Reliability Modeling of Rotary Systems Subjected to Imbalance

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(Received on October 31, 2012, revised on April 26, 2013)

Abstract: The focus of this paper is on reliability computation of rotary systems. Rotary systems reliability depends not only on the static design stresses but also on dynamic forces generated during operation. These systems are subjected to many sources of dynamic forces such as imbalance, misalignment, oil whirl, bend shaft, mechanical looseness and so on. The major source of problem of a rotary system, viz, the imbalance is considered in this paper. A model is proposed to incorporate the effect of imbalance mass on the reliability of rotary systems. The effect of rotating speed on reliability can also be assessed by this model. A stress-strength interference approach together with a simulation-based methodology is used for modeling and analysis of this complex relationship. Illustrative examples are provided for demonstrating the practical utility of the approach.

Keywords: Rotary system, reliability, simulation, imbalance, stress-strength interference.

1. Introduction:

Mechanical systems can be broadly classified into four categories, viz., stationary, reciprocating, sliding and rotating systems. The type of forces and the failure mechanisms acting on these systems are also of different nature. Each type of system, therefore, requires appropriate methods to identify and quantify dynamic forces in these systems. Traditionally, failure theories in mechanical systems are based on deterministic approaches like the factor of safety or safety margin [1]. However, the probabilistic stress-strength interference (SSI) approaches appear to be more appropriate for mechanical systems [13]. This research work addresses the type of additional forces generated in mechanical systems and their effect on reliability. The focus is on the rotary systems. This section provides a brief review on the failure mechanisms, failure models, stress-strength models, and mass imbalance problem in rotary systems.

1.1 Mechanical System Reliability

A detailed literature survey on mechanical system reliability is available in [2]. The recent developments are: a diagraph method [3] to compute the reliability of the interacting tribo-pair, function-cum-structure approach for the failure analysis [4], application of support vector mechanics to predict the vibration trend and degradation [5], an empirical model called stressor-based univariate monitoring method for process and equipment monitoring, fault detection, diagnostics, and prognostics [6], wavelet transformation approach for the fault detection in gear shaft [7], non-destructive defect assessment procedures namely MILS-UI and GILS-EKF-UI for fault diagnosis [8], and a model based fault identification in rotating machinery [9]. Some important works on failure
mechanisms and models for the mechanical systems are also available in literature. Failure mechanisms such as fatigue, diffusion, inter-diffusion, creep, corrosion, wear, radiation damage, large elastic deformation, yield, buckling, fracture were listed under the headings of over-stress failures and wear-out (degradation) failures are presented in [10] and 11]. These are classified into four types of failure models, viz., stress-strength interference (SSI), damage-endurance, challenge-response, and tolerance-requirement models [12].

A detailed and comprehensive treatment on using the SSI approach in mechanical systems is presented in [13]. In recent times, the SSI approach is applied for evolving the failure rate from the competing mechanisms of statistically increased load and degrading strength [14] and a model relating life of the component and the number of load applications is presented in [15]. Readers may also refer [2] and [16] for a critical review on these topics. Rotating systems are subjected to several problems such as mass imbalance, shaft misalignment, and the improper surface finish of the bearing among others [17]. The mass imbalance causes vibrations leading to unwanted motion of the rotors such as rotor-to-stator rubbing. This problem is discussed in detail by [8, 19, 20, and 21]. The forces generated due to the imbalance of rotor can be very large especially at high rpm of rotor [22]. Such forces can result in large vibrations and catastrophic failures. The problem of balancing in the rotating machinery is also well researched and documented. Balancing of rotary systems has grown as a specialized topic itself. However, any model relating the effect of imbalance on the reliability of rotary systems is not found in the literature. In view of these research gaps, an attempt has been made in this paper to develop a reliability model for the rotary systems with imbalance. The effect of imbalance is included in the SSI models and a simulation approach is used for numerical solution.

The remaining part of the paper is organized as follows: Dynamic problems in rotating systems with specific emphasis on imbalance are presented in section 2. The proposed model is presented in section 3 which is illustrated with a numerical example in section 4. Conclusions with scope of future work are presented in section 5.

