Process Monitoring and Feedforward Control for Proactive Quality Improvement

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Abstract: Process adjustment strategy is an important part of the process improvement methods, which is also called engineering process control (EPC), and it is often integrated with statistical process control (SPC) to improve the process control performance. While feedback control is used to compensate for the output deviation, feedforward control is a proactive control strategy based on a direct measurement of the disturbance, and it acts before the disturbance affects the system. Feedforward control is usually combined with feedback control for variation reduction. In this article, rationales for feedforward control are explained, and a new philosophy on its application is given. The feasibility condition for feedforward control application illustrated from a new disturbance decomposition viewpoint, and the validity of some disturbance models which work well for feedforward control is investigated. Some relevant issues on process monitoring, feedback control and feedforward control are discussed and addressed.

Keywords: Statistical process control, engineering process control, feedback control, feedforward control, mean square error, disturbance models

1. Introduction

“Control is a continuous endeavor to keep measures of quality as close as possible to their target values for indefinite periods of time”[1]. Process monitoring and process adjustments are two different techniques to achieve this goal, and they are part of statistical process control (SPC) and engineering process control (EPC) respectively. Shewhart charts, cumulative sum (CUSUM) charts and exponentially weighted moving average (EWMA) charts are frequently employed in SPC to find the special causes which should be further removed or eliminated. Unfortunately, sometimes the special causes are known but cannot be economically removed, so compensation must be made to prevent a process from wandering off the target, calling for feedback control and feedforward control in EPC. SPC has historical foundation in parts industry and EPC has originated in process industry. SPC and EPC were applied by people with different technical backgrounds for different control objectives. Box and Kramer [2] discussed the interface between SPC and EPC, and how they can be integrated efficiently.

While feedback control is a reactive process adjustment strategy based on the process output error, feedforward control is based on a direct measurement of the disturbance and it acts before the disturbance affects the system. Prevention is better than reactive correction, especially for the process with a large inertia. Feedforward control has wide applications in chemical, mechanical and management industries [3]. While feedback control has been extensively investigated [1-2], [4], the existing research on feedforward control from a statistical perspective is quite limited, thus further investigations on feedforward control are necessary and worthwhile.
The rest of this article is organized as follows. Section 2 illustrates a new perspective on feedforward control application based on disturbance decomposition, followed by two real life examples. Section 3 presents some basic disturbance models in SPC and EPC literature, and proposes some disturbance models that can be adjusted by feedforward control. In Section 4 we provide the feedback and feedforward control equations for these disturbance models, and some numerical results for a random step change disturbance model. Some discussions on relevant issues with process monitoring, feedback control and feedforward control are provided in Section 5, which shed light on these important questions. Concluding remarks are given in Section 6.

2. A New Perspective on Feedforward Control

Here we explain the basic ideas of feedback control and feedforward control using a black box in our control system, and illustrate feedforward control feasibility conditions from a new perspective, called disturbance decomposition.

![Figure 1: Disturbance Decomposition for Feedforward Control Loop](image)

The factors in a feedback control system can be categorized into three groups: control factor \( X \) whose levels can be adjusted, noise factor \( e \) which is the source of variation, and an output variable \( Y \), with transfer function \( Y = f(X, e) \), as illustrated by Figure 1(a). We interpret the feasibility condition for feedforward control from a disturbance decomposition viewpoint: if noise \( e \) can be further decomposed into a new observable but uncontrollable factor \( B \) and remaining noise \( e' \), with a new transfer function \( Y = f'(X, B, e') \), then feedforward control can be applied by adjusting \( X \) based on \( B \) to make \( Y \) closer to the target value, illustrated by a feedforward control loop on \( B \) in Figure 1(b). We call \( B \) informative disturbance variable and \( e' \) non-informative noise or background disturbance. Since unmeasured background disturbances are always present in any control system, feedforward control is usually combined with feedback control to achieve the highest possible efficiency. How effective this feedforward control can be depends on the magnitude of the informative disturbance variable \( B \) that can be taken out from the noise \( e \), and we will explain this later in detail.

