Optimal Tenuring Collection Times for a Generational Garbage Collector based on Continuous Damage Model

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(Received on February 06, 2013, revised on June 16, 2013)

Abstract: The processing time intervals for a generational garbage collector may be ephemeral enough to consider the objects that would survive increase with time continuously according to some probabilistic law. From such a viewpoint, this paper firstly answers for the weak points of cumulative damage model whose damage is additive at discrete times. Secondly, we take up a continuous damage model and apply such model to garbage collection policies. Costs for garbage collections are estimated and two models with tenuring collection times, where tenuring collection is made at time $T$ or at level $K$ for random collections and at the $N$th collection or at level $K$ for periodic collections, are proposed. Four cases of optimal tenuring collection times for every model are discussed analytically and numerically, and comparisons of these policies and some useful results are given.

Keywords: Degradation process, continuous damage, garbage collection, tenuring collection, minor collection.

1. Introduction

Cumulative damage models [1] play an important role in reliability theory: These models are considered as a sequence of shocks that occur randomly and give some amount of damage to a unit. The damage is accumulated to the current damage level and weakens the unit gradually, and the unit fails when the total damage exceeds a failure level. The reliability properties and various maintenance policies for damage models were summarized sufficiently in [2]. A variety of maintenance models subjected to shocks were studied extensively in [3-8]. Some applications of these damage models into computer science, such as backup policies for a database [9, 10], garbage collection policies for the memory management [11-14], were explored.

Motivated by [9-14], we found that the hypothesis of cumulative damage models may be so strong that they are not so practicable for some applications: (a) an amount of damage due to every shock may not have an identical distribution, and the effect of later shocks may be much more serious than former ones. In other words, expected number of shocks until failure in practice is much less than that in theory; (b) it is more practical to suppose that the total damage stored within the unit would increase with time itself, but not be in a constant level until next shock occurs; (c) referring to these applications studied in [9-14], due to high frequency of computer processes in the modern society, it may be not so valid to assume that update of data or garbage occurrence would follow an nonhomogeneous Poisson process, because time intervals of procedures might be very short and unclear enough.

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Firstly, to answer the above two weak points (a) and (b) of cumulative damage models, we consider the model where the total damage increases with time itself continuously according to some probabilistic law, which could be named as continuous damage model. By modifying $X(t) = x_0 + \mu t + \sigma W(t)$ [15, p.216], which is a Gaussian process, where $W(t)$ denotes a standard Brownian motion, $x_0$ is some initial degradation level, and $\mu$ and $\sigma$ are the drift and the variance, respectively, we suppose that the path of the total damage with time $t$ is

$$X(t) = \alpha t + \sigma B, \quad (\alpha, \sigma > 0),$$

(1)

where $x_0 = 0$ means that the initial state of the unit is as good as new and $B_t$ is distributed exponentially. So that we can know that total damage $X(t)$ increases with time $t$ strictly and undergoes some random positive change. We make such an assumption also because we cannot neglect the dependence between damage stored within the unit and damage caused by shocks, although some research papers combined the degradation process and shock process [16, 17], but they are independent.

Secondly, to answer (c), this paper focuses on optimization problems of a generational garbage collector using the continuous damage model obtained in equation (1). In memory management in computer science, generational garbage collection [18, 19] has been popular with programmers for the reason that it can be made more efficient. However, for every garbage collection, the manner of stop and copy pause all application threads to collect the garbage. The duration of time in which a garbage collector has worked is called pause time [20, p.148], which is an important parameter for interactive systems and depends largely upon the volume of surviving objects and the type of collections. The latest technical report [19] proposed some models by estimating the lifetimes of all objects, however, this kind of technique may be not so easy in practice, because for high frequency of random computer processes, the number of objects in the heap may be too great to count, also because of random factors of object lifetimes. References [11-14] improved above method by estimating the survivor rate of one group of objects rather than focusing on every object. To make the estimation cost for collections more easily, we get inspiration from [19] which asserted that garbage increases with time continuously, and apply equation (1) to estimate surviving objects at some time by statistical methods. It is true that increase in objects might be very unclear at discrete times for the high frequency of computer processes in the modern society.