Notation:

- \( d \) : diameter of the shaft
- \( F_{im} \) : force due to imbalance
- \( f_\alpha(\alpha) \) : density function of ultimate shear strength
- \( f_{\text{eff}}(\tau_{\text{eff}}) \) : density function of effective maximum shear stress
- \( M \) : bending moment
- \( M_{im} \) : bending moment due to imbalance force
- \( m \) : imbalance mass
- \( R \) : reliability
- \( r \) : distance between center of rotational axis and imbalance mass
- \( T \) : torque
- \( \alpha \) : ultimate shear strength
- \( \mu_\alpha \) : mean of the normally distributed ultimate shear strength
- \( \mu_{\text{eff}} \) : mean of the normally distributed effective maximum shear strength
- \( \sigma_\alpha \) : standard deviation of the normally distributed shear strength
- \( \sigma_{\text{eff}} \) : standard deviation of the normally distributed effective maximum shear strength
- \( \sigma_{\text{max}} \) : maximum normal stress
- \( \tau_{\text{eff}} \) : effective maximum normal stress
- \( \tau_{\text{max}} \) : maximum shear stress
- \( \tau_{\text{eff}} \) : effective maximum shear stress
- \( z \) : standardized normal variate
- \( \Omega \) : angular velocity
- \( \Phi(z) \) : standard normal cumulative probability value of \( z \)
2. Dynamic Problems in Rotary Systems

As stated earlier, the rotary systems are subjected to severe stresses generated by different types of dynamic problems. Imbalance is one of the most important problems. Components such as gears, pulleys, rollers, and blades mounted on the shaft lead to its imbalance over a period of time. A rotary system is said to be under imbalance if the center of rotation of the rotor is not coinciding with the axis of the rotation of the shaft. This generates large forces when the system rotates, especially at higher speeds. The quantity of the unbalance force depends both on the unbalanced mass, its distance from the center of rotational axis and the rotational speed \[22\]. The force produced due to imbalance is given by

\[ F_{im} = mr\omega^2 \]  

(1)

From the above equation, it can be found that even a little imbalance in the rotor generates huge force resulting in considerable stress. It is also very sensitive to rotational speed. Systematic study with adequate modeling will help to effectively address problems of imbalance. An attempt is made in the next section to develop a model to compute the additional stress due to imbalance and to compute the failure probability.

2.2 Reliability of the Rotary System under Imbalance

Several theories such as Rankine theory, Tresca or Guest’s theory, Saint Venant theory, Haigh’s theory, and Von Mises theory are available in the literatures for shaft design \[23\]. During the transmission of power as standalone transmitter or with the help of mounted gears, pulleys etc., the shaft is subjected to both twisting moment and bending moment. Therefore, when the shaft is designed, both moments are considered simultaneously, to compensate the elastic failures due to combined stresses. The Rankine’s (maximum normal stress theory) and Guest’s (maximum shear stress theory) theories are widely used in the literature \[23\].

The Maximum shear stress in the shaft, according to Guest theory \[23\] is given by,

\[ \tau_{max} = \frac{16\sqrt{M^2 + T^2}}{\Pi d^3} \]  

(2)

The Maximum normal stress in the shaft, according to Rankine’s theory \[23\] is given by,

\[ \sigma_{max} = \frac{32\left(\frac{(M) + \sqrt{M^2 + T^2}}{2}\right)}{\Pi d^3} \]  

(3)

Traditionally, the computed dimensions based on these theories are then multiplied with factor of safety (FS) to get the design dimensions. The value of FS varies depending on the type of load acting on the component, viz., steady load, live (dynamic) load or shock load \[23\]. Some times for dynamic loads, the factor of safety will be as high as 12 to 15 depending on the material used and the application. These FS are empirical assessment based on the observations and experience, and not the exact one \[1\]. The ill effects of FS are large dimensions, space, weight and cost.
Force due to Imbalance

