A Dual-Stochastic Process Model for Surveillance Systems with the Uncertainty of Operating Environments Subject to the Incident Arrival and System Failure Processes

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Abstract: Surveillance system is widely used today to enhance the security level of the protected area. Reliability of the entire surveillance system is a critical issue since the breakdown of such system would leave the monitoring area unobserved and encountered much higher risk under the attacks. This paper presents a dual stochastic-process model for predicting the reliability of surveillance systems consisting of many subsystems (units) with considerations of the environmental factors, skill of intruder to avoid detection, the intrusion/incident arrival process and subsystem failure process. Several numerical examples are presented to illustrate the proposed model.

Keywords: Reliability, operating environments, stochastic processes, surveillance systems

1. Introduction

The application of surveillance systems provide continuous observation to the monitored area and offering important reference for the monitoring centralized personnel to make prompt actions against threats or incidents in general. It is a great enhancement to the security level of the detected area. The installation of surveillance systems including cameras and alarm detectors etc. in private sectors as well as in public areas are dramatically increased over the last decade [1]. A surveillance system often consists of multiple video cameras sending signals to centralized monitoring center and recording devices for review and investigation. With the rapid progress in automated control, image processing and high performance computing, the multiple cameras are capable of working in a coordinate manner under the control of the intelligent software programs to enhance the performance of the entire system [2].

Since the wide implementation of the surveillance systems and the fact that either failure or deterioration in performance of the system may result in severe damage to the protected facility, the reliability estimation of the system and maintenance-inspection scheduling is worth receiving serious attention [3]. Two incidents are discussed here to emphasize the importance of modeling the dual stochastic process and maintenance scheduling for the surveillance systems of this study. On January 3rd, 2010, the Newark Liberty International Airport had a security breach that one man reached the secure sterile area through a checkpoint exit without being screened by airport security [4]. Due to the breakdown of the surveillance recording system, the airport authority failed to identify the inadvertent intruder until they got the footage from the redundant cameras two hours later. The incident caused hours of delay in flights and thousands of passengers to be rescreened before boarding [4]. Another bizarre incident [5] recently occurred on August 13, 2012 when a man ran out of gas of his jet ski at Jamaica Bay in New York. He climbed the 8-foot-high perimeter fence and walked across the two runways seeking for help, without being detected by the perimeter intrusion detection system, which should be given out
series of warnings under the circumstance [5]. Those lessons raise the requirement of more accurate models for assessing the reliability of such critical systems.

Various approaches have been studied to estimate the reliability of complex systems including k out of n systems. The redundancy of components not only enhances the system reliability but also makes the system tolerant to errors. Mathur [6] derived reliability models for N modular redundant systems (NMR) [7] and generalized the models to general modular redundant systems [8] in which there are 3 types of modules, i.e., minimal set, on-line and spare.

When discussing the system structure configuration, normally the assumption of independent component lifetimes is implied. However, if the components of the system are sharing in a common operating environment which is differ from the laboratory test environment, a type of induced dependency is introduced between the components, which is well discussed in [9]. In the work, the authors presented the expression of the reliability model for a two-component system sharing a common uncertain environment following a Gamma distribution. They also discussed the effect of ignoring the environmental factor and applying the independence assumption between components that would first overestimate then underestimate the reliability of the system. Pham [10] presented a systemability model in which he also derived the expression of systemability for series, parallel and k-out-of-n system configurations. This work is further applied to model an automatic packaging machine and a motorcycle drive system where real data sets are available for parameter estimation [11].

Several authors [12-17] have considered multiple failure processes in modeling the reliability of complex systems. For example, Pham and Xie [12] have developed a two-process model to help the inspectors and policy-makers to make decisions on which maintenance facility would need to inspect for their next visit. Several studies have been proposed to analyze the competing risks in which two processes, the degradation path and the random shock arrival process, are typically taken into consideration [13-16]. Some of the works considered the two processes independent, but a recent study [17] has considered the connection between the degradation processes and the shocks where the aging of the system will cause itself more vulnerable to the shocks while the shocks will accelerate the degradation, if not destroy the system immediately.