This paper considers a pause time goal that is called time cost or cost for simplicity for a generational garbage collector. Our problem is to obtain optimal tenuring collection times that minimize the expected cost rates. Following sections are organized as: Section 2 gives working schemes and costs for a generational garbage collector. Section 3 discusses the case when garbage collections occur at a nonhomogeneous Poisson process. Section 4 discusses models when collections occur at periodic times. Numerical examples are computed in Section 5 and concluding remarks are given in Section 6.

2. Working Scheme and Cost

For a generational garbage collector, detailed working schemes that have been introduced in [11-14] are given as following steps:

1. New objects are allocated in Eden space.
2. When the first minor collection occurs, surviving objects are copied from Eden into a survivor space.
3. When the second minor collection occurs, surviving objects from Eden and
survivor spaces are copied into the other survivor space.

4. In the fashions of 1-3, minor collection copies surviving objects between the two survivor spaces until they become tenured, i.e., tenuring collection occurs when some parameter meets the tenuring threshold, and then, the older or oldest objects are copied into Old space.

5. When Old space fills up, major collection of the whole heap occurs, and surviving objects from Old space are kept in Old, while objects from Eden and survivor spaces are kept in a survivor space.

Although major collection spends much time, Old space will be filled with tenured objects slowly, especially when the tenuring threshold is high and the survivor rate is low, that is, major collection occurs rarely in this case. So that, this paper concentrates only on minor and tenuring collections and considers tenuring collections as renewal points of the collection processes, that is, after tenuring collection, the same collection cycle begins with step 1 to 1 to 2 to 3 to 4 ···. From [11-14], we have known that surviving objects that should be copied increase with the number of minor collections. That is, we can define:

1. The total surviving objects in Eden and survivor space at time \( t \) is \( X(t) = at + \sigma B_t \) with distribution \( \Pr\{X(t) \leq x\} = W(t, x) \).

2. \( c_s + c_w(x) \) is the cost suffered for every minor collection, where \( c_s \) is the constant cost suffered for scanning surviving objects and \( x \) is the amount of copied objects, \( c_k (c_k > c_s + c_w(K)) \) is the cost suffered for tenuring collection at level \( K \).

3. The expected cost of minor collection at time \( t \) is given by

\[
C(t, K) = \frac{1}{W(t, K)} \int_0^K [c_s + c_w(x)] dW(t, x),
\]

where \( C(0, K) = 0 \). It is clear that \( C(t, K) \) increases with \( t \) for any \( K \geq 0 \).

3. Random Collections

It is assumed that garbage collections occur at a nonhomogeneous Poisson process with an intensity function \( \lambda(t) \) and a mean-value function \( R(t) = \int_0^t \lambda(u) du \). Then, the probability that collections occur exactly \( j \) times in \((0, t]\) is

\[
H_j(t) = \frac{[R(t)]^j}{j!} e^{-R(t)}, \quad (j = 0, 1, 2, \ldots)
\]

Letting \( F_j(t) \) denote the probability that collections occur at least \( j \) times in \((0, t]\).

Then, \( F_j(t) = \sum_{i=j}^m H_i(t) \), where \( F_0(t) = 1 \).

Suppose that minor collections are made when the garbage collector begins to work, tenuring collection is made at a planned time \( T(0 < T \leq \infty) \), or when surviving objects have exceeded a threshold level \( K(0 < K \leq \infty) \), whichever occurs first. Then, the mean time to tenuring collection is

\[
TW(T, K) + \int_0^T t \bar{W}(t, K) dt = \int_0^T W(t, K) dt,
\]

and the expected cost suffered for minor collections until tenuring collection is
Therefore, the expected collection cost rate is

\[ C_i(T, K) = \frac{c_k - c_r}{\int_0^T W(t, K)dt} \left( \int_0^T \lambda(t)C(t, K)W(t, K)dt + \int_0^T \sum_{j=1}^\infty \int_0^T C(u, K)dE_j(u)\right) \]  

(4)

It can be seen that \( C_i(T, K) \) includes the following collection polices:

1. Tenuring collection is made at time \( T \) for a given \( K \), the reason why making such a policy is \( c_r < c_k \).
2. Tenuring collection is made at level \( K \) for a given \( T \). In this case, \( c_r > c_k \).
3. Tenuring collection is made only at time \( T \) or only at level \( K \). In these two cases, \( c_r = c_k \).

