Wavelet Analysis based Gear Shaft Fault Detection

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Abstract: Fault detection and diagnosis of gear transmission systems have attracted considerable attention in recent years, but there are very few papers dealing with the early detection of shaft cracks. In this paper, an approach to gear shaft fault detection based on the application of the wavelet transform to both the time synchronously averaged (TSA) signal and residual signal is presented. The autocovariance of maximal energy coefficients based on the wavelet transform is first proposed to evaluate the gear shaft fault advancement quantitatively. For a comparison, the advantages and disadvantages of some approaches such as using standard deviation, kurtosis and the application of the Kolmogorov-Smirnov test (K-S test), used as fault indicators with continuous wavelet transform (CWT) and discrete wavelet transform (DWT) for residual signal, are discussed. It is demonstrated using real vibration data that the early faults in gear shafts can be detected and identified successfully using wavelet transforms combined with the approaches mentioned above.

Keywords: Gear shaft fault detection, residual signal, wavelet transform, K-S test, standard deviation, autocovariance of maximal energy coefficients.

1. Introduction

There has been extensive research on the vibration behavior of cracked shafts and crack identification in rotating shafts [1-4]. However, all the papers have focused on the crack identification in a non-gear shaft, specifically in a rotor shaft. As summarized by Hamidi et al. [5], several publications have proposed a number of techniques such as the use of natural frequencies, mode shapes and frequency response functions for damage detection of rotor shafts. An autoregressive model-based technique to detect the occurrence and advancement of gear shaft cracks is proposed by Wang and Makis [6].

Recently, wavelet transform (WT), which is capable of providing both the time-domain and frequency-domain information simultaneously, has been successfully used in non-stationary vibration signal processing and fault diagnosis [7-10]. Only wavelet approaches are sometimes inefficient for picking up the fault signal characteristic under the presence of strong noise. In this paper, the autocovariance of maximal energy coefficients combined with wavelet transform approaches is firstly proposed for gear shaft crack detection. The results reveal that the method can enhance the capability of feature extraction and fault diagnosis for gear shaft. Residual signal is used as the source signal, and some wavelet transform approaches such as CWT and DWT are considered. Measures such as standard deviation, kurtosis and the K-S test are used as fault indicators.

The remainder of the paper is organized as follows. In section 2, a parallel gear transmission system is briefly introduced. Section 3 briefly describes residual signal based on time synchronous averaging. Section 4 provides a quick overview of wavelet transforms such as CWT and DWT. Fault indicators based on wavelet transform are considered in section 5.
In section 6, we describe briefly the experimental gear test rig, and summarize the data-processing techniques used in this study. The results are presented in section 7, followed by the conclusions in section 8.

2. The Parallel Gear Transmission System

The scheme of a gear transmission system is shown in Figure 1. The system consists of a pinion gear and a driven gear. The pinion gear has a smaller number of teeth than the driven gear. Normally, a gear transmission system is designed to reduce the angular velocity in order to increase the output torque. In such a speed reduction gear transmission system, the pinion is connected with an input shaft, and the driven gear is connected with an output shaft.


TSA technique is widely accepted as a powerful tool in the fault detection and diagnosis of the rotating systems. The technique attempts to isolate the raw vibration signal from the gearbox by reducing the effects of noises. Noises can be from the external environment or come from other gears from the same gearbox. Suppose that there is a discrete time series \( x(n) \) \( n = 0, 1, \ldots, N-1 \), which covers a number of revolutions of the gear. Then, the TSA signal is calculated using the following formula:

\[
y(n) = \frac{1}{L} \sum_{i=0}^{L-1} x(n - iK), \quad n = (L-1)K, (L-1)K + 1, \ldots, N-1
\]

where \( y(n) \) is the TSA signal, \( L \) is the number of revolutions to be averaged, \( K \) is the number of sampling points per revolution.

We can extract different TSA signals based on the different values of \( K \), which are dependent on the different gears and their shafts. In this paper, only the signals from the pinion gear shaft were extracted by the TSA method.

Residual signal is obtained by eliminating, from the FFT spectrum of the TSA signal, the fundamental and harmonics of the tooth-meshing frequency, subsequently applying the inverse Fourier transform and then reconstructing the remaining signal in the time-domain. So that the residual signal can be expressed as:
\[ z(n) = y(n) - g(n) \]  \hspace{1cm} (2)

where \( z(n) \) is the residual signal, \( g(n) \) is the signal composed of the eliminated components.