The stress caused by the imbalance force produces considerable bending moment and a negligible amount of torque in the shaft [24]. Therefore, the actual stress on the shaft is the sum of the effects of the bending moment due to force caused by mass imbalance and those due to operational forces. The combined effect of these stresses when exceeds the ultimate strength of the system, it fails. The effective stress can be computed by including the bending moment and torque due to imbalance to the operational bending moment and torque respectively. The equations (2 and 3) given by Rankine and Guest theories can thus be modified as:

\[ \tau_{\text{eff}} = \frac{16\sqrt{(M_u + M)^2 + T^2}}{\Pi d^2} \]  \hspace{1cm} (4)

\[ \sigma_{\text{eff}} = \frac{32}{\Pi d^2} \left( \frac{(M_u + M) + \sqrt{(M_u + M)^2 + T^2}}{2} \right) \]  \hspace{1cm} (5)

These equations can be used for developing the SSI models for reliability computation.

3. Reliability Model

It can be seen that several variables in the equations (4 and 5) are probabilistic. The variables such as radial distance between axis of the shaft and the location of unbalance mass, mass causing imbalance, ultimate tensile stress and ultimate normal stresses are probabilistic in nature. Variations in manufacturing, heat treatment, dimensions, environmental effects, human or machine accuracy, position and quantity of unbalance mass among several other factors contribute to this random nature of variables. Computation of the stress and strength distributions and the reliability of the system with such randomness become very difficult by the usual analytical methods of SSI technique. One possible method is to conduct laboratory tests to generate data. However, it requires huge resources and hence practically difficult. Therefore we propose to use the discrete event simulation technique to generate these variables and establish the strength distribution. This distribution is then used for reliability computation using the following equation (6) [25].

\[ R = \int_{-\infty}^{\infty} f_{\alpha}(\alpha) \left[ \int_{-\infty}^{\tau_{\text{eff}}} f_{\tau_{\text{eff}}}(\tau_{\text{eff}}) \, d\tau_{\text{eff}} \right] d\alpha \]  \hspace{1cm} (6)

As mentioned earlier, the ultimate material strength will also have stochastic nature due to variations in raw material quality, manufacturing process, heat treatment, surface defects, and several other causes. Therefore, we also propose to generate the ultimate stress values by simulation. Thereafter these two distributions are combined using the well known Stress-Strength interference (SSI) technique to compute the reliability. It is important to mention here that in several literatures, for example [26, 27] the time independent reliability is termed as reliability strength (or strength reliability). It is proposed also to replicate the simulations several times to correctly establish the best distributions and evaluate the parameters. As a special case, if the effective shear stress values and ultimate shear strength values obtained after the simulation cycles follow the normal distribution, equation (7) can be used to compute the reliability.

\[ R = 1 - \Phi(z) \]  \hspace{1cm} (7)

where, \( z \) is given by

\[ z = -\frac{\mu_{\tau_{\text{eff}}} - \mu_{\tau_{\text{eff}}}}{\sqrt{\varsigma_{\tau_{\text{eff}}}^2 + \varsigma_{\tau_{\text{eff}}}^2}} \]
Similar method can be applied for other distributions also. For more details refer [13, 25]. Important steps of the proposed methodology are presented below:

**Steps of the Proposed Methodology**

1. Define the rotary system and various parameters, including ranges of speed and unbalance expected.
2. Compute the total force due to the imbalance in the rotor using equation (1).
3. Convert the force of imbalance into the equivalent stress on the system by mathematical models. See equations (2, 3).
4. Modify standard stress theories by incorporating the additional stress due to imbalance. This modified equation is referred as effective stress equation in this work. See equation (4, 5).
5. Identify the random variables with probabilistic nature in the effective stress equation.
6. Identify the probability distribution function (pdf) of these variables and estimate the parameters of the distribution.
7. Assume the lowest speed and unbalance mass.
8. Generate random numbers and use them to generate values for each one of the random variables based on their pdf.
9. Simulate the derived stress equations for the generated random variables and fit the results into the suitable combined stress distribution. Estimate the parameters of this distribution.
10. Similarly find the pdf of strength distribution and estimate its parameters. In many cases strength distribution may also be assumed as following a normal distribution [27, 28].
11. Use the SSI theory by substituting density function for stress and density function for strength in the generalized SSI reliability equation (6).
12. Substitute the parameters of the distributions, in the equation obtained in steps 9 and 10 to compute the reliability.
13. Increase the unbalance mass and study the effect of this on reliability at different rotational speeds. (Repeat the steps from 8 to 13 until the upper limit of speed and unbalance mass is reached).
14. Establish matrix of safe limits for unbalance and rotational speeds for target reliability based on the results. Graphical presentation of results is very useful for analysis.