(1) \textbf{Optimal } T_i^*

When \( c_r < c_k \), we find an optimal \( T_i^* \) which minimizes \( C_i(T, K) \) in (5) for a given \( K \). Differentiating \( C_i(T, K) \) with respect to \( T \) and setting it equal to zero,

\[ (c_k - c_r)\left[h(T, K)\int_0^T W(t, K)dt - \bar{W}(T, K)\right] + \int_0^T \left[\lambda(T)C(t, K) - \lambda(t)C(t, K)\right]W(t, K)dt = c_r. \]

(6)

Letting \( L_i(T, K) \) be the left-hand side of (6), \( L_i(0, K) = \lim_{T \to \infty} L_i(T, K) = 0 \) and

\[ \frac{dL_i(T, K)}{dT} = (c_k - c_r)d\bar{h}(T, K) - \int_0^T C(T, K) - \lambda(T)\int_0^T W(t, K)dt - \lambda(T)\int_0^T \frac{dC(T, K)}{dT} \int_0^T W(t, K)dt dt. \]

Thus, if \( \lambda(t) \) increases with \( t \), then the left-hand side of (6) increases with \( t \) from 0.

Therefore, there exists a unique optimal \( T_i^* \) \((0 < T_i^* \leq \infty)\) which satisfies (6).

(2) \textbf{Optimal } K_i^*

When \( c_r > c_k \), we find an optimal \( K_i^* \) which minimizes \( C_i(T, K) \) in (5) for a given \( T \). Letting \( w(t, x) \) be the density function of \( W(t, x) \), i.e., \( w(t, x) = dW(t, x) / dx \). Then, differentiating \( C_i(T, K) \) with respect to \( K \) and setting it equal to zero,

\[ (c_r - c_k)\left[Q_i(T, K)\int_0^T W(t, K)dt - \bar{W}(T, K)\right] + \int_0^T \left[Q_i(T, K) - \lambda(t)C(t, K)\right]W(t, K)dt = c_k. \]

(7)

where
Optimum Replacement Policy for Continuous Damage Model and its Application to Garbage Collection

\[ Q(T, K) = \frac{w(T, K)}{\int_0^T w(t, K)dt}, \quad Q_2(T, K) = \frac{[c_s + c_{\mu}(K)]\int_0^T \lambda(t) w(t, K) dt}{\int_0^T w(t, K) dt}. \]

Letting \( L_2(T, K) \) be the left-hand side of (7), \( L_2(T, 0) = \lim_{K \to \infty} L_2(T, K) = 0 \) and

\[ \frac{dL_2(T, K)}{dK} = (c_r - c_k) \frac{dQ_1(T, K)}{dK} \int_0^T W(t, K) dt + \frac{dQ_2(T, K)}{dK} \int_0^T W(t, K) dt. \]

Thus, if \( Q_1(T, K) \) and \( Q_2(T, K) \) increases with \( K \), then the left-hand side of (7) increases with \( K \) from 0. Therefore, there exists a unique optimal \( K^*_1(0 < K^*_1 \leq \infty) \) which satisfies (7).

(3) Optimal \( t^*_1 \)

When \( c_r = c_k \), putting that \( K = \infty \) in (5), the expected cost rate is

\[ C_1(T) = \lim_{K \to \infty} C_1(T, K) = \frac{1}{T} \left[ \int_0^T \lambda(t) C(t; \infty) dt + c_r \right], \]

where \( C(t, \infty) = \int_0^T [c_s + c_{\mu}(x)] dW(t, x) = c_s + \int_0^T \tilde{W}(t, x) dc_{\mu}(x). \)

From (6), if \( \lambda(t) \) increases with \( t \), then an optimal tenuring collection time \( t^*_1 \) which minimizes (8) is given by a unique solution of the equation

\[ \int_0^T (\lambda(T) C(T, \infty) - \lambda(T) C(T, K)) dt = c_r. \]

(4) Optimal \( k^*_1 \)

When \( c_r = c_k \), putting that \( T = \infty \) in (5), the expected cost rate is

\[ C_1(K) = \lim_{T \to \infty} C_1(T, K) = \frac{\int_0^\infty \lambda(t) C(t, K) W(t, K) dt + c_k}{\int_0^\infty W(t, K) dt}. \]

