Reliability Based Methodologies for Optimal Maintenance Policies in Military Aviation

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Abstract: In Military aviation, depots confront situations of declaring certain components as High Failure Rate Components (HFRC) to shortlist them for reliability improvement and subjecting them to reviewed maintenance actions. Presently this is done mainly through intuition, experience or based on the number of unscheduled failures at repair depots. This paper develops a methodology by selecting three variants of the same aero engine as a case for deciding a threshold based on reliability, at which the component can be rendered HFRC. Further an optimization methodology based on downtime is also evolved to review the overhaul policy for the HFRCs. Generalized Renewal Process (GRP) models have been utilized to compute Maximum Likelihood Estimators (MLEs). The authenticity of all the developed models is tested through numerical examples and is validated with the existing field conditions.

Keywords: Generalized renewal process, repairable systems, Monte Carlo simulation, high failure rate component, military aviation.

1. Introduction

Repair actions in military aviation [1] may not fall under ‘as good as new’ and ‘as bad as old’ assumptions but could be somewhere between the two. Thus aviation associated aggregates are considered to be repairable systems [2, 3]. A repairable system may end up in one of the five likely states subsequent to a repair: (a) as good as new, (b) as bad as old, (c) better than old but worse than new, (d) better than new, (e) worse than old. Existing probabilistic models used in repairable system analysis, such as the perfect renewal process (PRP) and the non homogenous Poisson process (NHP), account for the first two states, correspondingly. In the concept of imperfect repair [4, 5, 6] the repair actions are unable to bring the system to as good as new state, but can transit to a stage that is somewhere between new and that of preceding to the failure. Because of the requirement to have more precise analyses and predictions, the GRP can be of great interest to reduce the modelling ambiguity resulting from the above repair assumptions.

Kijima and Sumita [7] recommended two possible probabilistic models to address a very broad assumption concerning the system repair state called general renewal process. Kijima model I assumes that repair is effective only for the last repair whereas Kijima model II assumes that repairs can restore cumulative wear out and damage up to the present time. Kaminskiy and Krivtsov [8] and Krivtsov [9] projected a fairly accurate solution to Kijima model I using the Monte Carlo (MC) simulation method. Yanez et al [10] combined MC simulation with numerical methods to develop MLEs for the Kijima model I. Mettas and Zhao [11] present a general likelihood function formulation for single and multiple repairable systems for estimation of GRP MLEs. Literature review reveals scope for further work, predominantly in applications of GRP particularly in the field of military aviation.

Presently, components are designated HFRC based primarily on the number of failures, declared high failure rate components and shortlisted for being undertaken for
reliability improvement measures. If the threshold on HFRCs does not have a sound scientific and mathematical foundation, the number of components being branded HFRC could be either underestimated or overestimated. If numerous components are declared HFRC, the organization faces needless maintenance load, and expenditure; if a small number of components are acknowledged as HFRC, the organization might lose opportunities to undertake them for reliability improvement measures. Consequently developing an appropriate scientific model for HFRC declaration is crucial for improving the overall performance of the organization.

The paper aims at developing a model for defining threshold for declaring components HFRCs by treating them as repairable systems, and using GRP models to estimate MLEs and other performance parameters. As a measure to develop a methodology for treatment of declared HFRCs, the paper also evolves a new maintenance policy by reviewing the Time Between Overhauls (TBOs) for the HFRCs. All developed models and methodologies have been appropriately tested and effectively demonstrated with the help of field failure data for the three variants of the same aero engine.

The paper is planned to progress as follows: section 2 covers assumptions and basic concepts related to GRP, section 3 explains the methodology, section 4 presents inferences & deliberations, and section 5 concludes the work.

2. Concepts Related to GRP

In this section we list out the assumptions and discuss the concepts related to virtual age, repair effectiveness and performance measures linked with GRP.