We first discuss some real examples with feedforward control from our disturbance decomposition viewpoint. In a water heater example where steam is used in the heat exchanger to heat the incoming cold water to maintain some target temperature of the hot water. Instead of only using the thermostat on the hot water outlet for feedback to adjust the amount of steam, a skillful operator could use a simple feedforward strategy to maintain the hot water temperature on target. The worker would compensate for changes
in inlet cold water temperature by monitoring it and in response to that, increasing or
decreasing the steam rate to counteract the change in the temperature of the incoming hot
water. Here the cold water temperature is our informative disturbance variable $B$ and it is
very easy to observe and measure.

Another example is the cruise control which enables a car to maintain a steady road
speed. When an uphill stretch of road is encountered, the car slows down below the set
speed; this speed error causes the engine throttle to be opened further, bringing the car
back to its original speed with feedback control. Feedforward control can be applied by
measuring the road slope, upon encountering a hill, then it would open up the throttle by a
certain amount automatically and anticipate the extra load. The car does not have to slow
down at all for the correction to come into play. Here the road slope is the informative
disturbance variable $B$.

3. Disturbance Models

The main objective of SPC provides an ongoing check on the stability of a process by
looking for the assignable cause which causes departure or deviation from the “in control”
process. In EPC, it is always desirable to adjust a process from time to time such that the
process output is as close to the target value as possible, and to achieve the minimum
mean square error (MMSE). The development and implementation of any control, either
for process monitoring or adjustment, requires a reasonably realistic representation of
disturbance model $t$, i.e., the output deviation from target that would occur if no
adjustment action were taken.

3.1 Basic Stationary and Non stationary Disturbance Models

In SPC, the simplest and most familiar disturbance model for a process in a state of
control is the white noise series,

$$z_t = a_t,$$  \hspace{1cm} (1)

where $\{a_t\}_{t=1}^{\infty}$ is a sequence of iid random variables that are normally distributed with
mean 0 and variance $\sigma^2$. The well known Deming’s funnel experiment has the similar
assumption on the disturbance model that is has a fixed mean and does not drift away by
itself. Then the best control strategy for this process is just to “leave it alone” without any
adjustment, since any control action will “temper” the process and increase the variability.

It is known that in process industries under EPC scope, a more general situation for
the disturbance model is the stationary auto-correlated process, such as (autoregressive) AR (1) model:

$$z_t - \phi z_{t-1} = a_t, \quad |\phi| < 1$$  \hspace{1cm} (2)

and (autoregressive moving average) ARMA (1,1) model:

$$z_t - \phi z_{t-1} = a_t - \theta a_{t-1}, \quad |\phi| < 1, \quad |\theta| < 1$$  \hspace{1cm} (3)

MacGregor [5] investigated an interesting modified Deming’s funnel experiment in which
the process mean follows an autocorrelated AR (1) model, and showed that it calls for an
active adjustment rule instead of the no control strategy for variation reduction.

However, the assumption of stationarity might be questionable for disturbances modeling
in many realistic control problems. As Deming [9] says, “no process, except in artificial
demonstrations by use of random numbers, is steady and unswerving”, implies that the
nonstationary disturbance model is more realistic in EPC. Compared with the stationary process, the nonstationary process does not have a fixed mean. The integrated moving average (IMA) model is the simplest, most common and appropriate nonstationary disturbance model [1-2], [6], [10]:

$$z_t - z_{t-1} = a_t - \theta a_{t-1}, \quad |\theta| < 1$$

In recent decades, with the fast development of some traditional and modern high tech industries, such as manufacturing, semiconductor, computer and software industries, the processes in these fields are becoming more and more complicated and are hybrids in nature. The disturbance models of these processes are often approximately additive models of a certain background disturbance [11], such as the IMA, AR(1), or ARMA(1,1) process, with some additional part added together, like a spike, a sustained mean shift, a ramp, an exponential rise to new levels, etc. [1], [6], [12-13].