From (7), if \( Q_2(\infty, K) \) increases with \( K \), then an optimal tenuring collection time \( k^*_1 \) which minimizes (10) is given by a unique solution of the equation

\[ \int_0^T [Q_2(\infty, K) - \lambda(t) C(t, K)] W(t, K) dt = c_k. \]

4. Periodic Collections

It is assumed that garbage collections occur at periodic times \( jT_0 (j = 1, 2, \cdots) \) for a given time interval \( T_0 (0 < T_0 < \infty) \) minor collections are made when the garbage collector begins to work, tenuring collection is made at the \( N \)th \( (N = 1, 2, \cdots) \) collection, or at first collection time when surviving objects have exceeded a threshold level \( K(0 < K \leq \infty) \), whichever occurs first. Then, the mean time to tenuring collection is

\[ NT_0 W(NT_0, K) + \sum_{n=0}^{N-1} \int_{jT_0}^{(j+1)T_0} [j + 1]T_0 dW(t, K) = T_0 \sum_{n=0}^{N-1} W(jT_0, K). \]

The expected cost suffered for minor collections until tenuring collection is
Therefore, the expected cost rate is
\[
W(NT_0, K) \sum_{j=0}^{N} C(jT_0, K) + \sum_{j=0}^{N} \int_{P_0}^{P_{j+1}} \sum_{i=1}^{N} C(iT_0, K) dW(i, K).
\] (13)

Therefore, the expected cost rate is
\[
C(N, K) = \frac{c_k - (c_k - c_\lambda) W(NT_0, K) + \sum_{j=0}^{N-1} C(jT_0, K) W(jT_0, K)}{T_0 \sum_{j=0}^{N} W(jT_0, K)}. \] (14)

(1) Optimal \( N_2^* \)

When \( c_\lambda < c_k \), we find an optimal \( N_2^* \) which minimizes \( C_2(N, K) \) in (14) for a given \( K \). From the inequality \( C_2(N+1, K) - C_2(N, K) \geq 0 \),
\[
(c_k - c_\lambda) \left[ Q_2(NT_0, K) \sum_{j=0}^{N} W(jT_0, K) - W(NT_0, K) \right] + \sum_{j=1}^{N} \left[ C(NT_0, K) - C(jT_0, K) \right] W(jT_0, K) \geq c_N,
\] (15)

where
\[
Q_2(NT_0, K) = \frac{W(NT_0, K) - W((N+1)T_0, K)}{W(NT_0, K)}.
\]

Letting \( L_2(N, K) \) be the left-hand side of (15),
\[
L_2(N + 1, K) - L_2(N, K) = \left( c_k - c_\lambda \right) \sum_{j=0}^{N} W(jT_0, K) \left[ Q_2((N+1)T_0, K) - Q_2(NT_0, K) \right] + \sum_{j=1}^{N} \left[ C((N+1)T_0, K) - C(NT_0, K) \right] W(jT_0, K).
\]
Thus, if \( Q_2(jT_0, K) \) increases with \( j \) and \( L_2(\infty, K) > c_\lambda \), then there exists a finite and unique minimum \( N_2^* \) \((1 \leq N_2^* < \infty)\) which satisfies (15).

(2) Optimal \( K_2^* \)

When \( c_\lambda > c_k \), we find an optimal \( K_2^* \) which minimizes \( C_2(N, K) \) in (14) for a given \( N \). Differentiating \( C_2(N, K) \) with respect to \( K \) and setting it equal to zero,
\[
(c_k - c_\lambda) \left[ Q_2(NT_0, K) \sum_{j=0}^{N} W(jT_0, K) - W(NT_0, K) \right] + \sum_{j=1}^{N} W(jT_0, K) [c_\lambda + c_m(K) - C(jT_0, K)] = c_k,
\] (16)

where
\[
Q_2(NT_0, K) = \frac{w(NT_0, K)}{\sum_{j=0}^{N} w(jT_0, K)}.
\]
Let $L_4(N, K)$ be the left-hand side of (16), $L_4(N, 0) = \lim_{K \to 0} L_4(N, K) = 0$ and
\[
\frac{dL_4(N, K)}{dK} = (c_K - c_N)\frac{dQ_T}{dK} + \sum_{j=0}^{N-1} W(jT_0, K) + \frac{dW}{dK} + \sum_{j=0}^{N-1} W(jT_0, K).
\]
Thus, if $Q_T(NT_0, K)$ increases with $K$, then the left-hand side of (16) increases with $K$ from 0. Therefore, there exists a unique optimal $K_0^*$ ($0 < K_0^* \leq \infty$) which satisfies (16).