2.1. Assumptions

We assume the following:

(a) Time to first failure distribution (TTFF) is known and can be estimated from the available data using MLEs of equations (8), (9) and (10). Because of its flexibility and applicability to various failure processes, the Weibull distribution is used as the distribution of time to first failure to perform the analysis. Every time a failure takes place, it is subjected to an imperfect repair and it attains a virtual age. The probability of next failure is conditioned on survival till the earlier virtual age attained after the imperfect repair. This continues and hence every failure appears as the first failure conditioned on the virtual age after the previous failure (as modelled in equation 1). This is the reason we call it TTFF distribution.

(b) The repair time is assumed to be negligible so that the failures can be viewed as point processes.

(c) The aero engine is subjected to imperfect repair on failing. The aero engine is assumed to be ‘as good as new’ after overhaul.

2.2. Virtual Age Concept (Kijima Model I)

Kijima [7] developed an imperfect repair model by using the notion of the virtual age \( A_n \) process of a repairable system. Parameter \( A_n \) represents the calculated age of the system immediately after the \( n^{th} \) repair occurs. If \( A_n = y \), then the system has a time to the \((n + 1)^{th}\) failure, \( X_{n+1} \), which is distributed according to the following cdf:

\[
F(x|A_n = y) = \frac{F(x+y) - F(y)}{1-F(y)}
\]

(1)

where \( F(x) \) is the cdf of the Time To First Failure (TTFF) distribution of a new system or component. It is assumed that the \( n^{th} \) repair would only compensate for the damage
accumulated during the time between the \((n-1)^{th}\) and the \(n^{th}\) failure. With this assumption the virtual age of the system after the \(n^{th}\) repair is:

\[ A_n = A_{n-1} + qX_n = qS_n, \quad n = 1, 2 \ldots \]

(2)

where \(q\) is the repair effectiveness parameter, \(A_0 = 0\). If the times between the first and second failure the second and third failure, etc, are considered, they can be expressed as:

\[ A_1 = qX_1 \]
\[ A_2 = q(X_1 + X_2) \]
\[ \vdots \]
\[ A_n = q(X_1 + X_2 + \cdots + X_n) \]

(3)

As per this model, \(q = 0\) leads to perfect repair (PRP), while \(q = 1\) leads to minimal repair (NHPP). The case of \(0 < q < 1\) corresponds to an imperfect repair (better than old but worse than new) while \(q > 1\) leads to worse or worst repair (worse than old). Consequently \(q\) can be actually interpreted as an indicator for representing effectiveness and quality of repair. Therefore \(q > 1\) indicates that repair has made the component/system serviceable but after some compromise in its performance.

### 2.3. Expected Number of Failures Estimation in GRP

As evident from section 2.2 the event of the next failure in repairable systems can be simulated in terms of Times To First Failure (TTFF) distribution conditioned appropriately on the virtual age. Monte Carlo simulation offers a numerical solution to the GRP using the TTFF Weibull distribution as follows. Let \(t_i\) be the time between \((i-1)^{th}\) & \(i^{th}\) failures and \(F(t_i)\) be the cumulative density function (cdf) of the TTFF. \(a\) is the scale parameter and \(b\) is the shape parameter. Then using [10]

\[
F(t_i) = 1 - \exp\left[a\left(\frac{X_1 + \cdots + X_i}{q}\right)^{b} - a(t_i + q\sum_{j=1}^{i-1}t_j)^{b}\right] \quad (4)
\]

\[
t_i = \frac{1}{a^{1/b}} \left[\ln\left(1 - F(t_i)\right)\right]^{1/b} - q\sum_{j=1}^{i-1}t_j, \quad \text{for } i=2,3,\ldots n \quad (5)
\]

Then with the help of Monte Carlo simulation [8] we estimate the expected number of failures between the first and second overhaul which we need to determine the optimal time to the second overhaul (equation 17).