In this paper, we present a framework to model the reliability of the surveillance systems with considerations of the environmental factors, skill of intruder to avoid detection, the intrusion/incident process and the subsystem failure process. The surveillance system often has complex structure, thus multi-unit system configuration should be taken into consideration instead of single unit system. Since the subsystems, i.e., cameras and the motion sensors, likely will be placed outdoor and stay close to each other, the uncertainty of common environment is worth to consider in the modeling that reflects the reality of reliability prediction. It is also worth to note that a careful analysis of the intrusion/incident arrival rate can provide useful information and, therefore, our proposed framework is worth the effort to study.

**Notation**

- $\lambda_I(t)$: the intensity function associated with the incident arrival process
- $\lambda_0$: baseline of the intensity function $\lambda_I(t)$
- $A(t, \theta)$: all factors that have impact on the intensity function $\lambda_I(t)$
- $T_s$: Time from the last detected incident
- $T_p$: A time interval in which the total number of detected incidents is counted
- $h(t)$: hazard rate function
- $\eta$: random common environmental factor
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\[ G(\eta) \] distribution function of the environmental factor

\[ R_i(t) \] reliability of subsystem \( i \)

\( \lambda \) intensity parameter of Weibull distribution

\( \gamma \) shape parameter of Weibull distribution

\( p(\omega_i) \) incident detection probability for subsystem \( i \)

\( R_{pi}(t) \) equivalent reliability of subsystem \( i \)

\( R_{ci}(t) \) traditional reliability where it only considers the subsystem failure process

\( T \) time length of one inspection period

\( R_{sf}(t) \) soft-failure probability by time \( t \)

\( F_{hf}(t) \) hard-failure probability by time \( t \)

\( R(t) \) Extended reliability of the surveillance system by time \( t \)

2. Description of the Surveillance System Framework

Consider a traditional reliability modeling where only the component failure process is being considered. This type of modeling is suitable for the products or systems processing demands continuously. Once the system fails, it can no longer provide any service to the customer to meet their persistent need. Applications for systems with continuous work load include car engines, power plant, and production lines.

For the non-continuous work load of systems such as alarm-detection units, nuclear power plants, airbag car system, medical monitoring control units, that require frequent surveillance by certified personnel who must identify potential problem through inspections, there is a chance that even though the system has already failed, but no incident or attack arrives since the failure until the system is being inspected and repaired to functional status. Because of the characteristic of this type of system that we discussed here, the reliability of the system responding to sparse discrete demand should be revised as to properly function when a pulse demand takes place. In other words, the reliability discussed in this paper which consists of two parts: the first is the reliability with respect to the component failure process subject to environmental factors, and the other is the system has actually failed but no incident arrives between the failure and the next inspection point after the failure.

To realize the estimation of the extended reliability, a two stochastic processes model is proposed with the application of the surveillance camera systems. The first process is a non-homogeneous Poisson process (NHPP) for the incident arrival. The second is a two-stage stochastic process indicating the status of the system (failure or functioning). The hidden failure that is not aware of by the certified personnel (or central stations) is the main focus in the modeling of this study and is considered as an innovation modeling approach. This type of failure can only be detected and fixed by periodic inspection. If the occurring failure raises the attention of the central stations, then immediately a maintenance action will be provided and the time to repair on-line is ignored. All the subsystems are periodically inspected for hidden failure and the failed subsystems are perfectly repaired [15].

Figure 1 illustrates a possible series of events of the two processes within an inspection cycle. With the assumption of perfect maintenance, we only need to consider a single inspection cycle in this work. A future research can be extended to consider the multiple inspection cycles subject to imperfect maintenance etc. However, in Fig. 2, there are n identical subsystems (i.e., cameras) are installed to enhance the system performance. For the k-out-of-n surveillance system to work, there are at least k subsystems must work. We assume that each of the working subsystem has the probability, \( p(\omega_i) \), of detecting an...
incident. This probability reflects the fact that there is a chance, although the subsystem is properly functioning, the subsystem cannot detect the attacker’s action which is \(1 - p(\omega_i)\). All subsystems are considered to work under a common random environment, which adds the dependency between the life-time of each subsystem.