4. **Wavelet Methodology** [12-13]

In this section, we first give a brief introduction of the CWT and then summarize the theory of DWT.

4.1 **Continuous Wavelet Transform (CWT)**

The wavelet transform is a linear transform which uses a series of oscillating functions with different frequencies as window functions, \( \psi(t) \), to scan, and translate the signal \( x(t) \). The wavelet transform, \( \text{CWT}(\alpha, \beta) \), of a time signal \( x(t) \) can be defined as:

\[
\text{CWT}(\alpha, \beta) = \frac{1}{\sqrt{\alpha}} \int_{-\infty}^{\infty} x(t) \psi\left(\frac{t-\beta}{\alpha}\right) dt
\]  \hspace{1cm} (3)

where \( \psi(t) \) is an analyzing wavelet and \( \psi^*(t) \) is the complex conjugate of \( \psi(t) \). \( \alpha \) is the dilation parameter for changing the oscillating frequency and \( \beta \) is the translation parameter.

4.2 **Discrete Wavelet Transform (DWT)**

By choosing fixed values \( \alpha = \alpha_0^j \) and \( \beta = k \beta_0 \), \( j, k = 0, \pm 1, \pm 2, \cdots \), we obtain for the DWT:

\[
\text{DWT}(j, k) = \sum_{j=0}^{\infty} \frac{1}{\sqrt{2^j}} \int_{-\infty}^{\infty} x(t) \psi\left(\frac{t-j2^j - k \beta_0}{\alpha_0^j}\right) dt
\]  \hspace{1cm} (4)

In particular, if \( \alpha \) and \( \beta \) are replaced by \( 2^j \) and \( 2^j k \), then the DWT is given by:

\[
\text{DWT}(j, k) = \sum_{j=0}^{\infty} \int_{-\infty}^{\infty} x(t) \psi\left(\frac{t-j2^j - k \beta}{\alpha_0^j}\right) dt
\]

5. **Fault Feature Extraction**

5.1 **Selection of Maximal Energy Coefficients for Fault Detection**

The signal processed by wavelet transform can be the raw vibration signal, the TSA signal or the gear motion residual signal. In this paper, the residual signal is used as the source signal to which the wavelet transform is applied. The procedure can be described as follows.

5.1.1 **Computing Threshold Wavelet Coefficients**

The threshold used here is determined by calculating the mean and the variance values of \( W(\alpha, \beta) \) for different scales, where \( W(\alpha, \beta) \) are the coefficients of WT. This value was chosen to effectively remove the noise. After determining the threshold, denoising and threshold wavelet coefficients are then computed. These are denoted as:

\[
M(\alpha_j) = \frac{1}{K} \sum_{k=1}^{K} W(\alpha_j, \beta_k)
\]
\[
\sigma(\alpha_i) = \left[ \frac{1}{K} \sum_{j=1}^{K} \left( W(\alpha_i, \beta_j) - M(\alpha_i) \right)^2 \right]^{1/2}
\]

\[
Thr(\alpha_i) = M(\alpha_i) + \sigma(\alpha_i)
\]

\[
\hat{W}(\alpha_i, \beta_j) = \begin{cases} 
\text{sign}(W(\alpha_i, \beta_j)) \cdot \left[ W(\alpha_i, \beta_j) - Thr(\alpha_i) \right] & \text{if } W(\alpha_i, \beta_j) \geq Thr(\alpha_i) \\
0 & \text{if } W(\alpha_i, \beta_j) < Thr(\alpha_i)
\end{cases}
\]

where \( K \) is the number of sampling points, \( M(\alpha_i) \) is the mean, \( \sigma(\alpha_i) \) is the variance, \( \hat{W}(\alpha_i, \beta_j) \) represent coefficients after denoising, sign is the signum function where \( \text{sign}(x) \) is -1 when \( x \) is negative, 0 when \( x \) is 0, and 1 when \( x \) is positive.

### 5.1.2 Computing Autocovariance of Maximal Energy Coefficients

First, we find the maximal energy coefficients at a special scale \( \alpha_{\text{max}} \) defined below, and then autocovariance of the special scale series using the following formulas:

\[
\hat{E}(\alpha_i) = \sum_{j=1}^{M} (\hat{W}(\alpha_i, \beta_j))^2
\]

\[
\left[ \hat{E}(\alpha_{\text{max}}) \right] = \max_{\alpha_i \rightarrow \alpha_{\text{max}}} \left[ \hat{E}(\alpha_i) \right]
\]

\[
\hat{R}_{WW}(m) = \sum_{j=1}^{M-m} \hat{W}(\alpha_{\text{max}}, \beta_j) \hat{W}(\alpha_{\text{max}}, \beta_j)
\]

\[
P(i) = \hat{R}_{WW}^2(i)
\]

Finally, these can be used as fault indicators combined with some statistical measures such as kurtosis, standard deviation (std) and the K-S test.