### 2.4. GRP MLEs

Differentiating Equation (4) with respect to \(t_i\), the following pdf is derived [10]:

\[
f(t_i) = ab\left[t_i + q\sum_{j=1}^{i-1}t_j\right]^{b-1} \exp\left[a\left(\frac{X_1 + \cdots + X_i}{q}\right)^{b} - a(t_i + q\sum_{j=1}^{i-1}t_j)^{b}\right], \text{for } i=2,3,\ldots n \quad (6)
\]

We can now derive the MLEs for parameters \(a\), \(b\) and \(q\) from the failure data, as brought out in the next segment.

#### 2.4.1. GRP MLEs for Failure-terminated Process

For the first failure we use the cdf and pdf of a Weibull distribution. For the succeeding failures we use Equations (4) and (6). The likelihood function is given by the expression [10]:
We differentiate the logarithm of the likelihood function above with respect to each of the three parameters \( a, b, \) and \( q \) and equate each derivative to zero, we get a system of three equations with three unknown variables \([10]\). For obtaining the values of \( a, b, \) and \( q \) we need to solve the equations shown below:

\[
\frac{\partial \ln(L)}{\partial a} = \left( b \frac{a}{b+1} \right) \left\{ \sum_{i=2}^{n} \left( t_i + q \sum_{j=1}^{i-1} t_j \right)^b - \left( q \sum_{j=1}^{i-1} t_j \right)^b \right\} + \left( b \frac{a}{b} \right) \left[ a \left( t_i + q \sum_{j=1}^{i-1} t_j \right)^b \right] = 0 \tag{8}
\]

\[
\frac{\partial \ln(L)}{\partial b} = \sum_{i=2}^{n} \left[ \frac{n}{b} \ln(t_i) - n \ln \left( \frac{1}{a^{i/b}} \right) - at_i^{1/b} \ln \left( t_i a^{1/b} \right) \right] + \sum_{i=2}^{n} \left[ \ln \left( t_i \right) + \ln \left( t_i + q \sum_{j=1}^{i-1} t_j \right) - a \left( t_i + q \sum_{j=1}^{i-1} t_j \right)^b \ln \left( t_i \right) \right] = 0 \tag{9}
\]

\[
\frac{\partial \ln(L)}{\partial q} = (b - 1) \times \sum_{i=2}^{n} \left( \frac{t_i}{t_i + q \sum_{j=1}^{i-1} t_j} \right)^b - \sum_{i=2}^{n} \left( t_i \right)^b \sum_{i=1}^{n} \left( q \sum_{j=1}^{i-1} t_j \right)^b \ln \left( t_i \right) = 0 \tag{10}
\]

3. **Methodology**

As discussed in section 1, presently the threshold for the number of failures for a component to be declared HFRC is significantly dependent on the intuition and experience of the relevant personnel. There is a clearly felt need for a scientific methodology for HFRC designation. We propose and demonstrate such a methodology below.

3.1. **Case Description**

We have chosen three variants of the same aero engine as a case to develop a methodology for determining HFRC and work out a new maintenance strategy consequently. The selected aero engines are turbojet with eleven stages compressor, an annular type combustion chamber, two stages gas turbine, with a variable area jet nozzle. As discussed before the aero engine is a repairable system which is sent to depot [1] for repair and can land up in any repair state. The aero engines are planted back into function after imperfect repair. Original equipment manufacturer (OEM) specified TBO for the aero engines is 550 hrs and total technical life (TTL) is 1800 hrs.

3.2. **Proposed Threshold Model for HFRC**

Whenever failures take place, the aero engine is exposed to imperfect repair at depot and planted back into function. We consider the time to the first overhaul (OH) be \( t_{1,oh} \) and \( t_i \) as explained in section 2.3. Prior to developing a threshold model for HFRCs our main concern is to estimate \( a, b, \) and \( q \) with the assistance of failure data for the first overhaul cycle. We work out the consequent Intensity function and reliability using equations (11) & (12).