Based on the assumption of the two processes model, the proposed surveillance system may result in three different states by the next inspection point as follows:

- **Working state**: The system still works (i.e., at least \(k\) subsystems are working);
- **Soft-failure state**: There are at least \((n-k+1)\) subsystems have failed during the inspection interval, but no incident has arrived till time \(T\). In this case the surveillance system is down, but we are lucky to reach the maintenance point without serious damage. This outcome is referred to as the soft failure;
- **Hard-failure state**: Undetected incident occurred during the surveillance system failure period.

Based on the reliability modeling, the inspection period \(T\) is determined to meet the system performance requirement. The extended reliability of the system is defined as the sum of the probabilities of the first two outcomes (working state and soft-failure state).

### 3. Mathematical Modeling

Consider a model consisting of two mutually dependent stochastic processes. One is a nonhomogeneous Poisson process (NHPP) for the incident arrival process; and the other is a two-stage stochastic process of the system failure process.

#### A. NHPP Incident Arrival Process:

We assume that the arrival of the incident follows a NHPP with intensity function \(\lambda_i(t)\) which has the following form:

\[
\lambda_i(t) = \lambda_0 e^{A(t, \theta)}
\]  

where \(\lambda_0\) is the baseline and \(A(t, \theta)\) is a function that incorporates the environmental effects on the intensity function. This type of parametric arrival rate estimation can be found in [12,16]. Those factors considered in this work include:

- Time from the last incident, \(T_s\);
- Number of incidents have occurred in the past \(T_p\) units of time prior to time \(t\),
  \[N_p = N(t \in T_p)\];

The larger value of \(T_s\) or the more number of incidents in the past would likely result to the higher intensity rate of the incident arrival. We define the function \(A(t, \theta)\) as follows
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\[ A(t, \theta) = \gamma_1(t - T_s) + \gamma_2 N_p \]  \hspace{1cm} (2)

B. Two-stage System Failure Process:

Here we consider a random variable \( \eta \) that represents the uncertainty of comment environments of each subsystem using the concept of systemability addressing the uncertainty of operating. The detail development of the systemability can be found in [10]. The mathematical formulation of the systemability is defined as:

\[ R_\eta(t) = \int \frac{e^{-\eta h(t)}}{\eta} \, dG(\eta) \]  \hspace{1cm} (3)

where \( h(t) \) is the hazard rate function of the subsystem and \( G(\eta) \) represents the distribution of the operating environment of random variable \( \eta \).

Let us assume that the subsystem lifetime follows a Weibull distribution, that is \( R_i(t) = e^{-\lambda_i t^{\gamma_i}} \) or \( R_i(t) = e^{-\lambda t^{\gamma}} \) for identical components. As in [10], we also considered the Gamma distribution for the random operating environment \( \eta \), that is \( \eta \sim \text{Gamma}(\alpha, \beta) \) where the probability density function (pdf) of \( \eta \) is given by

\[ f_\eta(x) = \frac{\beta \alpha x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)} \]  \hspace{1cm} (4)

The reliability function of each subsystem under the uncertainty of operating environments is defined as

\[ R_i(t|\eta) = e^{-\eta \lambda t^{\gamma}} \]  \hspace{1cm} (5)

Assuming that the incident detection probability is constant, \( i.e., p(\omega) = p \). In this paper, we assume that for the subsystem to function, it has to satisfy that the subsystem does not fail by time \( t \) and has successfully detected the incident. Thus the subsystem reliability can be expressed as

\[ R_{pi}(t|\eta) = pR_i(t|\eta) \]  \hspace{1cm} (6)