(3) Optimal $n_2^*$
When $c_N = c_K$, putting that $K = \infty$ in (14), the expected cost rate is
\[
C_2(N) = \lim_{K \to \infty} C_2(N, K) = \frac{1}{NT_0} \left( \sum_{j=0}^{N-1} W(jT_0, \infty) + c_N \right).
\]
From (15), an optimal tenuring collection time $n_2^*$ which minimizes (17) is given by a unique solution of the equation
\[
\sum_{j=1}^{N-1} [C(NT_0, \infty) - C(jT_0, \infty)] \geq c_N.
\]

(4) Optimal $k_2^*$
When $c_N = c_K$, putting that $N = \infty$ in (14), the expected cost rate is
\[
C_2(K) = \lim_{N \to \infty} C_2(N, K) = \frac{1}{T_0} \sum_{j=0}^{N-1} W(jT_0, K).
\]
From (16), an optimal tenuring collection time $k_2^*$ which minimizes (19) is given by a unique solution of the equation
\[
\sum_{j=1}^{N-1} W(jT_0, K) \geq c_K.
\]

5. Numerical Examples

We compute numerical examples of the models discussed above when $X(t)$ is given by (1) for $\mu = \sigma = 1$ and $B_t$ has an exponential distribution $1 - e^{-\lambda t}$. Next, select suitable parameters which can affect the optimal policies and resulting cost rates to compute each model numerically: (i) Initial parameters $K$ and $T$ are given to compute $T_0^*$ and $K_0^*$, $K$ and $N$ are given to compute $N_0^*$ and $K_0^*$. (ii) $c_k / c_T$, $c_T / c_K$, $c_k / c_N$ and $c_N / c_K$ mean that the effects of tenuring collection costs at time $T$ and level $K$ for random collection models and at $N$th collection and level $K$ for periodic collection models. (ii) $c_k / c_T(t = T, N, K)$ which mean that the effects of minor collection costs are selected to compute $T_0^*$, $K_0^*$, $N_0^*$, $K_0^*$, $k_2^*$, $n_2^*$, and $k_2^*$.

From Tables 1-6, we can obtain the following results:

1. Optimal tenuring collection times increase with the initial parameters and decrease
with minor or tenuring collection costs, however, the resulting cost rates have the opposite tendencies, that is, they decrease with the initial parameters and increase with minor or tenuring collection costs. Take $T_i^t$ and $C_i\left(T_i^t, K\right)/c_r$ in Table 1 for example: $T_i^t$ increases with $K$ and decreases with $c_k/c_r$ and $c_M/c_r$. Increasing in $K$, $c_k/c_r$ and $c_M/c_r$ mean tenuring collection time made at a given level $K$ is postponed, tenuring and minor collection costs are increased, respectively, so that tenuring collection times should be postponed for $K$ and be advanced for $c_k/c_r$ and $c_M/c_r$ to decrease the frequency of tenuring collections and to decrease the total minor collection costs. $C_i\left(T_i^t, K\right)/c_r$ decreases with $K$ and increases with $c_k/c_r$ and $c_M/c_r$ for the reason that the frequencies of tenuring collections are decreased and tenuring and minor collection costs are increased.

2. Compared with Tables 1 and 2, we can derive that $T_i^t \approx K_i^*$ and $C_i\left(T_i^t, K\right)/c_r \approx C_i\left(T, K_i^*\right)/c_k$, however, $C_i\left(T_i^t, K\right)/c_r$ are sometimes greater than and sometimes less than $C_i\left(T, K_i^*\right)/c_k$.

3. Compared with Tables 3 and 4, $N_i^* \approx K_i^*$ and $C_i\left(N_i^*, K\right)/c_n$ are always greater than $C_i\left(N, K_i^*\right)/c_k$.

