A Product Quality and Process Feasibility Modeling System

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(Received on February 16, 2012, revised on August 28, 2012)

Abstract: A quality modeling system is described to support manufacturing process development and quality control. The system relies on the main effects relating the process variables to the quality attributes. The extensive simplex method is used to derive the feasible universe of process settings satisfying all specification limits. Three different types of feasibility are analytically derived including 1) the global feasible set that establishes the extreme limits of feasibility by allowing all the process variables to vary simultaneously within their allowable range, 2) the local feasibility, which shows the immediate feasibility for each process setting holding other process variables at their current setting, and 3) the controllability that is indicative of the range that may be obtained for each quality attribute while holding other quality attributes at their current value. The described system explicitly considers both modeling uncertainty and uncontrolled variation; the specification limits may be automatically tightened by the magnitude of the confidence intervals and process standard deviations to ensure a desired level of confidence and robustness. An example is provided for an injection molding process with four process variables and three quality attributes.

Keywords: Statistical process control, design for six sigma, quality function deployment

1. Introduction

Design and manufacture of physical goods remains the largest single economic activity of most countries. The product development process relies upon successive iterations of synthesis and analysis that eventually converge to a manufactured product with a desirable level of cost and quality. Many design methodologies and quality control approaches have been developed to assist in product development and manufacturing [1]. The objective of this article is to provide a product and process development system that supports feasibility evaluation under process variation and modeling uncertainty. After an overview of related research, the system is presented with theory, example, and discussion.

2. Related Research

Quality Function Deployment: QFD was put forth in 1966 by Akao and Oshiumi using a fishbone diagram [2]. For more complex products, the fishbone was combined into a matrix to address technical trade-offs in the quality characteristics by adding a “roof” to the top of the matrix, which was then termed the “House of Quality” in 1979. Quality Function Deployment (QFD) converts customer demands (WHATS) into quality characteristics (HOWs) and systematically develops a quality plan for the deployment of the finished product. In practice today, QFD is used to translate the voice of the customers (VOC) into a set of design elements that can be deployed vertically top-down through a four-phase process: Product Planning, Part Deployment, Process Planning, and Process Control [3].
The QFD method is closely related to the House of Quality (HOQ). The HOQ starts with a list of objectives, or the WHATs that have to be accomplished. By refining the list into the detailed HOWs, the customer requirements are translated into the product characteristics. The relationship between WHATs and HOWs is generally denoted by symbols: strong, medium, and weak. The correlation matrix is a triangular table establishing the correlation between each HOW item; positive correlation means that one HOW supports another HOW, while a negative value means that the HOW adversely affects another. Trade-off resolution is accomplished by adjusting the values of the how much. A competitive assessment analysis enables a comparison of different designs, which may uncover gaps in engineering judgment. While QFD does not specifically address the modern design considerations like cost, design for manufacture, safety, and ecology, it remains an important synthesis methodology due to its structured layout and open architecture. Many researchers have developed auxiliary and complementary methods to extend QFD as documented in an extensive review [4].

Statistical quality control: SQC originated from the application of scientific principles by Shewhart in the 1920s with subsequent widespread dissemination by Deming in the 1950s [5]. The fundamental philosophy is that the statistical variation of a process must be considered when designing products to ensure fitness for use, and when selecting processes and quality programs to ensure consistency. One of the most common quality control techniques currently used in manufacturing is the control chart. The control chart is a graphical means of monitoring a process in real time by tracking the mean through an X-bar chart, variance through an R chart, and other quality characteristics.

However, many quality attributes are not directly or easily measurable. For instance, the dimensional measurement of a molded part requires a prolonged cooling time for the part to obtain its nominal size. In this case, the cost of accurately measuring the product approaches or exceeds the cost of the product itself. Furthermore, the time delay associated with the measurement may preclude the use of the measurement for real-time control. For this reason, it is very common for control charts to be applied to processing parameters rather than quality attributes. Such usage requires the establishment and maintenance of appropriate control limits for the process variables. One common practice is the empirical characterization of absolute limits on individual processing parameters, which are then tightened to ensure process capability [6]. Unfortunately, such an approach frequently ignores the interaction between multiple process parameters and quality attributes and so may arbitrarily be over or under conservative. This issue is significantly compounded when multiple quality attributes must be maintained within their respective specification limits.

More recently, statistical analyses and multi-attribute utility theory have led to the development of quality loss functions for optimization across multiple response surfaces [7]. While these approaches are based on scientific principles, the development of appropriate functions for multi-objective optimization remains challenging. Furthermore, numerical optimization may yield an “optimal” point for the process without providing the process engineer the necessary insight to achieve a process that is most preferred.