The methodology is illustrated by numerical examples in the next section.

**4. Numerical Example**

**Illustration 1**

Consider a shaft with a gear used for power transmission. It is rotating on two bearings as shown in figure 1. The shaft is made of 45C8 steel. All dimensions are shown in the figure. The maximum bending moment and the torque are estimated to be 3000 N-m and 10000 N-m respectively. From the Handbook on metals it is found that the ultimate mean values of the tensile strength and the shear strength are 700 MPa and 500 MPa respectively for the 45C8 steel. The range of speed considered for simulation is from 500 rpm to 17000 rpm and that of imbalance mass is from 0.025 kg to 0.25 kg. It is assumed that the unbalance mass never exceeds 0.25 kg due to periodic maintenance and balancing. Based on the experimental results one would like to establish the safe ranges of speed and imbalance mass for a given target reliability.
As discussed earlier, the imbalance mass causes additional stress due to force of imbalance apart from the other stresses due to the normal operations. This effective total stress is computed using the above proposed model by substituting the numerical values for the parameters involved. Then the reliability of the shaft is computed by using the simulation technique as discussed above.

In this example, the simulation is first done with unbalance mass of 0.025 kg and the speed 500 rpm. The values for the random variables (radius of the shaft, length of the shaft) are generated by simulation. The effective shear and tensile stresses are computed by the equation (4 and 5). These effective stress values are fitted to various distributions. It is found that the data can be best fitted by a normal distribution. The ultimate strength is assumed to be following normal distribution. The reliability is then computed using equation (7).

The above procedure is repeated for the different imbalance mass and speed. The simulation cycles are repeated for all generated random variables (simulated 25000 times).

Results and Discussion for Illustration 1

The results of the above example are presented in the figures 3 and 4. Variations of reliability with respect to imbalance mass and the rotational speed are shown in these figures. Several inferences can be made from these results. For an imbalance mass of 0.025 kg the reliability shows only negligible decline up to the rotational speed of 9500 rpm, and it drops considerably up to 10500 rpm. Thereafter, the reliability decreases steeply. Similar inference can also be made for other masses. Summary of these inferences are presented in the Table 1.

The Figure 4 shows that the imbalance mass is having negligible impact on the reliability up to the speed of around 3000 rpm for the given dimensions. If the rotor used for higher speed applications its reliability decreases very fast. If the imbalance can be limited by regular monitoring and correction, then the system can be highly reliable and safe to operate up to about 9500 rpm. This system can be safely operated at higher rpm if and only if it is balanced periodically.

<table>
<thead>
<tr>
<th>Imbalance Mass (Kg)</th>
<th>Safe RPM (R ≥ 0.99)</th>
<th>Unsafe RPM (0.95 ≤ R ≤ 0.99)</th>
<th>Critical RPM (R &lt; 0.95)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>≤ 8000</td>
<td>8000 – 9700</td>
<td>&gt; 9700</td>
</tr>
<tr>
<td>0.050</td>
<td>≤ 6000</td>
<td>6000 – 6800</td>
<td>&gt; 6800</td>
</tr>
<tr>
<td>0.100</td>
<td>≤ 4200</td>
<td>4200 – 4750</td>
<td>&gt; 4750</td>
</tr>
<tr>
<td>0.150</td>
<td>≤ 3500</td>
<td>3500 – 3750</td>
<td>&gt; 3750</td>
</tr>
<tr>
<td>0.200</td>
<td>≤ 3000</td>
<td>3000 – 3400</td>
<td>&gt; 3400</td>
</tr>
<tr>
<td>0.250</td>
<td>≤ 2600</td>
<td>2600 – 3050</td>
<td>&gt; 3050</td>
</tr>
</tbody>
</table>
Illustration 2
The case of an industrial fan is considered. This consists of a shaft (0.045 m diameter) fitted on a bearing at one side and three blades on the other side. All the dimensions are shown in the figure 2. The fan exerts a vertical load of 7200 N on the shaft at A. An electric motor of 20 kW is attached as the prime mover. The fan rotates at various speeds depending on the requirement. The shaft is made up of 45C8 steel.