3.2 Periodic Shift Disturbance Models

Periodic cycles very commonly exist in some high-value discrete-part manufacturing processes. Motivated by a feedstock change problem discussed by Box and Luceno [14], Shi and Kapur [15] investigated periodic variance shifts disturbance models, with IMA, AR (1) and ARMA(1,1) processes as background disturbance respectively:

$$z_t - z_{t-1} = a_t - \theta a_{t-1} + \delta_t, \quad |\theta| < 1$$

$$z_t - \phi z_{t-1} = a_t + \delta_t, \quad 0 < \phi < 1$$

$$z_t - \phi z_{t-1} = a_t - \theta a_{t-1} + \delta_t, \quad 0 < \phi < 1, \quad 0 < |\theta| < 1$$

where $\delta_t$ is the shift that occurs periodically at $t = T, 2T, 3T, ...$ and $m_t = \delta_t + \epsilon_t$ is the one-step-ahead estimator of $\delta_t$ at time $t - 1$ with error $\epsilon_t$. Assume

$$E(\delta_t) = E(\epsilon_t) = 0$$

$$Var(\delta_t) = \sigma_\delta^2, \quad Var(\epsilon_t) = \sigma_\epsilon^2$$

Usually we are concerned with the variance inflation $\sigma_\delta^2 > \sigma_\epsilon^2$, with a small estimation error $\sigma_\epsilon^2 < \sigma_\delta^2$. We call them model (1), model (2) and model (3) respectively.

This periodic shift disturbance model fits well with a realistic manufacturing scenario in which a production line works for producing very thin metallic films. In this process, feedstock material is frequently fed into the production line to produce the metallic films. After a certain number of metallic films are produced, a lot is formed and then a new lot is begins to form by subsequent metallic films. For such process, both within-lots variability and between-lots variably exist. Within each lot, the feedstock material is relatively homogeneous and a lack of uniformity appears between different lots, which affect the thickness of the producing metallic films. The within-lot variability can be modeled by an IMA, or AR(1), or ARMA(1,1) process, and as soon as enough batches of metallic film was produced, reliable parameter estimates can be made.

3.3 Random Step Change Disturbance Models

Process monitoring of sustained step shift disturbance models has been discussed in some SPC literature. Vander Wiel [6] investigated monitoring an IMA process with a sustained
level shift, \( z_t - z_{t-1} = a_t - \theta a_{t-1} + D_t \), with \( D_t = 0, 0 \leq t < t_i \) and \( D_t = s, t_i \geq t \), where \( t_i \) is the unknown change-point. Nembhard and Valverde-Ventura [16] proposed the cumulative score (Cuscore) statistics \( Q_t = Q_{t-1} + s \theta^{-1} e_t \) in their Cuscore control chart to detect a sustained step shift in the IMA disturbance.

Other researchers focused on the more challenging disturbance models subject to random step changes instead of the sustained shift, which are usually caused by variations in the physical conditions, such as the environmental temperature and raw material qualities. Chen and Elsayed [17] studied using an EWMA estimator to monitor the i.i.d normally distributed process with random step changes. In their model, the random step-change occurrence has a constant probability \( p \) which is independent of the prior history of the process, with size \( r = \tau / \sigma \), where \( \sigma \) is the standard deviation of the background normal process, and \( \tau \) is the standard deviation of the process mean. They proposed an EWMA estimator with closed-form expression for the optimal value of the weighted variable \( \lambda \), as a function of the estimates \( \hat{p} \) and \( \hat{r} \) derived from historical data.

Tsiamyrtzis and Hawkins [18] investigated the process monitoring of a mean drift model of AR (1) process subject to random step changes, in a Bayesian framework. They supposed that the process mean has a jump of size \( \delta \) that occurs with probability \( p \), and assumed that the prior information about the process mean is available. Then at each time when the new data comes, they get the posterior distribution for the process mean through Bayes theorem to check if the mean has drifted or not. If there is no significant change, they use this posterior as the prior for the next stage. Their model is suitable for some practical problems, for example, tool wear problems in which the wear incorporates a random step change (due, e.g., to tool chipping) as well as drift. They also generalized the model with assigning a prior distribution to the size of the jumps \( \delta \), with AR(1) process subject to random step changes [13].