**Extensive Simplex Method:** Assume a $1^{st}$-order or piecewise linear transfer function between the $m$ processing variables $x$ and the $n$ quality attributes $y$:

$$y = Ax + b$$

$$LCL \leq x \leq UCL$$

$$LSL \leq y \leq USL$$

(1)
where $LCL$ and $UCL$ are the allowable range of the processing variables and $LSL$ and $USL$ are the specification limits for the quality attributes. Given the function matrix, $A$, the feasible space of the system can be solved by the Extensive Simplex Method that efficiently traverses all extreme point of the feasible polyhedron[8]. In order to exhibit this highly dimensional feasible space, the polyhedron can be displayed as a set of two-dimensional projections as shown in Figure 1 for a model with three processing variables. These profile projections provide the extreme limits of system feasibility across all processing variables. In addition to these projections, section views are taken for three two-dimensional planes at the current control point as indicated by the intersection of the three centerlines. These projections provide an accessible view of the highly dimensional feasible universe, which dynamically changes with the current vector of process settings and system of transfer functions as the design and/or manufacturing process evolves.

![Figure 1: Projection and Sections of Feasible Space (Convex Hull)](image1)

![Figure 2: Product Quality and Process Feasibility Model](image2)

3. System Description

As an interactive system, the decision maker can view the entire representation in a perspective view as shown in Figure 2, or can rotate the view to various orthographic projections showing only the partial representation such as function matrix, process chart, quality chart, process window, or quality window. The core of this quality modeling system is the set of transfer functions relating the processing variables to the quality attributes. A set of graphs, referred to as the function matrix, graphically depicts these relationships which form the “center” of a “house.” In this “house”, there are two “walls”, the bottom edges of which are formed by the abscissa and ordinate axes corresponding to the process variables and quality attributes of the function matrix, respectively. The vertical edges of each “wall” correspond to elapsed time of the process and provide the means for providing historical charts for both the process variables and quality attributes. The “roofs” of the “house” are formed by two panels that include: 1) the process window with a set of graphs representing the correlations and feasible boundaries for multiple process variables, and 2) the quality window with a set of graphs representing the correlations and Pareto optimal sets for multiple quality attributes. The decision maker can readily observe the impact of changes to process variables on all quality attributes and
perform interactive optimization by adjustment of the control and specification limits. As the function models and parameters change across time, the described algorithms are constantly operating to maintain the fidelity of the representation.

**Function Matrix Development:** The function matrix, \( A \) in equation 1, may be derived from experimental methods (such as design of experiments and response surface methods), analytical models, or estimation based on experience.\(^1\) Response surface methods are advocated here since they are generally applicable, commonly practiced, yield validated models, and are also of a dimensionality that is compatible with the methods disclosed in this paper.\(^2\) In the event that the transfer functions are non-linear, then both global and local linear models are utilized in the developed system. Global linear models, while providing low fidelity of highly non-linear systems, are useful to identify the approximate feasible region of the decision making problem. Once the decision maker moves to a specific region of interest, local linear models are generated of higher fidelity with known confidence intervals.

**Specification Limits:** Denoting the \( i \)th quality attribute as \( y_i \), a typical specification can be expressed as \( LSL_i \leq y_i \leq USL_i \). Without loss of generality, a one-sided specification can be formed by substituting \(-\infty\) or \(+\infty\) to the unspecified limits. Suppose \( y_i = f_j(X) \), where \( X \) is the set of process variables, \( X = \{x_1, x_2, \ldots, x_j, \ldots, x_n\} \) and \( LCL_i \leq x_j \leq UCL_i \). The specification limits on the quality attributes should rationally drive the feasible range of the process variables. Similar to the method described by Roman et al. [10], the implemented approach tightens the specification limits on the quality attributes to compensate for random variation in the process variables as well as modeling uncertainty. For example, the lower specification limit may be adjusted for uncontrolled variation as:

\[
LSL_i^* = LSL_i + k \sigma_i
\]

where \( LSL_i^* \) is the original specification limit for the \( i \)th quality attribute, \( LSL_i^* \) is the active specification limit used for analysis, and \( k \) is the number of standard deviations by which the specification is tightened. For Six Sigma practice, \( k \) would be 6, which would imply three defects per million opportunities if the assumption of normal statistics is valid; it is suggested that the decision maker investigate different values of \( k \) from 0 to 9.

