Markov Modeling Approach for Survivability Analysis of Cellular Networks‡

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Abstract: Survivability is the capability of a system to fulfill its mission in a timely manner in the presence of failures, attacks and accidents. In this paper, quantitative assessment of survivability of cellular networks is conducted by developing an analytical model using Markov chains. A stochastic reward net model is then developed for the automated generation of CTMC and hence survivability metrics in terms of call blocking probabilities and excess delay due to failures are computed. Finally, numerical results are presented for the illustration of the proposed model.

Keywords: Call blocking probabilities, cellular networks, Markov chains, stochastic reward nets, survivability

1. Introduction

Quantification of survivability is of great concern in cellular networks since partial or complete breakdown of the networks due to component failures, software faults, malicious attacks or natural disasters such as earthquakes, storms and floods, significantly reduce the quality of service (QoS). Initial work on survivability was focused on the survivability of network databases (such as Home Location Register (HLR), Visitor Location Register (VLR) etc.) [1, 2]. In [3, 4], a survivability framework for cellular networks, with fault tolerant strategies such as self-healing ring technology, spare capacity allocation, traffic management and redundant component strategies, was proposed. Performance metrics, for example call blocking probabilities and excess delay due to failures, were identified as survivability metrics. ANSI T1A1.2, a group working on network survivability [5] emphasized on the time dependent or transient analysis of survivability metrics for survivability quantification. For this reason, we use the following definition of survivability given by ANSI T1A1.2.

Definition: Suppose a measure of interest $M$ has the value $M_0$ just before a failure occurs. Survivability behavior can be depicted by the following attributes: $M_0$ is the value of $M$ just after the failure occurs; $M_m$ is the maximum difference between the value of $M$ and $M_0$ after the failure; $M_r$ is the restored value of $M$ after some time $t_r$; and $t_R$ is the time for the system to restore the value $M_0$.

This definition is mathematically precise for the quantification of survivability. This definition was used in [6, 7] for computing the survivability metrics of telecommunication. In this paper, we extend their idea to present an analytical model based on continuous time Markov chains (CTMCs) for survivability analysis of fault tolerant cellular networks. The proposed CTMC model considers simultaneous occurrence of failures and their post-effects such as network congestion and frequent occurrence of

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wireless channel failures. Stochastic reward net (SRN) model is constructed from which the underlying CTMC is automatically generated and solved. We use the software package SHARPE [8] to solve the SRN model and compute the survivability metrics.

The rest of this paper is organized as follows. In Section 2, the basic architecture of cellular Base Station (BS) is presented. The survivability model is presented in Section 3. In Section 4, SRN models are constructed. Finally, the numerical results are discussed in Section 5. Conclusions are provided in Section 6.

2. Cellular Base Station Architecture

Cellular networks partition a given geographical region into a large number of cells. Each cell is served by a BS which communicates with base station controller (BSC). A BS consists of transmitters, receivers, associated computing unit, wired/microwave links to the BSC and mobile switching centre (MSC) and the power supply [9]. Figure 1 shows its generic architecture. Causes of faults in a BS could be hardware failures, software errors and bugs, power outages or natural disasters such as rain, storms, tornadoes, hurricanes etc., affecting external BS equipments. In [10], a technique for reporting faults of a BS to the BSC is presented. The network operators install special purpose fault detectors at the BS locations to test the functionalities and faults of the BS. These fault detectors detect the faults and report addresses for accessing the relevant fault messages to the fault management unit of the cellular network system. Depending on the type of failure, the corresponding effect or failure may be contained within a channel or may propagate to the entire BS. Thus, these failures of a BS are categorized into two groups:

Channel failure: Radio spectrum or wireless channels consist of access channels that are provided to mobile users for connectivity and a control channel that does the allocation of an access channel to a mobile user. These wireless channels are processed for fault detection, multi path detection, fading correction etc., by channel unit processors. Due to the faults in these channel elements, multi path detecting, fading, noise, interference etc., are not performed successfully. As a result, a wireless channel becomes unavailable. We consider it as a channel failure. With an occurrence of a channel failure, call carried by a failed channel is lost, while other calls are carried smoothly. In [10], a model was presented to detect the channel failures. Once a channel failure is detected, the fault management unit performs the necessary steps to restore from the channel failure. One such step is automatic software reconfiguration.