Since “a step change is more difficult to see when buried in an IMA than when buried in iid noise” [6] or a stationary background noise, we further generalize the background disturbance to be the nonstationary IMA model subject to random step changes,

\[
\begin{align*}
    z_t - z_{t-1} &= a_t - \theta a_{t-1} + D_t, \\
    D_t &\sim \begin{cases} 
    N(0, \sigma^2_a) & \text{with probability } 1 - p \\
    N(\delta, \sigma^2_\delta) & \text{with probability } p 
    \end{cases}
\end{align*}
\]

(7)

and \( \sigma_a \) is the standard deviation of the white noise \( \{a_t\}_{t \geq 1} \). Since we do not know the actual size of the step shift \( \delta \), we assume it is random with a certain prior distribution:

\[
\pi(\delta) \sim N(\mu_\delta, \sigma^2_\delta)
\]

(8)

where \( \mu_\delta \) and \( \sigma^2_\delta \) are the expected mean and variation of the step shift size \( \delta \) respectively. Both \( \mu_\delta \) and \( \sigma^2_\delta \) are assumed to be known, considering that they can be determined from previous engineering knowledge or experiences about the process. Small values of \( \sigma^2_\delta \) lead to informative settings while large values indicate a priori ignorance for the size of the step shift. The size of the step shift can be modeled by \( \delta \) and the uncertainty of the
step shift can be modeled by the standard deviation ratio \( r = \frac{\sigma_\delta}{\sigma_a} \).

Figure 2: IMA Disturbance Without and With Random Step Change

Figure 2(a) shows an IMA disturbance process of 200 data with \( \theta = 0.8 \), and Figure 2(b) shows the same IMA disturbance subject to random step change with occurrence probability \( p = 0.02 \). The shift size \( \delta \) has the prior distribution \( \pi(\delta) \sim N(\mu_\delta, \sigma_\delta^2) \) with \( \mu_\delta = 2 \) and \( \sigma_\delta = 3 \), so the standard deviation ratio \( r = 3 \).

There are 4 random step changes: a 5.25 upward step change in the 92th value of the IMA disturbance \( z_t \), a 2.08 downward step change in the 101th value, a 3.72 downward step change in the 150th value, and a 3.49 upward step change in the 184th value.

4. Control Equations and MSEO Results

4.1 Dynamic Model for Process Inertia

Assume \( X_t \) is the level of adjustment (input) variable, \( g \) is the process gain, i.e., the eventual change in the output \( y_t \) that is induced by a unit adjustment at \( X_t \), then it may take some time before the full effect of adjustment is experienced at the process output, which can be modeled by a first-order dynamic equation:

\[
Y_t = \delta Y_{t-1} + g(1-\delta)X_{t-1} + C
\]

where \( C \) is a constant, \( 0 \leq \delta < 1 \). For every unit change in the input \( X \), the proportion of the total change that occurs in the first time interval is \( 1-\delta \). The larger \( \delta \) corresponds to a slowly responding system with greater inertia. We mentioned earlier that feedback control tends to have low efficiency when a process has a great inertia, resulting in a large delay between the adjustment on the input and its effect experienced on the output.

When \( \delta = 0 \), the full effect of an adjustment made at time \( t \) is realized at the output \( y_t \) within the next unit interval, then the dynamic model becomes \( Y_t = gX_{t-1} \) + constant. This is called the pure-gain model (also the responsive system), and we make the simplified assumption that the system is responsive throughout this paper.

4.2 Control Equations for Periodic Shift Models

Suppose \( x_t = X_t - X_{t-1} \) is the adjustment made at time \( t \), \( e_t \) is the output error under control. For a responsive system, the feedback control equation for an IMA process with
\[ \theta \] is

\[ g_{x_t} = -G e_t \]  \hspace{1cm} (10)

where \( G \) (0 < \( G \) ≤ 1) is the damping factor. When \( G = 1 - \theta \), it provides the minimum mean square error (MMSE) control [1]. Suppose

