Availability Estimation of a Cooling Tower Using GSPN

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Abstract: This paper deals with the availability analysis of a Cooling Tower used in an air conditioning system. The system is modeled as a Generalized Stochastic Petri Net (GSPN) and analyzed using Monte Carlo Simulation method. The superiority of this approach over others is demonstrated. The proposed GSPN is a promising tool that can be conveniently used to model and analyze any complex systems. It is a promising tool for modeling and estimation of reliability measures of any process plants.

Keywords: Availability, reliability, Markov method, Petri net, process plants

1. Introduction

An important system like cooling tower must be operated at high levels of availability in view of not only a huge loss incurred due to loss of cooling capacity but also on their installation, operation and maintenance. In this context, a reliability analysis would not only give an estimate of system’s availability, but also propose a means of discovering potential combinations of events which might result in catastrophic failures and evaluating the probabilities of their occurrence. The assessment procedure should be able to evaluate other performance measures and include cost-related aspects. Some of the important modeling approaches used in reliability analysis are Network models, Fault Tree and Event Tree analysis (FTA and ETA), State-transition diagram and Petri Nets (PNs).

Network models are function-oriented. These models can tackle structural failures which lower the system performance. It is almost impossible to incorporate maintenance actions, software and human error and other cost-related aspects in network models.

Fault trees are event-oriented. The repair actions and the dependence between components cannot be easily incorporated in the model. Standby redundancies, time-delay conditions and other dynamic behaviour cannot be easily modeled using fault trees, since they are static in nature.

The biggest handicap of Markov models is the explosion of state space. Though it is possible to capture the dynamic behaviour and dependence between components in this formulation, state-space explosion limits its usage. When formulating a Markov model of a complex system, it is difficult to ensure that all the possible combinations of events in a subsystem have been considered. Moreover, it is very difficult to use state-transition diagrams for model validation.

Petri Net, a graphical and mathematical modeling tool is used for studying a complex system, which is concurrent, asynchronous, distributed, parallel and nondeterministic. The use of Petri Nets for reliability analysis simplifies the task of the modeler considerably. It involves drawing a net representing a model of the system and marking it with the corresponding firing times of the transitions. If algorithms to construct the set of all reachable markings of a PN were available and if tools to automate the process of finding the
probability of the markings could be built, then the analyst can concentrate more on reliability issues instead of writing and solving the equations for the underlying stochastic process. A systems approach is possible with PNs since hardware, software and human behaviour can be modeled using the same language. It is also possible to incorporate safety and fault tolerance requirements.

2. **Petri Nets**

As per [1] and [2], Petri Nets have, over the last four decades, attracted the attention of researchers in several areas ranging from computer science to social sciences. PN can be introduced either algebraically or graphically. They are defined algebraically in terms of the following elements [3].

A PN is a 5-tuple, $PN = (P, T, A, W, M_0)$, where

- $P = \{P_1, P_2, ..., P_m\}$ is a finite set of places
- $T = \{t_1, t_2, ..., t_n\}$ is a finite set of transitions
- $A \subseteq (P \times T) \cup (T \times P)$ is a set of arcs,
- $W$ is a weight function that takes values 1, 2, 3, ...
- $M_0$ is the initial marking.

A standard PN consists of a set of "places" $P$ drawn as circles, a set of "transitions" $T$ drawn as bars and a set of directed arcs $A$. An arc connects a transition to a place or a place to a transition. Place may contain "tokens", which are shown as dots. The "marking" or the state of a PN is defined by the number of tokens contained in each place and is denoted by $M$. The construction of a PN model requires the specification of the "initial marking" $M_0$.

A place is called an "input place" to a transition if an arc exists from it to the transition. A place is an "output place" if an arc exists from a transition to the place. A transition is said to be "enabled" when all its input places contain at least one token. If the enabled transition is "fired", it removes one token from each input place and deposits one token in each output place. The firing of a transition modifies the distribution of tokens in places and thus produces a new marking for the PN.

For a given initial marking $M_0$, the "reachability set" $S$ is defined as the set of all markings that can be reached from $M_0$ by a sequence of transition firings. In a Stochastic Petri Net (SPN), the firing time is an exponentially distributed random variable. Thus the marking sequence in a SPN obtained from the firings, is isomorphic to a continuous time Markov Chain. In a Generalized Stochastic Petri Net (GSPN), the transition firing rates can be instantaneous or random firing time based on some distribution. Therefore the set of transitions can be partitioned into a set of random timed transitions (with finite firing rate) and a set of immediate transitions. However, for any marking at which there are several enabled immediate transitions, a probability distribution must be specified, according to which firing of the transitions are selected.