**Intensity Function Equation**

\[
u(t, a, b, q) = \frac{f(t_i)}{1-f(t_i)} = a \times b \times (t_i + q \sum_{j=1}^{i-1} t_j)^{b-1}\] (11)
Reliability Equation

\[ R(t) = \exp\left[ a\left(q \sum_{i=1}^{t_i} t_j \right)^b - a\left(t_i + q \sum_{i=1}^{t_i} t_j \right)^b \right] \] (12)

We progress towards our first objective i.e. to find the threshold age, \( t^* \) at which the component should be stated HFRC. Let the total no. of aircraft in the fleet be \( A \) and the total no. of aero engines in the inventory be \( E \). Assuming a critical situation where all aircraft are required to deliver and none of them is desired to be unavailable due to requirement of aero engines, total no of aero engines required for a desired level of flying availability is decided by the organization and is say \( A+\Delta A \), where \( \Delta A \) is the number of additional aero engines to be made available in the inventory to meet any unserviceability of the aero engines which are part of the flying aircraft at any given point of time. We assume \( \Delta A \) is given to us and decided by the organization based on utilization of engines. Thus, \( E-(A+\Delta A) \) is the number of aero engines at the overhaul Depot either undergoing /waiting for repair/ overhaul. The probability of survival (P) of aero engines can be defined as the ratio

\[ P = \frac{\text{Total number of aeroengines required}}{\text{Total number of aeroengines in the inventory}} \] (13)

The least required reliability \( R^* \) is estimated as shown below

\[ R^* \leq \frac{A+\Delta A}{E} \] (14)

The aero-engine is supposed to be affirmed HFRC at the age \( t^* \) as soon as the reliability \( R(t^*) \) goes below \( R^* \). Therefore the proposed criteria for designating a component HFRC is:

\[ R(t^*) = \exp\left[ a\left(q \sum_{i=1}^{t_i} t_j \right)^b - a\left(t_i + q \sum_{i=1}^{t_i} t_j \right)^b \right] \leq \frac{A+\Delta A}{E} \] (15)

We demonstrate the above on a hypothetical data set. Let the total number of aircraft in the fleet be 75 and the total number of aero engines in the inventory be 190. Let the total no of additional aero engines to be available in the inventory to meet any unserviceability of the aero engines which are part of the flying aircraft at any given point of time be 25. Thus \( A=75 \), \( E=190 \), \( \Delta A=25 \), \( A+\Delta A=100 \). From equation (14), \( R(t^*) = 100/190 =0.5263 \). Subsequently, the age \( t^* \) at which \( R(t^*) \leq 0.5263 \) indicates the age at which the aero engine is supposed to be acknowledged HFRC. The threshold reliability and intensity function can be estimated thereafter. Hence we can plot reliability \( R(t) \) as a function of time by means of the best fit curve acquired from the data set \([t_i, R(t_i)]\). We plot the best fit curve in Figure (1) below.

![Figure 1: HFRC Threshold](image)
Declaring an aero engine HFRC at an age lesser than $t^*$ will not yield desired results. If the aero engines are declared HFRCs below $t^*$, then their number visiting the repair depot will be very high. Depots will face an additional task of undertaking HFRCs for reliability improvement besides meeting the commitments of planned overhaul tasks and unscheduled repairs.

3.3. Reviewed Maintenance Policy for HFRCs

In the preceding section 3.2, we develop the criteria to define a threshold to declare the aero engines HFRC. This paper reviews the present maintenance policy for the HFRCs by reviewing the present TBO of the aero engines. Hence the objective here is to estimate the time of next overhaul, $t_{2oh}$. We could obtain the GRP MLEs for $a$, $b$, $q$ from the failure data of the first overhaul cycle and using equations (8, 9 and 10). Subsequently we find the optimal time for the next overhaul $t_{2oh}$, of the aero engines, subject to breakdowns. We develop a model based on downtime for this analysis.