Infrastructure failure: Failures in components of a BS such as transmitters, receivers, computing unit, power supply and links to BSC are termed as infrastructure failures. The major reason for failure in these components is natural disasters such as storms, rainfall, tornadoes, earthquakes, hurricanes etc. The occurrence of a natural disaster may completely damage BS components while the other neighboring BS may still be operating. Besides the BS components, the fault management module may also be
affected. In this scenario, the network operator provides temporary repair facility. Consequently, efficiency of repairing facility is greatly reduced which causes infrastructure failures to occur more frequently. Since infrastructure failures have harder impact, higher priority is given for the recovery of these failures. Also, after the natural disaster has occurred, the transmitting power of a partially operating BS is increased so as to provide coverage to the calls in the nearby failed BS. This results in an increase in network traffic in the BS under consideration.

In this paper, we consider a partially operating BS with a partially affected repair facility. In the next section, we develop CTMC models for its survivability analysis.

3. Analytical Survivability Model

In this section, we present an analytical model to quantify network survivability by the transient performance metrics of call blocking probabilities and excess delay due to failures. Our approach involves the construction of CTMC for performance, availability and performability models. Then, the performability model is truncated appropriately to develop the survivability model.

The network traffic arriving at a BS is classified into real time service (RTS) calls (for example, new/handoff voice, new/handoff video on demand) and non-real time service (NRTS) calls (for example, file transfer, text messaging). In this work, we use guard channel scheme for giving high priority to RTS calls. In this scheme a fixed number of channels are reserved exclusively for RTS calls. We next present the performance model to determine the performance measures.

Let each cell contain \( N \) channels. According to fixed guard channel scheme, we assume \( 0 \leq g < N \) channels are reserved for RTS calls and remaining \( N-g \) channels can be allocated to both RTS and NRTS calls. We assume that both RTS and NRTS calls arrive according to Poisson process with rates \( \lambda_{RT} \) and \( \lambda_{NT} \) respectively. Let channel holding times for RTS and NRTS calls be exponentially distributed with rates \( \mu_{RT} \) and \( \mu_{NT} \) respectively. For simplicity, we assume \( \mu_{RT} = \mu_{NT} \). This restriction is relaxed while developing the SRN model in the next section.

Let \( N(t) \) be the number of busy channels at any time \( t \). With an assumption of exponential distributions for inter-arrival times and channel holding times of the calls, the stochastic process \( \{N(t), t \geq 0\} \) is a homogeneous CTMC. State transition diagram for this CTMC is shown in Figure 2. We define \( \lambda = \lambda_{RT} + \lambda_{NT} \) and \( \mu = \mu_{RT} = \mu_{NT} \).

**Figure 2: Pure Performance Model**

Let \( x_j = \lim_{t \to \infty} \Pr(N(t) = j) \) denote the steady state probability that the CTMC is in state \( j \), \( 0 \leq j \leq N \). Then, the steady state call blocking probabilities for RTS and NRTS calls, denoted by \( P_b^{BR} \) and \( P_b^{BN} \) respectively, are computed as follows:

\[
P_b^{BR} = x_N, \quad \text{and} \quad P_b^{BN} = \sum_{j=N-g}^{N} x_j.
\]

Note that we did not consider the dropping probabilities for RTS calls as a performance measure since dropping probabilities are computed for handoff calls and in our case the handoff calls are included in the RTS calls. Since the performance model does not consider the failures and repairs, it overestimates the system performance. To obtain the
realistic values for the performance metrics, we next present the availability model to determine the system availability.

Let the channel failure time and the infrastructure failure time be exponentially distributed with rates $\gamma$ and $\alpha$, respectively. Further, let the time to recover a channel failure (infrastructure failure), be exponentially distributed with rate $\delta$ ($\beta$). Higher priority is given for the recovery of an infrastructure failure, since it has prolonged and severe impact on the network performance. With assumptions of exponential distributions, the availability model of the BS is a homogeneous CTMC. The state transition diagram is shown in Figure 3.

Let $i$, $0 \leq i \leq N$ denotes the number of non-failed channels. Let $k \in \{0,1\}$ denotes whether an infrastructure failure has occurred in the system ($0$: no infrastructure failure in the system, $1$: infrastructure failure has occurred). The state $(i,k)$, $0 \leq i \leq N$, $k \in \{0,1\}$, represents $i$ non-failed channels are available at the BS and the system is in state of an (no) infrastructure failure if $k = 1$ (if $k = 0$).