\[ \bar{z}_t = (1 - \theta)(z_t + \theta z_{t-1} + \theta^2 z_{t-2} + \ldots) = \theta \bar{z}_{t-1} + (1 - \theta) z_t \]  \hspace{1cm} (11)

is the one-step-ahead EWMA forecast with a smoothing constant \( \theta \). 0 ≤ \( \theta \) < 1, then its MMSE control \( g_{x_t} = -G e_t \) with \( G = 1 - \theta \) can be written as \( gX_t = -\bar{z}_t \) [19], and the output error under feedback control is just the forecast error of EWMA forecast for the IMA disturbance model. The feedback control equations for the AR(1) and ARMA(1,1) processes are given by Box and Lumeno [1], and Tsung et al. [8], respectively.

Compared with feedback control, feedforward control is applied to compensate the shift \( \delta_t \) estimated at time \( t - 1 \), and it can be written as

\[ g_{x_{t-1}} = -l(m_t, m_{t-1}, \ldots) \]  \hspace{1cm} (12)

where \( l \) is a suitable model-based function of the shifts estimated at current and previous times. The feedforward control equations for model (1), models (2) and (3) are proposed by Box and Lumeno [14], Shi and Kapur [15], respectively, see Table 1. Here \( B \) is the backshift operator such that \( BX_t = X_{t-1} \). \( z_t \) is an EWMA forecast with smoothing constant \( \theta \), \( \hat{e}_{t-1} \) and \( \hat{m}_{t-1} \) are EWMA forecasts with smoothing constant \( \phi \).

<table>
<thead>
<tr>
<th>Model (1)</th>
<th>Feedback control equation</th>
<th>Feedforward control equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_{x_{t-1}} = -m_t ) ( t = T, 2T, 3T, \ldots )</td>
<td>( g_{x_t} = -G e_t ) ( \theta ), where ( G = 1 - \theta )</td>
<td>( gX_t = gX_0 - G \sum_{i=1}^{t} e_i )</td>
</tr>
<tr>
<td>Model (2)</td>
<td>( g_{x_{t-1}} = -(m_t - \hat{m}_{t-1}) ) ( t = 1, 2, 3, \ldots )</td>
<td>( g_{x_t} = -\phi(e_t - \hat{e}_{t-1}) )</td>
</tr>
<tr>
<td>Model (3)</td>
<td>( g_{x_{t-1}} = -(m_t - \hat{m}_{t-1}) ) ( t = 1, 2, 3, \ldots )</td>
<td>( g_{x_t} = -\phi - \theta(e_t - \hat{e}_{t-1}) )</td>
</tr>
</tbody>
</table>

The output mean square error (MSEO) is defined by

\[ MSEO = \lim_{t \to \infty} \frac{1}{t} \sum_{i=1}^{t} \text{var}(e_i) \]  \hspace{1cm} (13)

With the control equations in Table 1, the closed-form MSEO formulae for models (1), (2), (3) under feedback control and combined control (feedback and feedforward control) strategies, can be derived in strict mathematical proofs, and the MSEO table can be found in Shi and Kapur [15]. The derivation processes for the feedforward control equation and
MSEO formulae are nontrivial, and they are mainly determined by entertaining with the output error relationship, see Shi and Kapur [15] for details.

4.3 Control Equations for Random Step Change Models

For model (4), we introduce a process adjustment procedure based on feedback control and an added adjustment based on output errors monitoring. We propose feedback control $g_{x_t} = -Ge_t$, plus adjustments as soon as a possible random step change is detected, i.e., when an output error $e_t$ falls outside of the $3\sigma$ limits, with control equation

$$
g_{x_t} = \begin{cases} 
-(e_t - 3\sigma_a), & \text{if } e_t > 3\sigma_a \\
-(e_t + 3\sigma_a), & \text{if } e_t \leq 3\sigma_a 
\end{cases}
$$

This added adjustment is applied to compensate the possible random step change, and can be considered as quasi-feedforward control, and the overall combined control was called as quasi-feedback feedforward control. The rationale for this quasi-feedforward control is due to the reason that the feedback control equation is only for the IMA disturbance, and it is not sufficient when other disturbances are present (random step change here). The amount of the quasi-feedforward control is chosen to be the distance between output error $e_t$ and the $3\sigma$ limit on the same side of the control chart.

