Diagnosis Decision-Making using Threshold Interpretation Rule and Expected Monetary Value

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Abstract: Lack of information in dissolved gas analysis (DGA) pieces of evidence necessitates Dempster-Shafer theoretic approach for combining these pieces of evidence. The threshold ground probability assignment (THG) that firmly judge major fault condition is determined from DGA dataset prior to year 2009. A threshold interpretation rule is proposed. Four distinct scenarios resulted from the application of the interpretation rule inclusive of a scenario, which the system operator is uncertain about the condition of a power transformer. DGA dataset of all power transformers that experienced electrical and thermal failures in year 2009 is collected to validate the threshold interpretation rule. Six decision policies are introduced to map power transformer condition propositions to decision spaces for decision-making under uncertainties. Expected monetary value is utilized to assess each decision policy and to select the optimal decision policy.

Keywords: Dempster-Shafer theory, dissolved gas analysis, risk assessment, diagnosis, expected monetary value

1. Introduction

The dissolved gas analysis (DGA) \([1, 2]\) is based on the knowledge of the decomposition of mineral oil and insulation paper hydrocarbon molecules, into by-product of combustible gases (CG) such as hydrogen (H\(_2\)), methane (CH\(_4\)), acetylene (C\(_2\)H\(_2\)), ethane (C\(_2\)H\(_6\)) and ethylene (C\(_2\)H\(_4\)), as a result of electrical and thermal fault mechanism in a power transformer. However, normal operating transformer and some transformer materials may produce CG \([3]\), and complicate the interpretation of DGA.

The DGA produces several pieces of fault and stray gases evidence. However, the DGA pieces of evidence lack information about the exact condition of power transformer. First, the imprecision of laboratory equipment \([4]\) compromises the accuracy of DGA measurement. Second, the evidence may lack information whether the transformer is in normal operating condition or early stage of fault. Third, conflict between disparate pieces of DGA evidence causes difficulty to differentiate between incipient fault condition, and major fault condition that leads to power transformer's failure \([5]\). Fourth, existing fault recognition methodologies cannot identify a precise fault type \([1, 2, 4]\) at some range of DGA values. Therefore, Dempster-Shafer (DS) theoretic approach is suitable for the fusion of DGA pieces of evidence since it can represent various types of ignorance in the knowledge sources.

In reference \([6]\), DS theory is utilized to diagnose a power transformer’s fault type; and proposed several future work recommendations for real-life application. In reference \([7]\), a framework for root-cause based THB Interpretation Rule is introduced and compared with...
Dempster and Yager combination rule. Reference [7] also proposes to use Yager combination rule when the power transformer’s condition is uncertain. This paper expands the work done in [7]. We collected field DGA dataset for all power transformers that experienced internal thermal or electrical failure in year 2009 to validate the THG. Four distinct scenarios resulted from the application of threshold interpretation rule, inclusive of a scenario where the system operator is uncertain about the actual power transformer’s condition. Six decision policies for the mapping of power transformer’s condition belief masses are introduced. Through the utilization of expected monetary value, the decision-making of the system operator is more systematic than the application of Yager’s combination rule earlier proposed in [7].

2. Dempster-Shafer Theory and DGA Pieces of Evidence

2.1 Dempster-Shafer Theory

2.1.1 Frame of Discernment and Axioms

Let \( X = \{\text{MF, IF, N}\} \) be the frame of discernment of a power transformer’s condition. MF, IF and N represent major fault, incipient fault, and normal, respectively. “Major fault” denotes serious power transformer’s fault that leads to breakdown. “Incipient fault” denotes the early stage of a thermal or an electrical fault in a power transformer. “Normal” denotes the normal power transformer condition. The basic belief mass (bbm) function, is a function that maps a proposition \( (prop) \) of power transformer condition to a value within \([1,0]\), i.e., \( m : 2^X \rightarrow [0,1] \), which satisfies the following axioms: (1) \( m(\emptyset) = 0 \), (2) \( m(\emptyset) = 0 \) (\( \emptyset \) is the null set), and (3) \( \sum_{A \in 2^X} m(A) = 1 \). The power set \( 2^X \) (\( \{\{\text{MF}\}, \{\text{IF}\}, \{\text{N}\}, \{\text{MF, IF}\}, \{\text{MF, N}\}, \{\text{IF, N}\}, X, \emptyset\} \)) includes all possible \( prop \) of power transformer’s condition. The collection of positive-valued bbm (\( m(A) > 0 \)) and their corresponding values are termed as belief structure.