The variances on the quality attributes, \( \sigma_i \), are evaluated experimentally or estimated via the moment matching method\(^{(11)}\) from the variances on the process variables, \( \sigma_x \):

\[
\sigma_i = \sqrt{\sum_{j=1}^{n} \left( \frac{\partial f_j}{\partial x_j} \right)^2 \sigma_x^2}
\]

With regards to uncertainty, physical systems may exhibit complex behavior that is not known to a high degree of precision. High fidelity non-linear models can provide a reasonable estimate of the system behavior together with a characterization of the error. Since a linear function model is used to with the Extensive Simplex Method, the implemented approach further tightens the specification limits on the quality attributes to compensate for modeling uncertainty. The fitting statistics are utilized to generate

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\(^1\) The function matrix, \( A \), is sometimes modeled in a qualitative form such as a set of shapes such as \( \triangle \), \( \square \), and \( \Theta \) to indicate a weak, medium, or strong effect that the process settings have on the quality attributes. A system for derivation of specification limits for qualitative models is discussed elsewhere [9].

\(^2\) Systems with very high dimensionality, e.g., more than fifty variables, pose significant challenges as the solution of global feasibility is not a polynomial problem and requires extended computation times. The concept of partitioning is suggested whereby various process settings and/or quality attributes are locked at acceptable values and eliminated from the solution to reduce the dimensionality of the system.
confidence intervals on the predicted quality attributes. For the linear system of equation 1 derived from p experimental runs, the covariance matrix for a quality attribute is:

\[ C = s^2 (B' B)^{-1} \]  (4)

where \( B \) is the \( p \) by \((m+1)\) experimental design matrix, \( s^2 \) is the estimated residual variance, and \((B' B)^{-1}\) is the inverse of \(B' B\). The columns of \( B \) are indexed by \( j \) from 1 to \( m \). The first column consists of ones, while the remaining columns consist of the process parameters. Each row contains one experimental run, as indexed by \( h \) from 1 to \( p \). The variance on the estimate of the modeled quality attribute is:

\[ V(y) = \sum_{j=1}^{m} \sum_{h=1}^{p} x_h C_{ij} + 2 \sum_{j=1}^{m} \sum_{h=1}^{p} x_h C_{ij} \]  (5)

The specification limits for a quality attribute can be adjusted at varying 1-\( \alpha \) confidence levels using the student’s t distribution such as for the lower specification limit:

\[ LSL_i = LSL_i^* + (V(y))^\frac{1}{2} t(p - m - 1; 1 - \alpha / 2) \]  (6)

The effect of modeling uncertainty can be quite different than random variation. It is suggested that the decision maker investigate different values of \( \alpha \) from 0.5 to 0.001, and understand its effect on system behavior and the feasible ranges of the process variables. As the required level of robustness and confidence increase with \( k \) and 1-\( \alpha \) for a given application, the specification limits are significantly tightened for all quality attributes, sometimes to the point of infeasibility. In such cases, the decision maker may choose to loosen the requirements for \( k \) and \( \alpha \), adjust the performance specifications, or expand the system feasibility with the addition of other process variables into the system. As a default, the implemented system utilizes \( k \) equal to 3 and \( \alpha \) equal to 2%, which provides for approximately equal robustness to process variation and model uncertainty.

**Feasibility Evaluation:** The feasible set of the process variables is computed by the Extensive Simplex Method. This algorithm [8] provides the global feasible set including the convex hull of all feasible process settings and all achievable quality attributes. If there is no feasible set given the specification limits on the quality attributes and the allowable range of process variables, then the algorithm returns no solution. In such cases, it is recommended that the decision maker reduce the values of \( k \) and \( \alpha \), loosen the specification limits, broaden the allowable range of the process variables, and investigate additional process settings to increase the system controllability.

Three feasible sets are derived and presented in the system. The first set is the global feasibility, shown as the light grey regions in Figure 2. This set provides the extreme limits on the process variables and quality attributes if all process variables are changed simultaneously. The second set is the local feasibility, shown as the dark grey regions in Figure 2. This set represents the feasible range of each process setting that will deliver all quality attributes within specification when all other process variables are held at their current setting. The local feasibility is used by the decision maker to obtain a feasible solution. If all process settings are within their local feasible range, then the quality attributes are within the specification limits as adjusted for variation by \( k \) and uncertainty by \( \alpha \). The third set is the controllability, shown as the white regions in the quality chart and quality window of Figure 2. This set represents the achievable range of each quality attribute with all other quality attributes held at their current value, i.e., the controllability of the quality attributes. The controllability is an explicit and quantitative alternative to the correlation matrix in QFD [2] and coupling concepts in axiomatic design [12].
4. Example

Polymer processing is a challenging application for quality control due to the number and complex relations between the process settings and quality attributes. In this typical experimental study [13], a commercial polypropylene grade (MFI=3.5 g/10min) was injection molded in a rectangular cavity with dimensions 150mm in length, 80mm in width, and 3.2mm in thickness. The experimental trials were conducted with an 80 ton Arburg 270V injection molding machine.