It is called quasi-feedback control, since it is based on neither pure output error, nor the direct measure of the random step change; instead, it is based on the outliers in sequence $\{e_t\}_{t=1}^T$ that are outside of the $3\sigma$ limits under feedback control, and these outliers call for further remedial actions. Thus such added adjustment goes beyond feedback control and does not meet the requirement for feedforward control, so in some sense, it is in between the traditional feedback and feedforward control categories, which justifies such a name. Box and Kramer [2] discussed a similar process to feedback control to show that the feedback control does not necessarily conceal the nature of the disturbance that is being compensated. Here we go a further step beyond them by applying an added adjustment to continue improving the process.

4.4 MSEO Results for Random Step Change Models

Now we present some numerical results on the MSEO under combined control (quasi-feedback feedforward control) in our random step change model (4). Most industrial time series of IMA disturbances have $0.6 \leq \theta \leq 0.8$ [1], and we choose $\theta = 0.8$ and $\sigma_a = 1$ in model (4). We consider two occurrence probabilities $p = 0.02$ and $p = 0.05$ for the random step change, and choose some different combinations for the prior distribution of the step change $\pi(\delta) \sim N(\mu_\delta, \sigma_\delta^2)$. We investigate the expected size of the step shift $\mu_\delta$ from -0.3 to 0.3, for both upward change and downward change, and the uncertainty of the step shift $r$ from 1 to 3. Due to symmetry, we only need to include $0 \leq \mu_\delta \leq 3$ in our numerical results summary.

We choose the sample size $n = 200$ in each simulated disturbance process and make 10,000 iterations for each simulated disturbance process in our simulation. We apply two control methods: feedback control and combined control, for the same disturbance process data, and compare their MSEO results. In our numerical results
summary, we show both the MSEO under combined control and the improvement percentage for the quasi-feedforward control, which is calculated by

$$\frac{MSEO_{fb} - MSEO_{comb}}{MSEO_{fb}}$$ (15)

where $MSEO_{fb}$ is the MSEO under feedback control and $MSEO_{comb}$ is the MSEO under combined control.

Table 2: MSEOs under combined control for different $p, \mu_\delta$ and $\sigma_\delta$ in model (4) ($\theta = 0.8$)

<table>
<thead>
<tr>
<th>$\mu_\delta$</th>
<th>$\sigma_\delta$</th>
<th>p = 0.02</th>
<th>p = 0.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3</td>
<td>1.51 (20.6%)</td>
<td>2.30 (36.7%)</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>1.42 (16.1%)</td>
<td>2.14 (32.0%)</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1.40 (11.5%)</td>
<td>2.06 (25.2%)</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1.40 (15.5%)</td>
<td>2.01 (29.2%)</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1.30 (9.8%)</td>
<td>1.78 (21.5%)</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1.23 (4.6%)</td>
<td>1.63 (12.2%)</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>1.32 (11.7%)</td>
<td>1.81 (22.0%)</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1.20 (5.2%)</td>
<td>1.50 (12.0%)</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1.10 (1.0%)</td>
<td>1.27 (3.3%)</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>1.30 (10.2%)</td>
<td>1.73 (19.3%)</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>1.16 (3.7%)</td>
<td>1.42 (7.8%)</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1.05 (0.2%)</td>
<td>1.13 (0.6%)</td>
</tr>
</tbody>
</table>

The numerical results are shown in Table 2, notice that the improvement percentage is given in the parenthesis followed by the MSEO under combined control. Since there are three parameters $\mu_\delta$, $\sigma_\delta$ and $p$, we will fix each two out of the three parameter values, and compare the MSEO and the improvement percentage for different values of the other parameter.

Some useful conclusions can be drawn from Table 2 are:

1. For the disturbance model with the same prior distribution of the random step change $N(\mu_\delta, \sigma_\delta^2)$, the higher occurrence probability $p$ will achieve larger improvement percentage for the quasi-feedforward control. However, the disturbance with a larger $p$ will result in a larger MSEO under combined control.