4. Compared with Tables 1, 2 and 3, 4, it is clearly that random collections are better than periodic collections.

5. Compared with Tables 5 and 6, optimal collection times and the resulting cost rates are almost at the same level for four models, that is, $l_i^* \approx n_i^* \approx k_i^*$ and $C_i\left(k_i^*, K\right)/c_k = C_i\left(n_i^*, K\right)/c_k < C_i\left(l_i^*, K\right)/c_k$. It is clearly that when $1/ \lambda = T_i^t$, $C_i^1(K)$ and $C_i^2(K)$ become the same model. According to [11-14] and step 4 in Section 2, we also can name $l_i^*$, $k_i^*$, $n_i^*$ and $k_i^*$ as tenuring threshold of the collection processes.

Table 1: Optimal $T_i^t$ and $C_i\left(T_i^t, K\right)/c_r$ when $c_M/c_r = 0.1$ and $\lambda = 0 = \sigma = 1$.  

<table>
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<th>$c_k/c_r$</th>
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<th>$c_r/c_r$</th>
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<td>0.05</td>
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<td>$C_i\left(T_i^t, K\right)/c_r$</td>
<td>$T_i^t$</td>
<td>$C_i\left(T_i^t, K\right)/c_r$</td>
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Table 2: Optimal $K_i^*$ and $C_i(T, K_i^*)/c_x$ when $c_s/c_x = 0.1$ and $\lambda = \alpha = \sigma = 1$.

<table>
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<tr>
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<td>0.3512</td>
<td>4.48</td>
<td>0.4405</td>
<td>3.62</td>
<td>0.5503</td>
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</tr>
</tbody>
</table>

Table 3: Optimal $N_i^*$ and $C_i(N_i^*, K)/c_x$ when $c_s/c_x = 0.1$ and $T_s = \alpha = \sigma = 1$.

<table>
<thead>
<tr>
<th>$K$</th>
<th>$c_s/c_x$</th>
<th>$c_w/c_x$</th>
<th>$N_i^*$</th>
<th>$C_i(N_i^*, K)/c_x$</th>
<th>$N_i^*$</th>
<th>$C_i(N_i^*, K)/c_x$</th>
<th>$N_i^*$</th>
<th>$C_i(N_i^*, K)/c_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.01</td>
<td>0.05</td>
<td>0.1</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>2</td>
<td>5</td>
<td>0.5447</td>
<td>4</td>
<td>0.6205</td>
<td>4</td>
<td>0.7320</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>0.6842</td>
<td>4</td>
<td>0.7594</td>
<td>4</td>
<td>0.8031</td>
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<td></td>
</tr>
<tr>
<td>5</td>
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<td>0.9130</td>
<td>3</td>
<td>0.9870</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>8</td>
<td>6</td>
<td>0.4618</td>
<td>5</td>
<td>0.6079</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>0.3463</td>
<td>6</td>
<td>0.4721</td>
<td>5</td>
<td>0.6111</td>
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<td></td>
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<tr>
<td>5</td>
<td>6</td>
<td>0.3789</td>
<td>5</td>
<td>0.4901</td>
<td>4</td>
<td>0.6174</td>
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</tr>
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</table>

Table 4: Optimal $K_i^*$ and $C_i(N_i^*, K)/c_x$ when $c_s/c_x = 0.1$ and $\lambda = \alpha = \sigma = 1$.

<table>
<thead>
<tr>
<th>$N$</th>
<th>$c_s/c_x$</th>
<th>$c_w/c_x$</th>
<th>$K_i^*$</th>
<th>$C_i(N_i^*, K)/c_x$</th>
<th>$K_i^*$</th>
<th>$C_i(N_i^*, K)/c_x$</th>
<th>$K_i^*$</th>
<th>$C_i(N_i^*, K)/c_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.01</td>
<td>0.05</td>
<td>0.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>3.96</td>
<td>0.6119</td>
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<td>0.6968</td>
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<td></td>
</tr>
<tr>
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<td>3.58</td>
<td>0.6402</td>
<td>3.34</td>
<td>0.7032</td>
<td>3.07</td>
<td>0.7762</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2.92</td>
<td>0.7977</td>
<td>2.79</td>
<td>0.8503</td>
<td>2.64</td>
<td>0.9129</td>
<td></td>
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<td>2</td>
<td>7.54</td>
<td>5.62</td>
<td>0.4369</td>
<td>4.29</td>
<td>0.5556</td>
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<td></td>
</tr>
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<td>5.21</td>
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<td>4.15</td>
<td>0.5638</td>
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<td></td>
</tr>
<tr>
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<td>5.57</td>
<td>0.3739</td>
<td>4.73</td>
<td>0.4737</td>
<td>3.94</td>
<td>0.5787</td>
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</tbody>
</table>
6. Conclusions