In order to evaluate the influence of process conditions on part dimensions of the injection molded parts, four process variables were chosen: mold temperature \( (x_1) \), melt temperature \( (x_2) \), injection rate \( (x_3) \) and hold pressure \( (x_4) \). These variables have consistently been reported in the literature as the most important determinants for shrinkage. Other processing variables were kept constant for all the experimental conditions. The range of process settings was defined by the machine and material limits. The minimum and maximum melt temperatures were 200 and 260°C, respectively. The mold temperature limits were 28 and 90°C. The injection rate was adjusted between 10 and 110 cc/s; the minimum and maximum holding pressure were set at 20 and 60 MPa to avoid moldings with structural defects. A central composite design of experiments was implemented after which the dimensions in the length and width directions, \( y_1 \) and \( y_2 \), were evaluated by measurement of the molded part at room temperature.

Based on this data, response surfaces were derived to model the dependence of each of the part dimensions on the process conditions. An adaptive fitting algorithm was utilized to successively add the most significant term to the regression model. The algorithm provided a set of regression models with varying number of terms and corresponding \( R^2 \) and \( R^2_{adj} \) values. The model having the greatest \( R^2_{adj} \) was selected for continued analysis.

The models for length, width, and weight are:

For length: \( y_1 = 1.4465 + 0.1193 x_2 - 0.0182 x_4 + 0.0150 x_1^2 + 0.0081 x_3 x_4 + 0.0198 x_1 x_2 + 0.0088 x_3 x_4 \) \( R^2_{adj} = 0.99 \) (7)

For width: \( y_2 = 1.487 + 0.134 x_1 + 0.0218 x_4 - 0.0100 x_3 + 0.034 x_1^2 + 0.015 x_2^2 + 0.0111 x_1 x_2 + 0.0105 x_3 x_4 + 0.0055 x_3 x_4 \) \( R^2_{adj} = 0.94 \) (8)

For weight: \( y_3 = 32.197 - 0.205 x_1 + 0.002 x_3 + 0.017 x_4 + 0.462 x_1 x_2 + 0.081 x_1 x_3 - 0.024 x_1 x_4 + 0.012 x_1 x_3 + 0.091 x_1 x_3 - 0.102 x_1 x_3 - 0.020 x_1 x_3 - 0.025 x_1 x_3 - 0.006 x_3 x_4 + 0.005 x_3 x_4 \) \( R^2_{adj} = 0.98 \) (9)

Inspection of equations (7) to (9) indicates that the models for length and width have similar topology and parametric dependence on the process variables. The coefficients reveal that mold temperature, \( x_1 \), and pack pressure, \( x_4 \), are the most significant variables. These response surface models are internally consistent, and with an average \( R^2_{adj} \) of 0.95, are believed to be representative of the system behavior in this application.

The non-linear functions and the global linear functions are shown in Figure 3 with each of the process variables set at their mid-range value. It is observed that the global linear models provide a reasonable fit for many of the transfer functions (e.g., width as a function of mold temperature) but a poor fit for other transfer functions (e.g., part weight as a function of mold temperature). The allowable ranges for the process variables and the specification limits for the quality attributes are respectively shown in Figure 4 as the tick marks to the bottom and left of the function matrix. The current values of the design variables are shown by the vertical lines, resulting in the current values of the quality attributes as shown by the horizontal lines that intersect the transfer functions.

The global feasible set is plotted as the light shaded region in each graph, and indicates the ranges of the process variables and quality attributes that are feasible. As indicated, the
current set of process conditions falls outside the feasible region so this initial process is infeasible. However, since the feasible regions are known, it is a simple exercise to change the values of the processing variables to achieve a feasible process. In particular, Figure 3 indicates that the process can be made feasible by jointly reducing the hold pressure and the mold temperature.

The changes successively made to the process by the decision maker are tracked in Figure 4, in which point ① is the initial centered point corresponding to Figure 3. The whiskers on each point correspond to three standard deviations of the process. At point ②, the decision maker has reduced the mold temperature and hold pressure to obtain a feasible process, which is indicated by the vector of process settings being contained within the darker shaded region that represents the local feasible process. This darker region indicates the feasible range of each processing variable that will produce moldings that meet all specifications while holding other processing variables constant.

