stabilization to reduce the influence of the noise is indispensable, and we used a technique based on a Kalman filter for inverse analysis. The state equation and the observation equation for the inverse analysis can be written as follows:
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where, \( x_n \) is the track irregularity, \( y_n \) is the car-body acceleration, \( v_n \) is the measurement noise. Track irregularities are defined as the random walk model with the process noise \( w_n \). The symbol \( h \) denotes the impulse response, and the symbol \( L \) denotes the total number of impulse responses.

A Kalman filter is a well-known algorithm that estimates unknown state of the system from measurement signal containing noise. It is suitable for a real-time estimation. A procedure of the Kalman filter without input is shown below.

**Time Update Equations**

\[
\begin{align*}
\dot{x}_{n|n-1} &= Fx_{n|n-1}, \\
P_{n|n-1} &= FP_{n|n-1}F^T + GQG^T.
\end{align*}
\]

**Measurement Update Equations**

\[
\begin{align*}
K_n &= x_{n|n-1}H^T(HP_{n|n-1}H^T + R)^{-1}, \\
x_{n|n} &= x_{n|n-1} + K_n(y_n - Hx_{n|n-1}), \\
P_{n|n} &= P_{n|n-1} - K_nHP_{n|n-1}.
\end{align*}
\]

**4. Estimation Result of Track Irregularity**

**4.1 Evaluation Waveform**

The wavelength band which affects the running safety and ride comfort is important for track maintenance. Therefore, we are focusing on the 10m-wavelength track irregularity that affect the running safety, it can be treated as a 10m-chord versine (mid-chord offset) method. This method emphasizes the 10m-wavelength track irregularity, and it is used in real maintenance. The 10m-chord versine method can be calculated by actual track geometry by following as

\[
a(x) = b(x) - \frac{b(x+5) + b(x-5)}{2},
\]

where, \( a(x) \) is the 10m-chord versine, and \( b(x) \) is the actual track geometry.
4.2 Estimation Results

Figure 11 shows the estimation result of 10m-chord versine, also shown the measured track irregularity by using track geometry car. This estimation result is obtained from the simulated car-body acceleration with the measured track irregularity which is shown in Figure 8.

A broken line represents the velocity change point. The system noise variance is $\sigma_w^2=0.00006$ m$^2$, and the measurement noise variance is $\sigma_v^2=0.07$ (m/s$^2$)$^2$ in this simulation. Those values are chosen so that the calculation gives a satisfactory result.

A large estimation error can be seen at 40m length after velocity change, which is due to a transient response in step velocity change. Therefore, we need to exclude the 40m length for comparison and evaluation. We can see that the estimation result has a good accuracy in all velocity range.

\begin{align}
\text{Figure 11: Estimated Track Irregularity}
\end{align}

5. Evaluation of Estimation Result

5.1 RMSE (Root Mean Square Error)

The RMSE is the evaluation method that focuses on the difference between the estimated value and the measured value. In this study, we calculate the RMSE of 20m section, and it can be expressed by following as

\begin{equation}
RMSE = \sqrt{\frac{\sum_{i=1}^{N} (X_i - x_i)^2}{N}},
\end{equation}

where, $x_i$ is the estimated value, $X$ is the measured value, and $N$ is the number of the data. Figure 12 shows the RMSE of estimation result. It can be seen that the estimation performance is almost same in the different velocity sections.
Figure 12: RMSE of Estimation Result

5.2 MPC Metrics

This method calculates the MPC metrics using the Sprague & Geers correlation function [7]. The MPC metrics treats the two waves magnitude and phase. The magnitude component $M$ should be sensitive to difference in magnitude. The phase component $P$ should be sensitive to difference in phasing. The component $C$ is the combination with the magnitude and phase.

These characteristics of MPC metrics allow the analyst to identify the aspects of the curves that do not agree. For each component of MPC metrics, zero indicates that the two waves are identical. Table 1 shows the equations of each component and calculation results. The symbol $e_i$ and $m_i$ in equation represents the estimated value and measured value. From table 1, we can see that there is a high correlation between the measured value and estimated value at each different velocity.

It can be seen from Table 1 that the estimation performance in the phase is less than that in the magnitude. It is considered that the difference is due to the effect of transient response in the step velocity change. Thus, it is necessary to develop the estimation method in transient section to improve the estimation performance.