2. For the disturbance model with the same expected size of the random step change $\mu_\delta$ and the occurrence probability $p$, the larger uncertainty $r$ (equivalently $\sigma_\delta$) will achieve larger improvement percentage for the quasi-feedforward control. However, the disturbance model with a larger $r$ will result in a larger MSEO under combined control.

3. For the disturbance model with the same uncertainty of the random step change $r$ (equivalently $\sigma_\delta$) and the occurrence probability $p$, the larger expected random step change size $\mu_\delta$ will achieve larger improvement percentage for the quasi-feedforward control. However, the disturbance with a larger $\mu_\delta$ will result in a larger MSEO under combined control.
All conclusions are consistent with our intuition: the larger the $\mu \gamma \delta \mu r \delta p$, i.e., the more information on the random step change, which is the informative disturbance variable, can be decomposed from the total disturbance, the more improvement can be achieved from feedforward control application. However, the larger and more frequent random step change always potentially increases the process variability, even after adjusted by feedback and feedforward control, resulting in a larger MSEO. This explains the reason for the interesting point that a larger improvement percentage always corresponds to a larger MSEO under combined control.

5. Some Comments on Process Monitoring and Feedback, Feedforward Control

There are some relevant issues on SPC and EPC that might cause misunderstanding, confusion and controversies from researchers and practitioners. Now we discuss these topics in greater detail and further illustrate the rationales behind our new perspectives on the process monitoring, feedback and feedforward control framework.

5.1 When SPC is not Enough

As we know, SPC provides an ongoing check on the stability of a process by using control charts to identify variation which are due to special causes. However, the process stability is not the only thing we need to care for, since a stable process might not be a capable process, for example, if the process mean is not on target, or if the process variation is too large. Under such cases, we need to change the process so it can meet the customer requirements, instead of maintaining the stability of this noncapable process. This calls for process adjustment strategies like feedback control and feedforward control to make the process on target and reduce variation.

5.2 When Feedforward Control Application is Possible?

Koontz and Bradspies [3] claimed that “even the most enthusiastic proponents of feedforward control admit that, if input variables are not known or unmeasurable, the system will not work.” Notice that their input variables just correspond to our informative disturbance variable $B$. However, the ideal assumption of absolute certainty on the informative disturbance variables have practical limitations, and sometimes parameter uncertainty should be treated as additional source of variability, possibly due to a poor understanding of the process behavior.

From our disturbance decomposition viewpoint, the above statement of Koontz and Bradspies [3] can be relaxed in some sense: as long as we have some knowledge on the informative disturbance variable $B$, either complete or partial knowledge, feedforward control or at least quasi-feedforward control is possible. For example, in model (4), without knowing the exact size of the random step change each time, we made the realistic assumption that with some empirical knowledge on the random step change, its occurrence probability $p$ is known, and its prior distribution is normally distributed with known mean and variance, following the assumption of Tsiamyrtzis and Hawkins [13].

5.3 Disturbance Nature and Change-point Identification in EPC

Box and Kramer [2] argued that the feedback control does not necessarily conceal the nature of the disturbance. However, sometimes feedback control does conceal it. This is due to the objective of EPC, and we will explain it as follows.

In SPC, when we monitor a stationary or nonstationary process, we can either
monitor the forecast errors through fitting a time series model, or monitor the original observations (disturbance) directly. For different disturbance models, no method is uniformly better than the other. When there is no advantage to using forecast errors, directly monitoring the raw data is much more convenient. However, when we need to adjust such a process in EPC, usually we cannot monitor a process without making any adjustment and wait until a signal appears; instead, we need to adjust the process from the beginning (usually by feedback control) and try to make it on target with minimum variation as much as possible, so we can only observe the process under control and the disturbance cannot be directly seen. When the size of the step change is small, or when there is only one change-point, the signal is more likely to be hidden in the process after adjustment. Therefore sometimes the nature of the disturbance might be concealed by feedback control.