2.1.2 Belief and Plausibility Functions

Let \( \text{Bel}(A) \) denotes the belief function and \( \text{Pl}(A) \) denotes the plausibility function assigned to \( \text{prop} \ A \). Belief function and plausibility function are defined as \( \text{Bel}(A) = \sum_{B \subset A} m(B) \) and \( \text{Pl}(A) = \sum_{A \subset B \neq \emptyset} m(B) \), respectively. The belief mass assigned to \( \text{prop} \) A lies within \([\text{Bel}(A), \text{Pl}(A)]\).

2.2 DGA Pieces of Evidence

Three pieces of faults evidence and three pieces of stray gases evidence are obtained from the DGA of transformer’s oil. Individual concentration of \( \text{CG} \) (\( \text{H}_2, \text{CH}_4, \text{C}_2\text{H}_2, \text{C}_2\text{H}_6 \) and \( \text{C}_2\text{H}_4 \)) denoted as Evidence 1, rate of increase (RGI) of total CG denoted as Evidence 2, Basic Gas Ratio [2] denoted as Evidence 3, \( \text{C}_2\text{H}_4 \) stray gas due to contamination from on-load tap changer denoted as Evidence 4, \( \text{C}_2\text{H}_6 \) stray gas denoted as Evidence 5 and \( \text{H}_2 \) stray gas denoted as Evidence 6.

2.2.1 Belief Structures for Evidence 1 and Evidence 2

The whole population of 132kV (90MVA and less) power transformers between year 2002 and 2008 were analyzed for assigning the bbm to \( \text{prop} \ \{\text{MF}\} \) for Evidence 1 and Evidence 2. The DGA dataset consists of 2,621 samples inclusive of twelve DGA dataset of failed transformers. The bbm assigned to \( \text{prop} \ \{\text{MF}\} \) for Evidence 1 and Evidence 2 is
modelled by using cumulative Weibull function. The imprecision of laboratory equipment [4] contributes to bbm assigned to prop X. m(X) is quantified by taking into account the tolerance of the measuring equipment. Bbm is also assigned to prop {IF, N} because the evidence could not distinguish between IF or N transformer's condition. The algorithms for assigning bbm to prop {MF} are described in [8]. The belief structures for Evidence 1 and 2 are given as \{m({MF}), m({IF, N}), m(X)\}. As individual concentration of CG or RGI approaches infinity, m({MF}) approaches 1, the belief structure reduces to m({MF})=1. On the other hand, as individual concentration of CG or RGI approach 0, m({MF}) and m(X) approach 0 and the belief structure reduces to m({IF, N})=1.

### 2.2.2 Belief Structures for Evidence 3

Basic Gas Ratio (BGR) uses three ratios of five CG, to indicate the existence of thermal fault or electrical fault within a power transformer. The power transformer’s faults are grouped into a few categories. The first category is the high energy electrical fault which consists of (1) low energy discharge (D1), (2) high energy discharge (D2) and (3) either low energy discharge or high energy discharge (D1/D2). The second category is the high temperature thermal fault, which consists of (1) thermal fault with temperature>700°C (T1) and (2) thermal fault with temperature between 300°C and 700°C (T2). The third category consists of other fault type such as (1) low temperature thermal fault (T3), and (2) partial discharge (PD). BGR may not indicate any fault in some range of gas ratios values, defined as uncertain (U) category, due to stray gas or normal gassing.