An infrastructure failure affects hardware supporting the control channel (in addition to the N access channels). The recovery mechanism, with probability $p$, recovers the hardware failure and the same control channel is able to function smoothly, while with probability $(1-p)$, it is unable to recover hardware failure. In this case, one of the access channels is modified to function as the control channel, and therefore, the number of access channels is reduced by one. This probability $p$ depends on the error handling and fault tolerance strategies incorporated in the BS.

We assume that when an infrastructure failure occurs, arrival of RTS and NRTS calls is blocked and the repair facility postpones the ongoing recovery of a channel failure until the infrastructure failure has been recovered. After the recovery of an infrastructure failure, the repair facility recovers the channel failure, if any, in the BS. Further, we suppose that no channel failure occurs during the recovery time of an infrastructure failure. This explains the reason for no transition from states $(i,1)$ to $(i-1,1)$, $0 < i \leq N$. The states $(i,0)$, $0 < i \leq N$ are up states since BS is operating with $i$ non-failed channels. The states $(i,1)$, $0 < i \leq N$ are down states because these states represent the existence of an infrastructure failure in the system and the BS is unable to provide services. On solving the system of equations for this CTMC, steady state probabilities are obtained, and the steady state availability is then evaluated.

3.1 Performability Model

To obtain performance measures, it is important to study performance changes along with failure and recovery behavior. By combining the CTMC of performance and availability model discussed above, the performability model is also a CTMC.
The state transition diagram is shown in Figure 4. Each state of the CTMC is labeled as \((i,j,k)\), where \(i \ (0 \leq i \leq N)\) denotes the number of non-failed channels, \(j \ (0 \leq j \leq i)\) represents the number of ongoing (RTS or NRTS) calls in the system and \(k \in \{0,1\}\) determines whether or not an infrastructure failure has occurred. The transition rate from state \((i,j,0)\) to \((i-1,j,0)\) is \((i-j)\gamma\). It corresponds to the failure in any one of the \((i-j)\) idle channels. Transition rate from the state \((i,j,0)\) to the state \((i-1,j-1,0)\) is \(j\gamma\) due to the failure in one of the \(j\) busy channels. The transition from state \((i,j,0)\) to the state \((i,j,1)\) represents the occurrence of an infrastructure failure. Let \(\pi_{i,j,k}\) denotes the steady state probability of state \((i,j,k)\). Note that in states \((i,i,0)\); \(0 \leq i \leq N\) and \((i,j,1)\); \(1 \leq i \leq N\), \(1 \leq j \leq i\), the incoming RTS calls are blocked since either all the available (non-failed) channels are busy, or the BS is in the state of the infrastructure failure. Hence, the steady state blocking probability for RTS calls before the occurrence of natural disaster, labeled as \(M_{BR}^{0}\), is

\[
M_{BR}^{0} = \sum_{i=0}^{N} \sum_{j=0}^{i} \pi_{i,j,0} + \sum_{i=1}^{N} \sum_{j=0}^{i} \pi_{i,j,1}. 
\] (1)

We next consider the conditions in which the arriving NRTS calls are blocked. First, in states \((i,j,0)\); \(0 \leq j \leq g, \ 0 \leq i \leq i\), the number of non-failed idle channels is less than \(g\) and these channels are reserved for higher priority RTS calls. Second, in states \((i,j,0)\); \(g+1 \leq i \leq N\), \(i-g \leq j \leq i\), the number of non-failed channels is greater than \(g\), but the number of busy channels is more than \((i-g+1)\) so the available, non-failed channels are reserved for RTS calls. Finally, in states \((i,j,1)\); \(1 \leq i \leq N\), \(0 \leq j \leq i\), BS is in the state of an infrastructure failure. The blocking probability of NRTS calls before the occurrence of a natural disaster, denoted by \(M_{BN}^{0}\), is hence given by:

\[
M_{BN}^{0} = \sum_{i=0}^{g} \sum_{j=0}^{i} \pi_{i,j,0} + \sum_{i=g+1}^{N} \sum_{j=0}^{i-g} \pi_{i,j,0} + \sum_{i=1}^{N} \sum_{j=0}^{i} \pi_{i,j,1}. 
\] (2)

### 3.2 Survivability Model

We next construct the survivability model and answer the question “How well a BS performs just after the occurrence of a natural disaster?” In a disaster struck area, few BSs are completely failed while some of them are partially operating. Calls from nearby
damaged BSs arrive in the coverage area of partially operating BS, thereby, increasing the network traffic load in this BS. Further, failures in this BS reduce the efficiency of repair facility, thereby increasing the occurrences of infrastructure failures. We suppose that say, N - n channels that have failed before the occurrence of a natural disaster could not be recovered and therefore n (0 < n < N) channels are available after the occurrence of a natural disaster. Out of these n channels, j channels are busy.