3.3.1. Revised TBO Model

The aim is to find optimal time for the next overhaul. According to the overhaul policy, overhaul will be done at a fixed time and the aero engines that fail are subjected to imperfect repair and then put back into the use. We establish the optimal interval to second overhaul to minimize the total downtime per unit time by using the following method.

We use the following notation for various parameters as mentioned below:
- $T_r$: Mean downtime necessary to carry out an imperfect repair.
- $T_{oh}$: Mean downtime required to perform an overhaul.
- $t_{2oh}$: Time to second overhaul
- $E_n(t_{2oh})$: Expected no of failures in the interval $(0, t_{2oh})$

The overhaul strategy is to carry out overhaul at $t_{2oh}$, irrespective of the age of the equipment, and imperfect repairs occur as many times as required in the interval $(0, t_{2oh})$

The overall downtime per unit time, for overhaul at time $t_{2oh}$, denoted by $D(t_{2oh})$ is

$$D(t_{2oh}) = \frac{\text{Expected downtime due to repairs} + \text{downtime due to overhaul cycle length}}{\text{cycle length}}$$

Expected downtime due to repairs = Expected number of failures in interval $(0, t_{2oh}) \times$ Mean time to repair $= E_n(t_{2oh}) \times T_r$. Therefore,

$$D(t_{2oh}) = \frac{E_n(t_{2oh}) \times T_r + T_{oh}}{t_{2oh} + T_{oh}}$$

We achieve $E_n(t_{2oh})$ from MC simulation with assistance of equation (5). We also carry out repetitive simulation to estimate the value of $t_{2oh}$ that minimizes $D(t_{2oh})$ specified by equation (17).

4. Inferences & Deliberations

Till now we could develop two models, one for deciding a threshold for HFRC and the other one for reviewing the current overhaul cycle for the HFRCs. But we need to verify and validate both the models by applying them on a real field failure data. In this section, we present the inferences and due deliberations associated with them after testing our methodology on all 3 variants of aero engines.
4.1. Results & Analysis: Variant 1

Variant 1 underwent 66 failures during the first overhaul cycle and the number of failures during the reviewed second overhaul cycle is 21. The results follow:

\[ a = 0.00022, \ b = 1.35, \ q = 0.75, \ t^* = 173 \text{ hours (h)}, T_{oh} = 6336 \text{ h}, T_r = 528 \text{ h}, t_{2oh} = 152 \text{ h} \]

Equations (8), (9) and (10) are applied to achieve the values of \( a, b, q \) as deliberated in section 2.4.1. \( T_{oh} \) is acquired from overhaul manual and \( T_r \) is estimated with the assistance of the repair data of preceding 10 years. Additionally, \( t^* \) & \( t_{2oh} \) are evaluated as per the methodology discussed in sections 3.2 & 3.3.1. Note that \( t_{2oh} < t^* \) which means that if the reviewed overhaul policy is followed, the aero engine will never fall in the domain of HFRC.

In Table 1 below, we present relative outcomes of both the overhaul cycles with the help of Reliability, MTBF & Availability figures at the end of each overhaul cycle, using our recommended overhaul policy. We also present the percent enhancement in these performance indices in the Table 1.

We have already discussed in section 3.2 regarding estimation of reliability and the intensity function. We now discuss the methodology to evaluate MTBF & Availability [2].

\[
MTBF(t) = \frac{1}{u(t, a, b, q)}
\]  
(18)

\[
A(t) = \frac{MTBF(t) + T_r}{MTBF(t)}
\]  
(19)

<table>
<thead>
<tr>
<th>Cycles</th>
<th>R(t)</th>
<th>MTBF(t)</th>
<th>A(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I OH ((t = t_{1oh}))</td>
<td>0.0095</td>
<td>117.28</td>
<td>0.1818</td>
</tr>
<tr>
<td>II OH ((t = t_{2oh}))</td>
<td>0.5960</td>
<td>286.69</td>
<td>0.3519</td>
</tr>
<tr>
<td>% Enhancement</td>
<td>98.4</td>
<td>59.1</td>
<td>48.34</td>
</tr>
</tbody>
</table>

We monitor from Table 1 that by means of our novel overhaul policy we gained almost 98% improvement in Reliability & 48% improvement in Availability which is quite appreciable.