At point ③, the decision maker switches to a higher fidelity model of the process that has been locally linearized about the current processing conditions. At this point, there is a significant change in the modeled behavior of the system, and a discontinuity in the
feasible process is observed. The decision maker now views the updated feasible processing regions for the current processing conditions, and makes adjustments to the processing variables to simultaneously achieve process feasibility while maximizing process consistency. Point \( \bullet \) indicates the final set of processing conditions.

Figure 5 provides the function matrix corresponding to point \( \bullet \). Figure 5 differs from Figure 3 in several ways. First, a different set of processing conditions has been selected, resulting in a feasible process as indicated by the cross hairs contained within the darker shaded region that represents the local feasible process. Second, this function matrix utilizes local linearization about the current set of processing conditions as opposed to the global linear models initially used in Figure 3. It is observed that the linear model intersects the non-linear model at the operating point, but may significantly diverge at process conditions far from the operating point (e.g., length is poorly modeled with this local linear model at high mold temperatures). Third, the confidence intervals \( (\alpha=0.02) \) have been added as dashed lines; the behavior of the quality attributes falls outside the confidence intervals of the linear model only for part weight and length as a function of the mold temperature. Fourth, the variation intervals corresponding to three standard deviations have been added as dotted lines; a robust process has been achieved.

Figure 6 provides a chart plotting the expected values of the quality attributes with their confidence intervals (solid lines) and variation limits (dotted lines). The dark region corresponds to a feasible process in which the expected values of all quality attributes are within their corresponding specifications. The lighter region indicates the feasible range of each quality attribute while holding other quality attributes at their current value. For this application, the part weight may be changed substantially while holding other attributes constant. By comparison, the length and width may only be slightly adjusted.

Points \( \circ \) to \( \bullet \) of Figure 6 correspond to the previous discussion for the process chart provided in Figure 4. Initially, the part weight and length are outside of specification. At point \( \circ \), the decision maker has adjusted the process to bring the process within specification. At point \( \bullet \), the decision maker adopted the local model of the quadratic response surface while trying to select a robust process; the process is infeasible since the part weight is above specification and the width is below specification. However, the confidence and variation intervals have been reduced compared to the global linear models. At point \( \bullet \), the decision maker has selected a set of process conditions that results...
in a length of 1.41% with an asymmetric process capability index, $C_{pk}$, of 1.33, a width of 1.46% with a $C_{pk}$ of 4.4, and a part weight of 32.02 g with a $C_{pk}$ of 1.36.

The decision window for the set of processing conditions of point 1 is provided in Figure 7, which shows the feasible conditions for each pair of process variables. These process window graphs are of significant value to the quality or process engineer who wishes to understand the feasibility of the process. In this application, for instance, the hold pressure and melt temperature is of critical importance and has the most restricted feasibility.

Any operating point within the lighter shaded region can result in a nominally feasible process by changing multiple process variables. For instance, it may be desirable for economic or other quality requirements to increase the mold temperature and reduce the hold pressure as indicated by the arrow. While this change results in an unacceptable process, feasibility could be restored by changing the injection rate and/or melt temperature.

The quality window for the set of processing conditions of point 1 is provided in Figure 8, and provides the local and controllable feasible spaces for each pair of quality attributes. The darker region indicates the feasible set of each pair of quality attributes while holding other quality attributes within specification. The lighter region indicates the feasible set of each pair of quality attributes while holding other quality attributes at their current value. The dotted ellipse about each operating point represents three standard deviations of uncontrolled variation; the dashed ellipse represents the confidence region ($\alpha=0.02$) for the linear model for each quality attribute. The confidence intervals can be larger in size than the variation intervals. For a confident and robust process, both the variation and confidence intervals should be contained within the feasible region. While the selected process is largely adequate, it is observed that a potential issue is the uncertainty of the length attribute, which may result in production of molded parts that fall outside of specification due to lack of model fidelity.
5. Conclusions
The quality modeling system was motivated by QFD’s capability to contemplate the relationship between process settings and quality attributes through multiple stages of the product development process. Unlike QFD, however, the described system supports the development, use, and refinement of validated analytical models to dynamically and accurately identify the feasible range of process settings and quality attributes from dominating constraints. The ability to interact with the process settings while modifying constraints provides valuable insights into the nature of the system and assists the formation of an acceptable solution.

Acknowledgment: The Extensive Simplex Method was developed through research supported by National Science Foundation Grant # 9702797.

References

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