### 5.2 K-S test and Some Measures for Wavelet Transform

In statistics, the K-S test is used to determine whether two underlying probability distributions differ, or whether an underlying probability distribution differs from a hypothesized distribution [14]. Recently, the K-S test has been found to be an extremely powerful tool in the condition monitoring of rotating machinery [15]. The K-S test - based signal processing technique compares two signals and tests the hypothesis that the two signals have the same probability distribution. Using this technique, it is possible to determine whether the two signals are similar or not.

More specifically, the K-S test considers the null hypothesis that the cumulative distribution function (CDF) of the target distribution, denoted by \( F(x) \), is the same as the cumulative distribution function of a reference distribution, \( R(x) \). The K-S statistic \( K \) is then the maximum difference between the two distribution functions, which can be used as the fault indicator.

In this paper, the coefficients of wavelet transform in the healthy state are chosen to represent the reference distribution, and the K-S test is performed to compare coefficients of the wavelet transform of other data files with the reference file.

In order to compare the effectiveness to indicate fault occurrence, other statistical measures such as kurtosis which is a statistical parameter commonly used to assess the
peakedness of a signal, and standard deviation are also considered in the following sections.

6. Experimental Set-up

Typically, vibration data are collected from accelerometers located on the transmission housing. The vibration data used in this paper were obtained from the mechanical diagnostics test-bed (MDTB) in the Applied Research Laboratory at the Pennsylvania State University [16-17]. It is functionally a motor-drive train-generator test stand. The gearbox is driven at a set input speed using a 22.38kW, 1750 rpm AC drive motor, and the torque is applied by a 55.95kW, 1750 rpm AC absorption motor. The MDTB is highly efficient because the electrical power generated by the absorber is fed back to the driver motor. The gearboxes are nominally in the 3.73~14.92kW range with ratios from about 1.2:1 to 6:1. The system can be seen in Figure 2.

Each data file was collected in a 10s window which covers 200000 sampling points in total. The time interval between every two adjacent data files is 30 minutes. The sampling frequency is 20 kHz. The signals of the MDTB accelerometers are all converted to digital data format with the highest resolution. Among all accelerometers located in the MDTB, the single axis shear piezoelectric accelerometer data A03 for axial direction presents the best quality data for state diagnosis of the gearbox. Therefore, the data recorded by this accelerometer is selected in this study. In this paper, we have only extracted and analyzed the signals of the input gear shaft with period $K = 686$ (sampling frequency*60/gear speed = 20000*60/1750 = 686).

7. Results and Discussion

Several data files (194~195,197,199~206,208~212,214, 217~218, 223, 225, and 228~231) of A03 from the test run #13 have been randomly selected to investigate the gear shaft (21 teeth pinion gear) fault. The gear shaft ran from the healthy state to the state of completely broken (See Figure 3) at 300% output torque (188.14 Nm). The duration of the whole experiment was 15.5 hours. The gear shaft states were unknown during the running period when the data files were collected. The shaft was inspected after completing the experiment. There are a number of different real and complex valued functions that can be used in analyzing wavelets. After a thorough investigation and analysis of the results, we have found that the Daubechies wavelet with order 4 is most
effective for processing the vibration data considered in this paper.

![Figure 3: Broken Gear Shaft in Test Run #13 [16]](image)

7.1 CWT for Gear Shaft Crack Detection

7.1.1 CWT based on the TSA Signal and Residual Signal

CWT is often graphically represented in a time-scale plane. However, using the relationship between frequency and scale, and by transforming the time of one wheel revolution to 360 degrees of wheel angular location, the results of CWT amplitude maps can be displayed in the angle-frequency plane.

We have investigated all selected files of the test run #13 with CWT applied to the corresponding TSA and residual signals.

![Figure 4: CWT based on TSA (a) and Residual Signal (b) for File 194](image)
Figure 5: CWT based on TSA signal (a) and Residual Signal (b) for File 214

Figure 6: CWT based on TSA Signal (a) and Residual Signal (b) for File 217

Figure 7: CWT based on TSA Signal (a) and Residual Signal (b) for File 218
The following results can be observed from the plots:

(1) In the healthy state of the gear shaft (Figures 4-6), the TSA signatures vary very regularly, oscillating along the center line. There are 21 signature periods in one revolution, corresponding to 21 teeth.