The application of BGR requires the calculation of 90% typical values of DGA of a common electricity network. A fault is insignificant if the DGA parameters are less than 90% typical value [2]. The whole population of power transformers were analyzed for assigning the bbm to prop {MF} for Evidence 3. The imprecision of laboratory equipment [4] assigns bbm to prop X. BGR criterion [2] produces one of the three cases as follows:

(a) First Case - BGR do not identify any fault type (uncertain category) but one of the CG or RGI exceeds the 90% typical value. The evidence implies the assignment of a portion of mass to prop {IF, N}. The belief structure is m({MF})+m({IF,N})=1. However when equipment tolerance is taken into account and the DGA measurement infers a fault, the belief structure is reassigned to m({MF})+m({IF,N})+m(X)=1.

(b) Second Case - BGR identifies a fault type and one of the CG or RGI exceeds the 90% typical DGA values. Since BGR identifies a fault, some portion of mass is assigned to prop {IF}. The belief structure is given by m({MF})+m({IF})}. The belief structure is re-assigned to m({MF})+m({IF})+m(X)=1 when DGA measurement due to equipment tolerance infers a different fault than the original fault. On the other hand, the belief structure is re-assigned to m({MF})+m({IF,N})+m(X)=1 when DGA measurement due to equipment tolerance infers an uncertain or normal category.

(c) Third Case - All individual gases or RGI are less than 90% typical values, which indicates normal condition. No failure occurs when DGA value is less than 90% typical DGA value. The belief structure is given by the following equation, m({N})=1. The belief structure is reassigned to m({MF})+m({IF,N})+m(X)=1 when BGR infers a fault type or uncertain category after taking into account the tolerance in the measuring equipment.
2.2.3 Belief Structures for Evidence 4 and Evidence 5

Evidence 4 and 5 are represented by the ratios $C_2H_2/H_2$ [1] and $C_2H_6/C_2H_4$ [3]; which indicates that the transformer is contaminated with CG from on-load tap-changer (OLTC), and catalytic combustible gas production from transformer material, respectively. When existence of stray gases is indicated by the stray gas ratios, knowledge about non-existence of major fault is obtained. However, the knowledge lacks information to assign a belief mass to $prop \{N\}$ or $prop \{IF\}$. The belief structure is shown as:

$$m(\{IF,N\}) = 1$$

(1)

If the gas ratios do not infer stray gas existence, the belief structure is shown as:

$$m(X) = 1$$

(2)

2.2.4 Hydrogen Stray Gas (Evidence 6)

Certain power transformers are made from materials that produce $H_2$ stray gas. A group of power transformers, which contains high value of $H_2$ gas, but did not fail, is identified from the DGA data set. The power transformers are made from similar design materials by the same manufacturer. We denote them as group A. Another group, which consists of failed transformers, is denoted as group B. Two samples t-test with unequal variances was utilized. It is significant at 95% confident level ($p$-value = 0.139 > 0.05), the mean value $H_2$ gas (in ppm) of group A (mean = 594.8, standard deviation = 510.24, sample size = 25), is equal to, the mean value $H_2$ gas (ppm) of group B (mean = 449.42, standard deviation = 289.3537, sample size = 12). This result implies that the system operator may wrongly diagnose a power transformer with $H_2$ stray gas as a power transformer having major fault. $H_2$ stray gas (Evidence 6) existence cannot be indicated by any gas ratio interpretation scheme. Therefore, for power transformers that catalytically produces $H_2$ stray gas, the concentration of $H_2$ is omitted from representing Evidence 1 for the combination with other belief structures.

3. Threshold Interpretation Rule

3.1 Ground probability assignment function

The pieces of evidence are fused by calculating the GPA functions. The steps taken to calculate the GPA of the pieces of evidence (from evidence 1 to evidence 6) are illustrated in Figure 1.

Choose the belief structure with highest bbm assigned to $prop \{MF\}$ amongst $CH_4$, $C_2H_6$, $C_2H_4$, $C_2H_2$ and $H_2$ belief structures to represent Evidence 1.

Choose the belief structure with highest bbm assigned to $prop \{MF\}$ amongst $CH_4$, $C_2H_6$, $C_2H_4$, $C_2H_2$, and $H_2$ belief structures to represent Evidence 1.