Thus the reliability of the entire k-out-of-n surveillance system in terms of the uncertainty common operating environment can be formulated as

\[ R_c(t|\eta) = \sum_{j=k}^{n} \binom{n}{j} R_{pi}(t|\eta)^j \left(1 - R_{pi}(t|\eta)\right)^{n-j} = \sum_{j=k}^{n} \binom{n}{j} p^j e^{-j\eta \lambda t^{\gamma}} \left(1 - p e^{-\eta \lambda t^{\gamma}}\right)^{n-j} \]  \hspace{1cm} (7)

Hence the reliability of k-out-of-n surveillance system with respect to the operating environments is given by:

\[ R_c(t) = \int_{\eta} R_c(t|\eta) dG(\eta) \]  \hspace{1cm} (8)

Since \( \eta \) follows a Gamma distribution, a close-form function of eq. (8) can be obtained using the Laplace transform [11]. The failure distribution function of k-out-of-n systems under the common operating environment is given by

\[ F_c(t) = 1 - R_c(t) \]  \hspace{1cm} (9)
3.1 System Probability States

a. Working state: The probability that the system is working by time \( t \) which is defined as in (8):
\[
R_c(t) = \int R_c(t | \eta)dG(\eta)
\]

b. Soft-failure state (fail-safe mode): The probability that the system will be in soft-failure state. That is
\[
R_{sf}(t) = \int \Pr[N(t) - N(\tau) = 0] dF_c(\tau)
\]
By definition, the soft failure requires that the system fails between two consecutive inspections. Assuming that the failure happens to be at the moment time \( \tau \) where \( 0 \leq \tau \leq t \) and in the remaining time interval \([\tau, t]\) there will be no incident arrive during this period and is given by
\[
Pr[N(t) - N(\tau) = 0] = e^{-\int_{\tau}^{t} \lambda_s(x) dx}
\]
Equation (10) yields
\[
R_{sf}(t) = \int_{0}^{t} e^{-\int_{\tau}^{t} \lambda_s(x) dx} dF_c(\tau)
\]

c. Hard-failure state: The probability of an incident occurs during the period of surveillance system failure which is defined as
\[
F_{hf}(t) = 1 - R_c(t) - R_{sf}(t)
\]
By definition, \( F_{hf}(t) \) is the probability that the system encounters an incident and the surveillance system has already failed before the next inspection takes place.
In this study, we define that the extended reliability of the surveillance system by time \( t \) is given by
\[
R(t) = R_c(t) + R_{sf}(t) = 1 - F_{hf}(t)
\]
Once the inspection interval \( T \) is scheduled based on the system requirement, one can simply replace \( t \) with \( T \) in order to calculate the outcome probability at selected inspection interval.

4. Numerical Example

Let us consider a 2-out-of-3 surveillance system structure to illustrate the proposed model. A list of all the parameters is given as follows for numerical example:
For the NHPP arrival rate: \( \lambda_0 = 0.01, \gamma_1 = 0.005, T_s = 500, \gamma_2 = 0.002, N_p = 5. \)
For the subsystem lifetime Weibull distribution: \( \lambda = 0.0001, \gamma = 1.5. \)
For the Gamma distribution of random variable \( \eta \): \( \alpha = 2, \beta = 3. \)
The incident detection probability: \( p = 0.9. \)
Then, from equations (1), (7-8) and (12), we have:
\[
\lambda_s(t) = 0.01 e^{0.005(t+500)+0.01}
\]
\[
R_c(t | \eta) = 3[(0.9)e^{-\gamma_0.0001t^{1.5}}]^{\frac{3}{2}} - 2[(0.9)e^{-\gamma_0.0001t^{1.5}}]^{\frac{3}{3}}
\]
\[
R_c(t) = 3(0.9)^{\frac{3}{2}}[\frac{3}{3 + 2(0.0001)t^{1.5}}]^{\frac{3}{2}} - 2(0.9)^{\frac{3}{3}}[\frac{3}{3 + 2(0.0001)t^{1.5}}]^{\frac{3}{2}}
\]
\[
R_{sf}(t) = \int_{0}^{t} e^{-\frac{0.01}{0.005} \cdot 0.005(t+500)+0.01 \cdot \frac{0.01}{0.005} \cdot 0.005(t+500)+0.01} dF_c(\tau)
\]
The probability of the surveillance system ending in each state, i.e., working, soft-failure, and hard-failure, by the next inspection time point is plotted in Fig. 3, with the variation of different inspection interval lengths. The dash line represents the reliability, \( R_c(t) \), or probability of observing the system in working status by the end of the inspection
period. The solid line represents the extended reliability of the entire surveillance system by adding both the traditional reliability of k out of n system and the probability of having soft-failure by the end of the inspection period using eq. (14). The dash-dotted line represents the hard-failure probability by the end of the period, \( F_{hf}(T) \). For example, we wish to determine the inspection interval time \( T \) where the probability that the system encountering hard failure must be less than 0.1, \( i.e., F_{hf}(T) < 0.1 \). Then from Fig. 3 and eq. (13), the result of the inspection interval time \( T \) that satisfies the system failure requirement would be \( T = 194 \) hours in this case.