3. **System Modelling and Analysis**

3.1 **System Overview**

The cooling tower taken up for the study is based on the principle of evaporative cooling. Some of the components of cooling tower namely Frame work, drift eliminators, the water distribution system, fan, cold water basin etc. The schematic diagram of the cooling tower is shown in the Fig. 1. The important components are described below.
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Figure 1: General Arrangement of a Cooling Tower

*Framework:* The structure enclosing the heat transfer process reinforced suitably to carry the other main items.

*Drift Eliminators:* There are positioned in the outlet air stream so as to prevent water droplets from being carried away from the tower by the air stream.

*Fan:* Correct selection of fan according to the tower duty is of major importance; volumetric air flow rate, the fan pressure developed and noise from the motor and fan impeller must all be considered according to the duty and location of the tower.

*Water Distribution System:* Water entering the tower must be spread as evenly as possible over the cross section of the tower using the distribution headers and spray nozzles.

*Packing (also called as Fill):* Consists essentially of a system of baffles which allows the progress of warm water through the tower and ensures maximum contact between water droplets and cooling air by maximizing surface area and minimizing water film thickness.

*Cold Water Basin (also called as Tank or Sump):* The point at which the cold water is collected before return to the process.

The construction of functional block diagram for the cooling tower is the first step towards its availability analysis. First, the components that can cause unavailability of each subsystem are identified. The reliability data for these components are taken from published sources and from the in-house records of the plant. Each component of the subsystem is considered to be in one of two states: good or complete failure. The redundancies are taken into consideration in calculating subsystem reliability parameters such as MTBF and MTTR. The failure of a component may cause system failure depending upon the functional configuration of the system. A common-cause failure may also occur due to deficiency in equipment design, operation and/or maintenance error and/or an external catastrophe.
3.2 GSPN specification

The failure mechanism and repair process model of the cooling tower is given in Fig. 2. The initial marking of the net contains tokens in the places $P_0$ to $P_7$ and $P_{17}$. This indicates that subsystems 0 to 7 are working initially. The token in the place $P_{17}$ indicates that the system is working normally. Tokens in the places $P_0$ and $P_{17}$ may enable the transition $t_0$, which corresponds to the failure of the subsystem 0. If the transition $t_0$ is fired, then it removes a token each from places $P_0$ and $P_{17}$ and deposits a token each in the places $P_8$ and $P_{16}$. The token in the place $P_8$ indicates the component 0 is in the failure mode and the one in the place $P_{16}$ indicates the system is in failed state. The token at $P_8$ can enable the transitions $t_8$. The transition $t_8$ corresponds to the repair completion of the subsystem 0. If the transition $t_8$ fires then it removes a token each from the places $P_8$ and $P_{16}$ and deposits a token each in the places $P_0$ and $P_{17}$. This means that the component 0 is repaired and the system starts working normally. The failure and repair actions for the other subsystems are represented in a similar manner.

In this model the presence of a token in the place $P_{17}$ indicates that the system is in good state. Its complete failure is indicated by the presence of a token in the place $P_{16}$.

If $T_o$ and $T_f$ are the respective mean times a token is available in the places $P_{17}$ and $P_{16}$ then the availability of the Lube oil system is given by,

\[
\text{Availability} = \frac{T_o}{T_o + T_f}
\]

Here, $T_o$ is equivalent to the MTBF of the Cooling tower and $T_f$ is equivalent to its MTTR.

![Figure 2: GSPN Model of a Cooling Tower](image-url)
3.3 Generation of Reachability Tree

The first step in the analysis of PNs is the generation of the reachability tree. This is a set of markings that are possible from the initial marking. The nodes of the reachability tree represent the markings of the net, the root representing the initial marking. The directed edge from one marking to another indicates the firing of the corresponding transition. The analysis of the reachability tree will generate a lot of information about the system and a close examination enables verification of PN as a valid representation of the system being modeled. Thus, it is used for checking whether the model is a good representation of the system. The reachability tree is generated as follows.

Beginning with the initial marking, transitions which are enabled by this marking are identified and new markings that result from the firing of each of the enabled transitions are generated. Each new marking is added to the tree and the directed edges from the markings are drawn. The algorithm for generating the reachability tree is given below. The set of reachable markings along with its arc sets and Reachability graph generated using the algorithm for the lube oil system are provided in Fig. 3 and Fig. 4. The entire algorithm is implemented in Excel and VBA.

\[
m = \text{total number of markings} \\
i = 1 \text{ and } m = 1 \\
\text{while } i \leq m \text{ do} \\
\quad \text{for } j = 1 \text{ to } t \text{ do} \\
\quad \quad \text{if } j \text{ is enabled by marking } i \text{ then} \\
\quad \quad \quad \text{generate new marking } M_{\text{temp}}(k) \text{ and} \\
\quad \quad \quad \text{for each } k, \text{ do} \\
\quad \quad \quad \quad \text{if } M_{\text{temp}}(k) \text{ is not already in the tree, then} \\
\quad \quad \quad \quad \quad \text{m = m + 1} \\
\quad \quad \quad \quad \quad M_w = M_{\text{temp}}(k) \\
\quad \quad \quad \quad \quad \text{edge } (M_i, M_w) = j \\
\quad \quad \quad \text{endif} \\
\quad \quad \text{endfor} \\
\quad \text{endif} \\
\text{endfor} \\
i = i + 1 \\
\text{endwhile}
\]
3.4 GSPN Simulation