Combine belief structure of Evidence 1 with belief structures of Evidence 2, Evidence 3, Evidence 4 and Evidence 5

**Figure 1**: Flow-Chart of combining Power Transformers Pieces of Evidence

Let us denote $m_1$, $m_2$, ..., $m_p$ as the bbm for belief structures of Evidence 1, Evidence 2,..., Evidence “p”, respectively. Let $F_p$ denotes the set of focal $props$ for $m_p$. Let us denote the mapping $q: 2^X \rightarrow [0,1]$ as ground probability assignment (GPA) function. The GPA assigned to $prop \ A, A \in 2^X$, is calculated by using equation (3) as follows:
The calculated GPA do not satisfy axiom (2) because \( q(Ø) \neq 0 \). Several combination rules were proposed to convert gpa to bbm [9,10,11,12].

3.2 Threshold Ground Probability Assignment (THG)

The whole population of DGA dataset of power transformers was used for the calculation of GPA. We selected 44 DGA dataset according to 44 highest magnitude of the calculated \( q(\{MF\}) \) for the analysis of the behaviour of gpas, as shown in Figure 2.

\[
q(A) = \sum_{A_1, A_2, \ldots, A_k \subseteq A} m_i(A_i) m_{A_i}(A_2) \cdots m_{A_i}(A_k), A \subseteq 2^A
\]

(3)

The calculated GPA do not satisfy axiom (2) because \( q(Ø) \neq 0 \). Several combination rules were proposed to convert gpa to bbm [9,10,11,12].

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The first stage shows that \( q(\{IF\}) \) or \( q(\{IF,N\}) \) dominates the GPA value. As the condition deteriorates in the second stage, \( q(Ø) \) increases in value, indicating increased of contradiction between the pieces of evidence. In the third stage, \( q(\{MF\}) \) increases significantly in comparison to the first and second stage. The THG is defined as the minimum \( q(\{MF\}) \) of power transformers that failed due to high energy electrical fault or high temperature thermal fault. Let \( q_{\text{electrical fault},i}(\{MF\}), (i = 1, \ldots, N) \), be the values of GPA assigned to \( \text{prop \{MF\}} \) corresponding to “i”th power transformer failure due to high energy electrical fault and \( q_{\text{thermal fault},k}(\{MF\}), (k = 1, \ldots, K) \); be the values of GPA assigned to \( \text{prop \{MF\}} \) corresponding to the “k”th power transformer failure due to high temperature thermal fault. Then, THG = \( \min [q_{\text{electrical fault},i}(\{MF\}), q_{\text{thermal fault},k}(\{MF\})] \).

3.3 Threshold Interpretation Rule Algorithm

The interpretation rule uses the THG to calculate the portion of \( q(Ø) \) that is distributed to \( \text{prop \{MF\}} \), to indicate a firm judgment about the critical condition of power transformer. Let us denote \( t(Ø) \) as the maximum portion of \( q(Ø) \) that is to be distributed to \( \text{prop \{MF\}} \). \( t(Ø) \) is calculated as:

\[
t(Ø) = 0.5 - \text{THG}
\]

(4)

The conversion from gpa to bbm is shown by equations (5), (6) and (7).

\[
m(\{MF\}) = q(\{MF\}) + Y
\]

(5)

\[
m(C) = q(C), \quad C \subseteq \{MF\} , Ø
\]

(6)

\[
m(X) = q(X) + Z
\]

(7)

where \( Y \) is the portion of \( q(Ø) \) that is distributed to \( \text{prop \{MF\}} \) and \( Z \) is the portion of \( q(Ø) \) distributed to the frame of discernment. Two situations exist. First, \( q(Ø) \geq t(Ø) \). In this situation, \( Y = t(Ø) \) and \( Z = q(Ø) - t(Ø) \). Second, \( q(Ø) < t(Ø) \). Since \( q(Ø) \) is less than \( t(Ø) \), the maximum portion of \( q(Ø) \) that can be assigned to \( \text{prop \{MF\}} \) is \( q(Ø) \) itself. In this situation, \( Y = q(Ø) \) and \( Z = 0 \).
4. System Operator's Decision Making Scenarios

Let Y denotes the system operator's decision spaces, Y={SD, IS, GO}, where “SD” denotes the decision to shut down the transformer, “IS” denotes the decision to increase DGA sampling frequency and plan for maintenance, and “GO” denotes the decision to continue the transformer’s operation. Let \{P(GO), P(IS), P(SD)\} denote the belief structure for decisions GO, IS and SD. The combined belief structure \{m({MF}), m({IF}), m({N}), m({IF, N}), m(X)\} is converted to decision spaces’ belief structure \{P(GO), P(IS), P(SD)\}. This conversion resulted in six decision policies (DP\_j, j = 1,...,6), as shown in Table 1.