![Figure 3: Probability of System States Plot within One Inspection Cycle](image)

5. Sensitivity Analysis

The proposed model includes several parameters. For the sensitivity analysis, we vary various parameters in the following manner so that each case corresponds to a physical modification of the surveillance system framework. The system performance can be evaluated by the maximum inspection interval length that keeps the hard-failure probability less than 0.1. The longer interval length indicates the better performance of the surveillance system. In Fig. 4, we modify the incident arrival rate \( \lambda_I(t) \) by increasing the baseline rate \( \lambda_0 \) from 0.01 to 0.05 which corresponds to the higher rate of attack in reality. As a result, the probability of system having a soft failure by the next inspection decreases, represented by the gap between the dash and solid lines. In Fig. 5, we increase the incident detection rate parameter \( p \) from 0.9 to 0.98. This change indicates that the subsystem is likely to detect the incident when it is functioning. Similarly, the reliability of the entire surveillance system as well as subsystem obviously increase as the failure rate of subsystem in the Weibull distribution decreases by half (\( i.e., from \( \lambda = 0.0001 \) to \( \lambda = 0.00005 \)) as shown in Fig. 6. The reliability of the surveillance system for various values of \( \beta \) are shown in Fig. 7. As for Fig. 8, we present the reliability measures of the 2-out-of-4 surveillance system configuration. By adding one redundant subsystem from 2-out-of-3 (see Fig. 3) to 2-out-of-4 (Fig. 8), obviously the entire system performs better in terms of reliability measure. Thus to enhance the reliability of the surveillance system in general, the following actions can be considered: reduce the incident arrival rate; use a reliable subsystem (\( i.e., camera, sensors \)) in the field; reduce the probability of having the
subsystem working under harsh environments; and add more redundancy to the surveillance system.

**Figure 4:** Reliability Plot with Change in $\lambda(t)$: $\lambda_0 = 0.01 \rightarrow \lambda_0 = 0.05$

**Figure 5:** Reliability Plot with Change in $p$: $p = 0.9 \rightarrow p = 0.98$

**Figure 6:** Reliability plot with change in $R_i(t)$: $\lambda = 0.0001 \rightarrow \lambda = 0.00005$

**Figure 7:** Reliability Plot with Change in $G(\eta)$: $\beta = 3 \rightarrow \beta = 2$
6. Conclusion

In this work, a surveillance system reliability model is presented with consideration of a dual stochastic-dependent process: incident arrival process and system failure process. The framework of the surveillance systems can be applied to other applications as modification can be easily conducted following the mathematical modeling in Section 3. One can adopt different system configurations, consider different environmental effects based on the collected data and evaluate intruder’s effort to avoid being detected using different mechanisms. The quantitative evaluation of the reliability and soft-failure and hard-failure probabilities with the variation of the inspection interval length is derived and illustrated with numerical examples and several sensitivity analyses. Possible actions to enhance the reliability of the surveillance system are also discussed.

References


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