Table 1: MPC Metrics

<table>
<thead>
<tr>
<th>Equations</th>
<th>Magnitude</th>
<th>Phase</th>
<th>Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M = \sqrt{\sum \frac{e_i^2}{m_i^2} - 1}$</td>
<td>0.045</td>
<td>0.147</td>
<td>0.154</td>
</tr>
<tr>
<td>$P = \frac{1}{\pi} \cos^{-1} \left( \sqrt{\frac{\sum e_i m_i}{\sum m_i}} \right)$</td>
<td>0.011</td>
<td>0.224</td>
<td>0.225</td>
</tr>
<tr>
<td>$C = \sqrt{M^2 + P^2}$</td>
<td>0.017</td>
<td>0.185</td>
<td>0.186</td>
</tr>
<tr>
<td>270km/h</td>
<td>0.008</td>
<td>0.232</td>
<td>0.233</td>
</tr>
</tbody>
</table>

6. Conclusions

This paper describes a track irregularity estimation technique using Kalman filter from car-body acceleration. Track geometries were defined as the random walk model with the external input and the process noise. A Kalman filter was designed to estimate the unknown track geometry. The impulse response generator for different running velocity was employed.

Estimation results for step velocity change showed that the track geometry can be estimated with good accuracy for track condition monitoring. The step velocity change is not realistic but we need to consider such situation when we estimate the track geometry in acceleration or deceleration. Thus, the estimation method in acceleration or deceleration should be developed next.
In this study, an accelerometer is set on the car-body just above the rear bogie so that the effect of bending mode of the car-body can be reduced. It is necessary to confirm the effectiveness of the proposed method using real car-body acceleration in different vehicle velocity, and to estimate other track geometry including cross level. It is also necessary to discuss the robustness of the method on uncertainty regarding the suspension properties and car-body mass as the future work.

References


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Hitoshi Tsunashima is a Professor in the Department of Mechanical Engineering of Nihon University. His main research expertise includes modelling and simulation of aircraft, automobile, railway vehicles, applications of advanced control for railway vehicle, condition monitoring of railway, multi-body dynamics, human factors and ergonomics. His major research activities are condition monitoring of railway system, application of multiple model approach for vehicle state estimation and control, measurement of brain function using NIRS, Brain-Computer interface (BCI) using NIRS. PRIZES: 1992 Vincent Bendix Automotive Electronics Engineering Award (Best paper award in automotive electronics, SAE), 2007 Best paper award of 15th symposium on Transportation and Logistics, The Japan Society of Mechanical Engineers, Best presentation award of 2006 symposium on Automotive Engineering,
Appendix: Coefficient Matrices in Equation (1)

Equations of motion of the vehicle model in vertical direction can be written as follows

\[
\begin{align*}
M &= \text{diag}\{m, I_x / l_x^2, m, I_x / l_x^2, m, I_x / l_x^2\}, \\
C &= \begin{bmatrix}
2c_e & 0 & -c_e & 0 & -c_e & 0 \\
0 & 2c_e & -c_e & 0 & c_e & 0 \\
-c_e & -c_e & 2(c_e + c_s) & 0 & 0 & 0 \\
0 & 0 & 0 & 2c_s & 0 & 0 \\
-c_e & c_e & 0 & 0 & 2(c_e + c_s) & 0 \\
0 & 0 & 0 & 0 & 0 & 2c_s
\end{bmatrix}, \\
D &= \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}, \\
K &= \begin{bmatrix}
2k_e & 0 & -k_e & 0 & -k_e & 0 \\
0 & 2k_e & -k_e & 0 & k_e & 0 \\
-k_e & -k_e & 2(k_e + k_s) & 0 & 0 & 0 \\
0 & 0 & 0 & 2k_s & 0 & 0 \\
-k_e & k_e & 0 & 0 & 2(k_e + k_s) & 0 \\
0 & 0 & 0 & 0 & 0 & 2k_s
\end{bmatrix}, \\
E &= \begin{bmatrix}
k_e & k_e & 0 & 0 \\
k_e & k_e & 0 & 0 \\
k_e & k_e & 0 & 0 \\
k_e & k_e & 0 & 0 \\
k_e & k_e & 0 & 0 \\
k_e & k_e & 0 & 0
\end{bmatrix}.
\end{align*}
\]