### Table 1: Mapping of Belief Masses

<table>
<thead>
<tr>
<th>Belief Mass</th>
<th>DP_1</th>
<th>DP_2</th>
<th>DP_3</th>
<th>DP_4</th>
<th>DP_5</th>
<th>DP_6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m({MF})))</td>
<td>P(SD)</td>
<td>P(SD)</td>
<td>P(SD)</td>
<td>P(SD)</td>
<td>P(SD)</td>
<td>P(SD)</td>
</tr>
<tr>
<td>(m({IF})))</td>
<td>P(IS)</td>
<td>P(IS)</td>
<td>P(IS)</td>
<td>P(IS)</td>
<td>P(IS)</td>
<td>P(IS)</td>
</tr>
<tr>
<td>(m({N})))</td>
<td>P(GO)</td>
<td>P(GO)</td>
<td>P(GO)</td>
<td>P(GO)</td>
<td>P(GO)</td>
<td>P(GO)</td>
</tr>
<tr>
<td>(m({IF,N}))</td>
<td>P(IS)</td>
<td>P(IS)</td>
<td>P(IS)</td>
<td>P(GO)</td>
<td>P(GO)</td>
<td>P(GO)</td>
</tr>
<tr>
<td>(m(X))</td>
<td>P(SD)</td>
<td>P(SD)</td>
<td>P(GO)</td>
<td>P(SD)</td>
<td>P(IS)</td>
<td>P(GO)</td>
</tr>
</tbody>
</table>

Four decision-making scenarios resulted. The first three scenarios exhibit system operator's firm judgment of a power transformer’s condition, and the corresponding decisions. The fourth scenario exhibits system operator's ignorance in the power transformer’s condition, that is, when Bel({MF}), Bel({IF}) and Bel({N}) are assigned belief mass less than 0.5.

#### 4.1 Scenario 1: Bel({MF}) \geq 0.5 and Threshold Interpretation Rule Validation

In year 2009, four power transformers had internal thermal and electrical major faults. These major faults occurred on March 21, 2009, April 10, 2009, July 26, 2009, and August 13, 2009. DGA data set of these power transformers is analyzed. The combined belief structure are calculated according to equation (7), (8) and (9) to validate whether THB Interpretation Rule could firmly indicate power transformer’s major fault condition. The belief interval of power transformer’s condition is shown in Table 2.

### Table 2: Belief Interval for Scenario 1

<table>
<thead>
<tr>
<th>Fault</th>
<th>[Bel({MF}), Pl({MF})]</th>
<th>[Bel({IF}), Pl({IF})]</th>
<th>[Bel({N}), Pl({N})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fault 1</td>
<td>[0.5833, 0.8538]</td>
<td>0.1287 - 0.4166</td>
<td>0.0000 - 0.2880</td>
</tr>
<tr>
<td>Fault 2</td>
<td>[0.5095, 1.0000]</td>
<td>0.0000 - 0.4905</td>
<td>0.0000 - 0.4905</td>
</tr>
<tr>
<td>Fault 3</td>
<td>[0.5031, 0.5450]</td>
<td>0.4550 - 0.4969</td>
<td>0.0000 - 0.0419</td>
</tr>
<tr>
<td>Fault 4</td>
<td>[0.5460, 0.9995]</td>
<td>0.0005 - 0.4540</td>
<td>0.0000 - 0.4535</td>
</tr>
</tbody>
</table>

Table 2 shows that Bel({MF})\(\geq\) max(Pl({IF}), Pl({N})) for the four power transformers that were in major fault condition in year 2009. The results validate that the THB Interpretation Rule has positively identifies the power transformers that experienced major fault in year 2009. Discussed in Section 4, the bbm assigned to prop {MF} is mapped to prop {SD} for decision-making. The bbm mapped to prop {SD} > 0.5, and the bbm mapped to either prop {IS} or {GO} is < 0.5. The decision for system operator is to shutdown the power transformer.