![Figure 3: Reliability of the Shaft with Imbalance at Different Speeds](image)

In this example the simulation is first done with unbalance mass of 0.05 kg and for the various speeds. The effective shear and tensile stresses are computed by the equation (4 and 5). These effective stress values are fitted to various distributions and it is found that the data can be best fitted by a normal distribution. The ultimate strength is assumed to be following normal distribution. The reliability is then computed using equation (7). This procedure is repeated for the different imbalance mass and speed. The numbers of simulation cycles are repeated for all generated random variables (simulated 300,000 times). The results are presented in the next section.

![Figure 4: Reliability of the Shaft with respect to Mass Causing Imbalance](image)

**Results and Discussion for Illustration 2**
The results of the above illustration 2 are presented in the Table 2. The graphical representations of results are shown in figure 5. Variations of reliability with respect to imbalance mass and the rotational speed are shown in this figure.
Table 2: Safe and Critical RPM of Rotor

<table>
<thead>
<tr>
<th>Imbalance Mass (Kg)</th>
<th>Safe RPM (R ≥ 0.99)</th>
<th>Unsafe RPM (0.95 ≤ R ≤ 0.99)</th>
<th>Critical RPM (R &lt; 0.95)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>≤ 4800</td>
<td>4800 – 4700</td>
<td>&gt; 4700</td>
</tr>
<tr>
<td>0.10</td>
<td>≤ 3430</td>
<td>3430 – 3445</td>
<td>&gt; 3445</td>
</tr>
<tr>
<td>0.15</td>
<td>≤ 2800</td>
<td>2800 – 2810</td>
<td>&gt; 2810</td>
</tr>
<tr>
<td>0.20</td>
<td>≤ 2400</td>
<td>2400 – 2430</td>
<td>&gt; 2430</td>
</tr>
<tr>
<td>0.25</td>
<td>≤ 2160</td>
<td>2160 – 2180</td>
<td>&gt; 2180</td>
</tr>
<tr>
<td>0.40</td>
<td>≤ 1705</td>
<td>1705 – 1725</td>
<td>&gt; 1725</td>
</tr>
<tr>
<td>0.50</td>
<td>≤ 1530</td>
<td>1530 – 1545</td>
<td>&gt; 1545</td>
</tr>
</tbody>
</table>

Figure 5: Reliability of the Shaft with Imbalance at Different Speeds

It can be seen from Figure 5 that for the given dimensions, the imbalance mass is having negligible impact on the reliability up to the speed of around 4800 rpm (for m = 0.05 Kg) and 1700 rpm (for m = 0.4 Kg). If the rotor system with imbalance mass used for higher speed applications its reliability decreases considerably. If the imbalance can be limited by regular monitoring and correction, then the system can be highly reliable and safe to operate at higher speeds. Therefore it is suggested that this system needs periodic monitoring and balancing. Similar figures can be drawn for other combinations of speed and mass for reliability analysis and decision making.

5. Conclusions

Imbalance in rotating systems results in huge dynamic forces at higher rotational speeds, leading to failure and unreliability. Development of models relating imbalance can help finding the safe and critical levels of speed for a known imbalance mass and vice versa.

In this paper, a reliability model for the generalized rotary system subjected to imbalance is presented and illustrated with examples. The model can be effectively used along with simulation methodology to predict the operational reliability of a rotor system given the imbalance present and its rotational speed. If the reliability target is specified, safe ranges of operational speed and allowable imbalance mass (imbalance tolerance) can be established for an existing or new system using this model. Such study can also help us to take necessary actions, including maintenance and balancing, to keep the imbalance within the threshold limit for a given or required rotational speed.

The approach presented has strong theoretical basis. As a future scope, experimental methods may be tried to conduct similar studies using standard test setups like a
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machinery fault simulator. The experimental results can be compared with the theoretical results for better understanding of the complex system.

References


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