We have firstly pointed out three weak points of cumulative damage models whose damage is only caused by random shocks. To answer these weak points, continuous damage model in equation (1) has been proposed, which shows that damage stored in the units would increase with time itself and undergo some random positive change due to shocks. The idea has been motivated by application of damage models into garbage collections in computer science. From Section 2 to Section 5, this paper has focused on optimization problems of tenuring collection times for a generational garbage collector, using the models given in equation (1). Obviously, we could monitor the increasing path of surviving objects in practice, then an interesting point is to estimate collection cost in equation (2). We have discussed random and periodic collection models for a generational garbage collector according to its working schemes. Optimal tenuring collection times have been obtained and their numerical examples have been given to explain model formulations.

Acknowledgement: This work is partially supported by the Grant-in-Aid for Scientific Research (C) of Japan Society for the Promotion of Science under Grant No. 22500897;

Table 5: Optimal $t^*_i$, $C_s(t^*_i)/c_s$, $k^*_i$ and $C_k(k^*_i)/c_k$ when $\lambda = \alpha = \sigma = 1$.

<table>
<thead>
<tr>
<th>$c_w/c_i$</th>
<th>$c_s/c_v = 0.1$</th>
<th>$c_s/c_v = 0.1$</th>
<th>$c_s/c_v = 0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t^*_i$</td>
<td>$C_s(t^*_i)/c_s$</td>
<td>$k^*_i$</td>
<td>$C_k(k^*_i)/c_k$</td>
</tr>
<tr>
<td>0.01</td>
<td>14.24 0.2424</td>
<td>14.15 0.2413</td>
<td></td>
</tr>
<tr>
<td>0.03</td>
<td>8.28   0.3486</td>
<td>8.15 0.3444</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>6.46   0.4233</td>
<td>6.30 0.4151</td>
<td></td>
</tr>
<tr>
<td>0.07</td>
<td>5.49   0.4851</td>
<td>5.32 0.4722</td>
<td></td>
</tr>
<tr>
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<td>4.68 0.5114</td>
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</tr>
<tr>
<td>0.10</td>
<td>4.62   0.5546</td>
<td>4.44 0.5239</td>
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</tr>
</tbody>
</table>

Table 6: Optimal $n^*_i$, $C_s(n^*_i)/c_s$, $k^*_i$ and $C_k(k^*_i)/c_k$ when $T_i = \alpha = \sigma = 1$.

<table>
<thead>
<tr>
<th>$c_w/c_i$</th>
<th>$c_s/c_v = 0.1$</th>
<th>$c_s/c_v = 0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n^*_i$</td>
<td>$C_s(n^*_i)/c_s$</td>
<td>$k^*_i$</td>
</tr>
<tr>
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<td>14.15 0.2413</td>
</tr>
<tr>
<td>0.03</td>
<td>9   0.3627</td>
<td>8.15 0.3444</td>
</tr>
<tr>
<td>0.05</td>
<td>7   0.4476</td>
<td>6.30 0.4151</td>
</tr>
<tr>
<td>0.07</td>
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<td>5.32 0.4722</td>
</tr>
<tr>
<td>0.09</td>
<td>5   0.5826</td>
<td>4.68 0.5114</td>
</tr>
<tr>
<td>0.10</td>
<td>5   0.5946</td>
<td>4.44 0.5239</td>
</tr>
</tbody>
</table>
Optimum Replacement Policy for Continuous Damage Model and its Application to Garbage Collection

National Natural Science Foundation of China (70471017, 70801036); Humanities and Social Science Research Foundation of China (05JA630027).

References

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