Fortunately, process adjustment usually does not require the identification of the change-point, for example, the EWMA forecast (feedback control) filters out the noise and gives a clear picture of how the true mean level varies [17]. In other words, as long as the process under control has small variation (i.e., MSEO), then it is a good control, and usually we do not need to worry about how the process looks like without control. However, in some particular circumstances we can benefit substantially from the change-point identification in EPC, and this leads us to the next discussion topic.

5.4 The Role of Process Monitoring in the Integration of SPC and EPC

Process adjustment itself usually is insufficient if we have high uncertainty on the disturbance model, for example, when the disturbance has multiple change-points, the size of the step change is unknown or under high uncertainty. In such circumstances, process adjustment needs to be combined with process monitoring to achieve higher efficiency.

In model (4), we only know the occurrence probability of the random step change and its prior distribution, without knowing the exact change-points locations. Control chart for process monitoring can be used as a supplemental tool for locating the possible change-point, serving for the further process improvement function. This idea of "process monitoring serves for better process adjustment" was advocated by Tucker [20], who argued that "the development of a proper monitoring function in the presence of feedforward and/or feedback control is (should be) a central research issue in the quest for continuous quality improvement".

Since in SPC, false alarms and failure of detection always exist due to the type I and type II errors, so similar problems exist in the integration of SPC and EPC as well. Intuitively, with larger $p \delta \mu \sigma$ and $r$ in our model (4), our quasi-feedback feedforward control tends to have potentially higher efficiency.

5.4 Disturbance Decomposition and Cuscore Charts

While this disturbance decomposition viewpoint is relatively new in EPC for feedforward control application, similar ideas can be found on some disturbance models in SPC. Sometimes people have experience on how the process will change so that the signal patterns are anticipated, and Cuscore chart is devised to detect such anticipated systematic signals hidden in certain noise for process monitoring function. Box and Luceno [1] discussed a variety of different signal and noise combinations for Cuscore chart monitoring. Very naturally, the signal and the noise monitored by Cuscore chart in SPC just parallel the informative disturbance variable $B$, and the background disturbance $e'$ in EPC from our disturbance decomposition viewpoint.
5.5 Disturbance Decomposition and Robust Parameter Design

One of our motivations on disturbance decomposition viewpoint in EPC comes from robust parameter design. Now we explain how the rationale behind our feedforward control idea can be unified in both robust parameter design and EPC framework.

It is well known that robust parameter design can be successful only if the control factors $X$ are interacting with the noise factors $e$, which is often called “signal and noise interaction”. This implies that after disturbance decomposition, the control factors $X$ should interact with the informative disturbance variable $B$ as well as the noninformative noise $e'$, while $B$ and $e'$ are independent of each other. It can be proved that these conditions are satisfied in our disturbance decomposition model as well. We only investigate model (1) here, since model (2) and model (3) can be similarly justified.

We show that $X$ interacts with $\delta$ and the IMA noise. Since feedforward control is applied to compensate the effect of $\delta$, so different levels of $\delta$ call for different adjustment amount $x_i$ (equivalently $X_i$), with control equation $gx_i = -m_i = -(\delta_i + e_i)$, thus $X_i$ definitely interacts with $\delta_i$. Similarly feedback control is intended to compensate the IMA process, with control equation $gX_i = -\bar{z}_i$, where the EWMA forecast $\bar{z}_i$ is a function of the current and all the previous $z_i$'s, so $x_i$ (equivalently $X_i$) and $z_i$ are interacting to each other as well. Finally, since model (1) is an additive model, so the decomposed informative disturbance variable $\delta_i$ and the IMA noninformative noise are independent of each other.

6. Conclusions

In this paper, proactive feedforward control is recommended to be combined with the traditional reactive feedback control for process improvement, and the feasibility condition of feedforward control is illustrated from a disturbance decomposition viewpoint. Some disturbance models, including the periodic shift models and random step change models are discussed, and their feedback and feedforward control equations are proposed. Some issues on process monitoring, feedback control and feedforward control are discussed, which provide some insights on the necessity of feedforward control, the relationship between process monitoring and process adjustment are investigated, and the rationale for our disturbance decomposition viewpoint is further illustrated.

References


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