#### 4.2 Scenario 2: Bel({IF})> 0.5

The second scenario depicts the situation when the system operator has a firm judgment that the power transformer is having an incipient fault. The belief interval of an operating transformer condition dated 13th of August 2005 is shown as in Table 3.
Table 3: Belief Interval for Scenario 2

<table>
<thead>
<tr>
<th>Bel({MF}), Pl({MF})</th>
<th>Bel({IF}), Pl({IF})</th>
<th>Bel({N}), Pl({N})</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.2071, 0.2071]</td>
<td>[0.5873, 0.7929]</td>
<td>[0.00000, 0.2056]</td>
</tr>
</tbody>
</table>

Table 3 shows that Bel({IF}) > max(Pl({MF}), Pl({N})). Therefore, it is judged that the power transformer’s condition is incipient fault. Discussed in Section 4.0, the bbm assigned to prop {IF} is mapped to prop {IS} for decision making. The bbm mapped to prop {IS} > 0.5, and the bbm mapped to either prop {SD} or {GO} is < 0.5. Since the bbm mapped to prop {IS} is greater than the bbm mapped to either prop {SD} or {GO}, the decision is to increase DGA sampling frequency and plan for transformer’s maintenance.

4.3 Scenario 3: Bel({N}) > 0.5

The third scenario depicts the situation when the system operator has a firm judgment that the power transformer is in normal working condition. The belief interval of an operating transformer condition dated 17th of September 2007 is shown in Table 4.

Table 4: Belief Interval for Scenario 3

<table>
<thead>
<tr>
<th>Bel({MF}), Pl({MF})</th>
<th>Bel({IF}), Pl({IF})</th>
<th>Bel({N}), Pl({N})</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.0084, 0.0084]</td>
<td>[0.0000, 0.0000]</td>
<td>[0.9916, 0.9916]</td>
</tr>
</tbody>
</table>

Table 4 shows that Bel({N}) > max(Pl({MF}), Pl({IF})). Therefore, it is judged that the power transformer’s condition is normal. Discussed in Section 4, the bbm assigned to prop {N} is mapped to prop {GO}. The bbm mapped to prop {GO} > 0.5 and the bbm mapped to prop {IS} and prop {SD} is less than 0.5. Since the bbm mapped to prop {GO} is greater than the bbm mapped to either prop {SD} or prop {IS}, the system operator’s decision is to continue the power transformer’s operation.

4.4 Scenario 4: Ignorance in the Power Transformer’s Condition

Ignorance in the power transformer’s condition occurs when Bel({MF}), Bel({IF}) and Bel({N}) are assigned with a mass value less than 0.5, thus m(X) and m({IF, N}) causes uncertainty in the actual condition of power transformer. Scenario 4 is represented in the second stage of Figure 2, which corresponds to the transition between transformer’s condition spaces. A stricter information seeking methodology that requires a planned-shutdown, denoted as IS*, is proposed. A planned shutdown reduces the possibility of losses. In addition, {IS*} requires off-line diagnostic methodology and investigation of historical record of stray gas for a more thorough information gathering. Suppose we consider DP6 in Table 1, represented by the mapping fDP6: 2^x → Y as:

\[
\begin{align*}
  f_{DP6}(\{MF\}) &= SD, \\
  f_{DP6}(\{IF\}) &= IS^*, \\
  f_{DP6}(\{N\}) &= GO \\
\end{align*}
\]

(8)

Let \{P(GO/DP6), P(IS*/DP6), P(SD/DP6)\} denotes the belief structure for decisions GO, IS* and SD when DP6 is considered. From equation (8), the belief structure of the decision spaces is obtained as:

\[
\begin{align*}
  P(SD/DP6) &= m(\{MF\}) \\
  P(IS^*/DP6) &= m(\{IF\}) \\
  P(GO/DP6) &= m(\{N\}) + m(\{IF,N\}) + m(X) \\
\end{align*}
\]