(2) In the broken state of the gear shaft (Figure 9), there is a very large variation in the whole waveform of the TSA signal. The TSA signal fluctuation induced by gear shaft crack does not show a sharp impulse, but a hump in the shape of the waveform. Also, the curve deviates far from the center line which is induced by the shaft eccentricity due to shaft crack. However, as there is no peak impulse in the curve, which is often induced by tooth fault, we can identify the fault as a gear shaft fault rather than a gear tooth fault or some other fault.

(3) In the corresponding CWT plots, the waveform of the mean amplitude of CWT based on TSA signal behaves just like the waveform of the TSA amplitude. In the healthy states, there is little fluctuation in the waveform, which is expected and it is due to small imperfections in the gear shaft. However, there are evident amplitude fluctuations in the gear shaft faulty states (Figure 9), and this can be explained by the bigger impact caused by faulty states (broken shaft).

(4) In the residual signal, most of the vibration energy generated by the gear meshing action has been removed, so the amplitude values of residual signal are relatively small.
The residual signals and CWT based on residual signals appear unorderly, and no special symptoms can be found in the maps of healthy states and small fault states. So, these maps cannot be used to indicate and to prognosticate the gear shaft fault advancement quantitatively.

### 7.1.2 CWT based on Residual Signal for Gear Shaft Crack Detection

In order to detect the gear shaft fault, some fault feature extracting indicators such as standard deviation, kurtosis and the K-S test using the wavelet coefficients of residual signal are computed and investigated in this section. The plots based on CWT and on the \( \{P(i)\} \) values (Equation (6)) for residual signal can be seen in Figures 10-12.

![Scatterplot of Std vs File](image1)

**Figure 10:** Std of CWT (a) and Std of \( \{P(i)\} \) Values (b) for Residual Signal

![Scatterplot of Kurtosis vs File](image2)

**Figure 11:** Kurtosis of CWT (a) and Kurtosis based on \( \{P(i)\} \) Values (b) for Residual Signal

![Scatterplot of K vs File](image3)

**Figure 12:** K of CWT (a) and K based on \( \{P(i)\} \) Values (b) for Residual Signal

The behavior of the standard deviation, kurtosis and K-S test of amplitude of CWT maps over gear shaft full lifetime are shown in Figures 10-12, respectively. The following results can be drawn from the plots:

1. Both plots in Figure 10 based on std present the same trend, but there are evident differences. First, the values of standard deviation based on \( \{P(i)\} \) are far larger than those
of CWT, since \( P(i) \) calculations are carried out using the square of a sum of squares. Also, starting from data file 194 to file 217 (Figure 10a), the values of std oscillate with a decreasing trend, followed by a dramatic increase for file 218 which is caused by early gear shaft fault (small crack). After data file 218, the values have a gradual increase with fluctuation. The values of std in Figure 10b remain constant with little fluctuation between data files 194 and 217, but a sudden increase occurred in data file 218, indicating the first stage of the fault development. After that, the values tend to decrease, then increase again until the occurrence of the catastrophic fault when gear shaft is broken (data files 230 to 231). We can conclude that std based on \( P(i) \) values is a better indicator of fault presence than std of CWT.

2) The values of kurtosis based on both CWT and \( P(i) \) values over full gearbox lifetime are plotted in Figure 11a and 11b, respectively. Kurtosis is used in engineering for the detection of fault symptoms because it is sensitive to impulses in signals. Obviously, the sharper the impulse in a signal, the greater the value of the kurtosis. However, from Figure 11, we can observe that the values of kurtosis oscillate irregularly. The kurtosis value of the residual signal is not proportional to the advancement of the gear shaft fault, particularly when the gear shaft is involved in a fault. Thus, kurtosis values based on residual signal are unable to diagnose early gear shaft fault.

3) For a comparison, the K-S test is also considered for the gear shaft fault detection. The data file 194 was used as the reference signal. The results are shown in Figure 12. Although the values have an increasing trend, the K-S test applied to CWT based on residual signal cannot diagnose early gear shaft fault, there is no obvious jump or sudden increase of K value.