At the beginning of the simulation run, the algorithm identifies all the enabled transitions from the initial marking. The firing time for each transition is determined by sampling from exponentially distributed firing intervals. The minimum firing time is selected and the corresponding transition is fired. The system moves to the next marking. The state of the system (good or complete failure) is ascertained. Failed subsystem, if any, will undergo repair. After repair the subsystem is as good as new. These events are simulated for 10 years.
The algorithm for the simulation is given below:

```
marking = initial marking
for j = 1 to t do
  firing_time(j) = -1
while (simulation run not ended) do
  for j = 1 to t do
    if transition j is enabled, then
      if firing_time(j) < 0 then
        generate firing_interval
        firing_time(j) = clock + firing_interval
      endif
    else (if not enabled)
      firing_time(j) = -1
    endif
  endfor
  find minimum firing_time(t)
  fire transition t
  reset firing_time(t) = -1
endwhile
```

In order to reduce variance, thetic and anti-theitic variation reduction technique was adopted. Each simulation run was carried out using thetic random numbers and anti-theitic random numbers. The average value is taken as the result of one simulation run. Like this 30 runs are carried out to get steady state results (Fig. 5).
4. Results and Discussion

Now the GSPN model shall be used to study the effects of the various component failure rates on the availability of the system. The system availability of the cooling tower was found to be 0.96713 in 10 years of its simulated life. It is estimated that 2 to 3 failures in a year which will call for repair actions. Frame works, nozzles and fills are the major reasons for unavailability. Planned maintenance and a close monitoring as well as maintenance actions are required to minimize these failures. The system availability graph and subsystem critically index graph are attached here with (Fig. 5 and Fig. 6).

![Cooling Tower Availability](image1)

**Figure 5:** Steady State Availability of Cooling Tower

![Failure Criticality Index](image2)

**Figure 6:** Failure Criticality Index of Components

The proposed GSPN model has been successfully used for the estimation of the availability of the system. Any changes in the system configuration such as redundancy or replacement of a component by a more reliable one can easily be incorporated into the model and their effects analyzed. It is also possible to analyze the system when different maintenance strategies and repair policies are adopted.
5. Qualitative Comparison of Various Modelling Methods for Availability Studies

A number of modeling approaches such as network, fault tree, Markov and Petri Nets have been developed for the computation of reliability characteristics of complex technical systems. These models are either structure-oriented or event-oriented. The structure-oriented models allow us to tackle structural failures that cause undesirable deviation from the expected performance. Network models are the best examples for this category. Event-oriented ones can, not only model hardware failures but also model undesirable situations that may develop due to error in software, operation or maintenance. The nature of the problem, the objectives and the size play a vital role in selecting a model.

This study has been devoted to the estimation of reliability / availability of complex systems. Model, suitable for real-world complex problems, have been proposed. Markov models are capable of including all the real-world complexities, but the state space explosion limits its usage. Petri Net, a mathematical modeling tool, is adequate for the development of methodologies for prediction and evaluation of RMA of the system. GSPNs are used to find the availability of the cooling tower system. This is an effective modeling tool which has immense potential for reliability studies. Using this, one can satisfy or at least try to satisfy all the reliability requirements.

6. Conclusion

The use of PNs for modeling complex systems for the purpose of availability assessment is demonstrated. The superiority of the GSPN over other approaches such as FTA and Markov models is brought out. The numerical estimates of the availability of the cooling tower system are obtained by simulating the GSPN. The proposed model can be conveniently used for modeling, analyzing and evaluating any complex stochastic systems.

References

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**G. Thangamani** is a faculty in Indian Institute of Management Kozhikode. He obtained his Ph.D. from IIT Madras and his Ph.D. thesis was entitled "Models for Assessment of Reliability Measures for Complex Systems". He earned his Bachelor’s degree in Mechanical Engineering with Honours for scholastics achievements and did his M.E. in Industrial Engineering from Anna University, Chennai. He started his career as a faculty in Pondicherry Engineering College, Pondicherry for 4 years. Later on, he was associated with companies such as Whirlpool and GE. His current research interests are in Complex System Reliability Prediction, Business Process Modeling, Analysis and Reengineering, Six Sigma, Lean approaches in manufacturing / service systems and product innovation and technology management. He has several publications in international journals and conferences.