(9)

The masses P(GO/DP6), P(IS*/DP6) and P(SD/DP6) for other “j” decision policies are obtained in similar way as DP6.
4.4.1 Expected Monetary Values for Each Decision Policy

References [14] and [15] use expected monetary value (EMV) to select the optimal combination rule for a type of safety policy. In this research, the objective of applying EMV is to quantify the risk and select the optimal decision policy. A financial outcome is obtained when the system operator makes a “real-life” decision. In order to differentiate between the actual decision made by the system operator and the decision policies, we denote GO$^{ac}$, IS$^{ac}$, and SD$^{ac}$ for actual “GO” decision, actual “IS” decision and actual “SD” decision, respectively. For example, a system operator receive positive monetary gain when decision GO$^{ac}$ matches the transformer’s actual condition N$^{ac}$ and makes a negative monetary gain (loss) when his decision GO$^{ac}$ does not match the actual transformer’s decision IF$^{ac}$ or MF$^{ac}$.

Let Z denotes the monetary gain and losses. The pay-off between system operator’s decision and the condition of a power transformer is shown as in Table 5.

<table>
<thead>
<tr>
<th>Transformer’s Condition</th>
<th>System Operator Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>GO$^{ac}$</td>
<td>IS$^{ac}$</td>
</tr>
<tr>
<td>MF$^{ac}$</td>
<td>Z$^{MF,GO^{ac}}$</td>
</tr>
<tr>
<td>IF$^{ac}$</td>
<td>Z$^{IF,GO^{ac}}$</td>
</tr>
<tr>
<td>SD$^{ac}$</td>
<td>Z$^{SD,GO^{ac}}$</td>
</tr>
<tr>
<td>N$^{ac}$</td>
<td>Z$^{N,GO^{ac}}$</td>
</tr>
</tbody>
</table>

Let us consider DP$^6$. By using pay-off notations in Table 5, the EMV for decision GO, IS, SD and DP$^6$ are calculated as in equation (10), (11) and (12).

\[
Z_{GO,DP^6} = P(MF / GO) \cdot Z_{MF,GO^{ac}} + P(IF / GO) \cdot Z_{IF,GO^{ac}} + P(N / GO) \cdot Z_{N,GO^{ac}} \quad (10)
\]

\[
Z_{IS,DP^6} = P(MF / IS^*) \cdot Z_{MF,IS^{ac}} + P(IF / IS^*) \cdot Z_{IF,IS^{ac}} + P(N / IS^*) \cdot Z_{N,IS^{ac}} \quad (11)
\]

\[
Z_{SD,DP^6} = P(MF / SD^*) \cdot Z_{MF,SD^{ac}} + P(IF / SD^*) \cdot Z_{IF,SD^{ac}} + P(N / SD^*) \cdot Z_{N,SD^{ac}} \quad (12)
\]

The pay-off between system operator’s decision and the decision policies is shown as in Table 6.

<table>
<thead>
<tr>
<th>Decision Policies</th>
<th>System Operator’s Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>GO</td>
<td>Z$^{GO,DP_1}$</td>
</tr>
<tr>
<td>IS</td>
<td>Z$^{IS,DP_1}$</td>
</tr>
<tr>
<td>SD</td>
<td>Z$^{SD,DP_1}$</td>
</tr>
</tbody>
</table>

By using the pay-off notations in Table 6, the EMV for DP$^6$ is calculated as in equation (13) and (14).

\[
E(Z / DP_6) = P(GO / DP_6) \cdot Z_{GO,DP_6} + P(IS^* / DP_6) \cdot Z_{IS^*,DP_6} + P(SD / DP_6) \cdot Z_{SD,DP_6} \quad (13)
\]

Substituting equation (9) into equation (13), the EMV for DP$^6$ is re-arranged as:

\[
E(Z / DP_6) = (m(MF)) \cdot Z_{MF,DP_6} + (m(IF)) \cdot Z_{IF,DP_6} + (m(N)) \cdot Z_{N,DP_6} \quad (14)
\]

EMVs for other DPs are calculated using similar method. The DP that maximizes the EMV is selected for the mapping of belief masses, shown as equation (15).

\[
\max_{1 \leq i \leq 6} (E(Z / DP_i)) \quad (15)
\]

4.4.2 A Numerical Example for Scenario 4

Let us consider a parallel system configuration. In a parallel system, a power transformer is connected in parallel to another power transformer; and one transformer can
cater for the load in the system if the other transformer is shut down. The belief structure of the DGA evidence for this particular transformer is shown in Table 7.