4.2. Results & Analysis: Variant 2

Variant 2 faced 142 failures during the first overhaul cycle. We act upon similar analysis and the results are put forth below. The number of failures until the reviewed overhaul cycle is 20.

\[ a = 1.64 \times 10^{-16}, \ b = 4.87, \ q = 4.3, \ t^* = 94.63 \text{ h}, T_{oh} = 6336 \text{ h}, T_r = 440 \text{ h}, t_{2oh} = 50 \text{ h} \]

<table>
<thead>
<tr>
<th>Cycles</th>
<th>R(t)</th>
<th>MTBF(t)</th>
<th>A(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I OH ((t = t_{1oh}))</td>
<td>0.0000</td>
<td>3.912*10^{-6}</td>
<td>0.1818</td>
</tr>
<tr>
<td>II OH ((t = t_{2oh}))</td>
<td>0.7703</td>
<td>274.65</td>
<td>0.38</td>
</tr>
<tr>
<td>% Enhancement</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 2: Relative Outcome of the Two Overhaul Cycles
The abnormally low values of $a$ and MTBF is attributable to an extreme wear out and very poor quality of repair on variant 2 aero engines experienced during the first overhaul cycle as evident from the estimated values of $b=4.87$ & $q=4.3$. This comes under the category of worse repair. Recall that higher the value of $q$, the worse is the quality of repair. From the Table 2 above we observe noteworthy enhancement in all performance indices.

4.3. Results & Analysis: Variant3

Variant 3 had 61 failures during the first overhaul cycle. Our analysis yields the following:

$$a=0.001275, b=1.08, q=0.12, t^*=278 \text{ h}, T_{dh}=6336 \text{ h}, T_r=520 \text{ h}$$

Since $b=1$, the failures are mostly random; hence reviewing the overhaul policy is not desirable. Further we scrutinize that even the repair effectiveness factor ($q$) is best in this case. The superior values of $b$ & $q$ for this variant arrive as no revelation. The variant 3 aero engine has been actually developed after incorporating considerable knowledge from the failure modes investigation of the earlier variants.

5. Conclusion

From the foregoing it is quite clear that at some point of time of equipment exploitation we need to define a threshold for the equipment, to be considered under the domain of a HFRC. In this paper we endeavor to develop an appropriate model for deciding a threshold for deeming a component HFRC. We went a step beyond by evolving another methodology for providing a suitable treatment for the declared HFRCs in form of reviewing the present maintenance policy. Both the models are developed by selecting the three variants of the same aero engine as case and considering them as repairable systems.

The repairs are considered imperfect and a comparatively less explored and utilized method in the field of repairable systems reliability analysis, GRP MLEs have been used to estimate all performance indices. The methodology not only provides shape and scale parameters but also renders an index to assess repair efficacy. Investigating these indicators individually and together provide an ample insight into the current maintenance practices. The proposed methodology in case of first two variants reveal that the aero engine has been rendered HFRC earlier than their OEM approved TBO. In view of the fact that the TTL of the aero engines is 1800 h, a dire need to review the TBO exists so that the aero engines do not come into the sphere of influence of HFRCs. We also present a methodology to review the TBO for the next overhaul cycle. We detect noteworthy enhancement in all the performance parameters in the revised maintenance policy.

A promising expansion of our work could be the development of threshold models based on availability and intensity function. Further cost benefit & failure modes analysis of the revised maintenance policy (that has more overhauls & less breakdown repairs) could be carried out for better insight and performance.

References


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