### 7.2 DWT for Gear Shaft Crack Detection

#### 7.2.1 DWT based on the TSA Signal and Residual Signal

In discrete wavelet analysis, the details which give identity of the signal are the low-scale, high-frequency components, and approximations which indicate overall behavior are the high-scale, low-frequency components. Since the process is iterative, it can be continued indefinitely in theory. In practice, we select a suitable number of levels based on the nature of the signal. In this paper, we consider three levels of decomposition. The DWT coefficients for some typical files based on the TSA and residual signals are shown in Figures 13-18 considering the lowest level of DWT decomposition.

![Figure 13: DWT based on TSA Signal (a) and Residual Signal (b) for File 194](image-url)
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Figure 14: DWT based on TSA Signal (a) and Residual Signal (b) for File 214

Figure 15: DWT based on TSA Signal (a) and Residual Signal (b) for File 217

Figure 16: DWT based on TSA Signal (a) and Residual Signal (b) for File 218
From Figures 13-18, we obtain the following results:

1. The amplitude values of coefficients of DWT for the healthy state are little smaller than those for the unhealthy state, but there is no evident difference for different data files.
2. Like CWT, these maps of DWT cannot be used to identify the early gear shaft fault. They must be combined with fault feature extracting indicators such as standard deviation, kurtosis and the K-S test.

### 7.2.2 DWT based on Residual Signal for Gear Shaft Crack Detection

In this section, we perform similar analysis as in section 7.1.2 using DWT. The results are summarized in Figures 19-21.

![Figure 17: DWT based on TSA Signal (a) and Residual Signal (b) for File 229](image)

![Figure 18: DWT based on TSA Signal (a) and Residual Signal (b) for File 231](image)

![Figure 19: Std of DWT (a) and Std of \{P(t)\} Values (b) for Residual Signal](image)
The following results can be obtained from Figures 19-21:

1. The values of the std of DWT (Figure 19a) oscillate with a slightly decreasing trend before data file 218, but a sharp jump occurs in file 218, revealing that the gear shaft crack may have occurred at that time. After data file 218, the values increase gradually, then fluctuate with a slightly decreasing trend. However, the trend of the Figure 19 is different from that of Figure 10, which shows the development of the gear shaft crack until the shaft is broken, since DWT requires a far smaller amount of work compared to CWT. Nevertheless, by using std for DWT based on residual signal, it can still be detected that an early fault occurred in the gear shaft starting with data file 218. The values of std in Figure 19b remain almost constant with limited fluctuation between data files 194 and 217, then abrupt change occurs in data file 218, and after that, the process shows the same behaviour as the process in Figure 19a.

2. Using the residual signal, the values and waveform of kurtosis show obvious differences for DWT when compared with CWT, but the same conclusion can still be obtained that kurtosis based on residual signal is unable to diagnose early gear shaft fault.

3. We can observe that the trend of the K-S test value K (Figure 21) is similar to that for std (Figure 19), which is a clear indication of the fault presence. Therefore, we can conclude that K-S test applied to DWT based on residual signal can indicate the occurrence of gear shaft early fault.

8. Conclusions

In this paper, the approach using the autocovariance of maximal energy coefficients combined with wavelet transform has been proposed for gear shaft fault detection using gear shaft vibration signal data. Several indicators such as std, kurtosis and the value of the K-S test statistic K have been calculated and analyzed in detail. The main results can be summarized as follows:

1. For both CWT and DWT, the statistical measure kurtosis is unable to reveal the occurrence and advancement of gear shaft cracks.
(2) The standard deviation of residual signal as an indicator over full gear shaft lifetime is able to diagnose early gear shaft fault and shaft fault advancement. We have also found that the std based on \( P(i) \) values is a good indicator of the presence of faults. Considering the amount of work required for CWT, DWT also proves to be an efficient method.

(3) With the DWT based on residual signals, the K-S test statistic K is able to detect the gear shaft crack occurrence, its advancement, and the faulty state effectively. However, it has been shown that for the CWT based on residual signal, the K value is incapable of revealing the occurrence of the gear shaft crack clearly.

(4) It can be concluded from the analysis that the gear shaft was in a healthy state during data files 194 to 217, there is an indication of a crack occurrence in data file 218, and the gear shaft can be diagnosed as being in the faulty state after data file 218. The diagnosis indicates that the impending fault using the method presented in this paper can be identified earlier than the inspection performed at the actual shutdown time of gearbox due to shaft cracks estimating fault occurrence between data files 230 and 231.

In this paper, we have employed the feature extraction approach based on the application of the autocovariance of maximal energy coefficients combined with wavelet analysis to gear shaft fault detection. It has been demonstrated using real vibration data that the faults in gear shafts can be early detected and identified successfully using this approach.

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References


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