Table 7: Belief Structures of Individual Evidence for Scenario 4

<table>
<thead>
<tr>
<th>Evidence</th>
<th>(m{\text{MF}}), (m{\text{IF}}), (m{\text{N}}), (m{\text{IF,N}}), (m{X})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>([0.7830, 0.0000, 0.0000, 0.1477, 0.0693])</td>
</tr>
<tr>
<td>2</td>
<td>([0.0017, 0.0000, 0.0000, 0.9981, 0.0002])</td>
</tr>
<tr>
<td>3</td>
<td>([0.0020, 0.0000, 0.0000, 0.9980, 0.0000])</td>
</tr>
<tr>
<td>4</td>
<td>([0.0000, 0.0000, 0.0000, 0.0000, 1.0000])</td>
</tr>
<tr>
<td>5</td>
<td>([0.0000, 0.0000, 0.0000, 0.0000, 1.0000])</td>
</tr>
</tbody>
</table>

The combined belief structure is \(\{m\{\text{MF}\}\), \(m\{\text{IF}\}\), \(m\{\text{N}\}\), \(m\{\text{IF,N}\}\), \(m\{X\}\)\} = \([0.4995, 0.0000, 0.0000, 0.2162, 0.2843]\). Supposed that for all "j" decision policies \((j = 1, \ldots, 6)\), the gain associated with the decision to shut down a power transformer is \(Z_{\text{SD,DP}_j} = \$300K\) due to prevention of accident that may involve the other parallel transformer and delay the re-start-up of the power supply to the load. The negative gain associated with the decision to allow the power transformer to continue operating, that is \(Z_{\text{GO,DP}_j} = -\$500K\) and the negative gain associated with seeking new information is \(Z_{\text{IS},DP_j} = -\$100K\). The calculated EMV for each DP is shown below:

Table 8: Expected Monetary Value of Each Decision Policy

<table>
<thead>
<tr>
<th>Decision Policy</th>
<th>DP₁</th>
<th>DP₂</th>
<th>DP₃</th>
<th>DP₄</th>
<th>DP₅</th>
<th>DP₆</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Monetary Value</td>
<td>$213524</td>
<td>$99809</td>
<td>-$13905</td>
<td>$12704</td>
<td>$13334</td>
<td>-$100381</td>
</tr>
</tbody>
</table>

In Table 8, DP₁ gives the maximum EMV and DP₁ is selected for the mapping of the bmm. The corresponding belief structure is \(\{P(\text{SD}/\text{DP}_1), P(\text{IS}*/\text{DP}_1), P(\text{GO}/\text{DP}_1)\} = \{0.7838, 0.2162, 0.0000\}\). This resulted in \(P(\text{SD}/\text{DP}_1) > P(\text{IS}*/\text{DP}_1) > P(\text{GO}/\text{DP}_1)\). Therefore, the most probable decision is to shut-down the power transformer.

5. Conclusion

We discussed the validation of threshold interpretation rule by using the power transformers which failed in year 2009. We also demonstrated four distinct scenarios that resulted from the application of the threshold interpretation rule for system operator’s decision-making. When the system operator is uncertain about the actual state of power transformers, six decision policies are introduced for the mapping of belief masses assigned to the transformer’s condition spaces to system operator’s decision spaces. Subsequently, we applied expected monetary value to quantify the gain and loss of each decision policy. The system operator’s action is based on the decision policy that can optimize the gain that a system operator may obtain. In this manner, we assess the risk of each decision policy when we are uncertain about the actual state of a power transformer.

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References


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