To capture the above mentioned consequences, the CTMC of the survivability model is constructed from the CTMC of the performability model in the following way: First, only the states indicating the number of non-failed channels less than or equal to n (the rows with states \((i,j,k)\): 0 \(\leq\) \(i\) \(\leq\) \(n\), 0 \(\leq\) \(j\) \(\leq\) \(i\), \(k\) \(\in\) \(\{0,1\}\)) are included. Second, the rows with states \((i,j,k)\) for \(n<i\leq N\), 0 \(\leq\) \(j\) \(\leq\) \(i\), \(k\) \(\in\) \(\{0,1\}\) are truncated because we consider that the repair facility is unable to recover \(N-n\) failed channels after a natural disaster has occurred. Finally, force all the transitions from states \((n,j,0)\) to \((n,j,1)\), 0 \(\leq\) \(j\) \(\leq\) \(n\), marked as dashed arcs, to exhibit that an instantaneous transition has taken place to an infrastructure failure state, just after the occurrence of a natural disaster. The state transition diagram of survivability model is shown in Figure 5. In this survivability model, arrival rate \(\lambda_{RT}\) (\(\lambda_{NT}\)) of RTS (NRTS) calls also include RTS (NRTS) calls admitted by a BS from the nearby failed BSs. When an infrastructure failure occurs, the recovery probability p is reduced because some faults are induced in the recovery facility. To obtain the transient survivability measures, we first assign initial probabilities to each state shown in Figure 5. Let \(p_{i,j,0}(0) = 0\), 0 \(\leq\) \(i\) \(\leq\) \(n\), 0 \(\leq\) \(j\) \(\leq\) \(i\), as we assume that the system is in the state of an infrastructure failure after the occurrence of a natural disaster. Next, for 0 \(\leq\) \(i\) \(<\) \(n\), the initial probabilities \(p_{i,j,1}(0)\) are set as 0. Finally, for \(i=n\), the initial probability \(p_{n,j,1}(0)\), 0 \(\leq\) \(i\) \(\leq\) \(n\), is given as:

\[
\pi_{n,j,0} = \sum_{i=0}^{n} \pi_{n,i,0}
\]

where \(\pi_{n,j,0}\) is the steady state probability of the state \((n,j,0)\) of the CTMC shown in Figure 4.

Let \(M_{BR}^{BR}(t)\) and \(M_{BN}^{BR}(t)\) respectively denote the blocking probabilities of RTS and NRTS calls at time \(t\) after the occurrence of a natural disaster. It is observed that an incoming RTS call is blocked when BS is in a state of an infrastructure failure or non-failed idle channels are not available at BS. Summing up transient probabilities of the states corresponding to the above two cases, the transient blocking probability \(M_{BR}^{BR}(t)\) for RTS calls is given by:

\[
M_{a}^{BR}(t) = \sum_{i=0}^{n} p_{i,j,0}(t) + \sum_{i=0}^{n} \sum_{j=0}^{i} p_{i,j,1}(t).
\]

Here, \(p_{i,j,0}(t), 0 \leq i \leq n, 0 \leq j \leq i, k \in \{0,1\}\) is the transient probability of the state \((i,j,k)\) in the CTMC shown in Figure 5. We next obtain the expression for \(M_{BN}^{BR}(t)\). An NRTS call is blocked in one of the following cases:

- BS is in a state of an infrastructure failure.
- \(n \leq g\): In this case, all idle channels are reserved for RTS calls.
- \(n > g\): If an incoming NRTS call finds \(n-g\) or more channels busy, it is blocked. Then, \(M_{BN}^{BR}(t)\) is obtained by summing the time-dependent probabilities of corresponding states and is given as follows:
3.3 Excess Delay due to Failures

Excess delay due to failures \( (EDF) \), measures the delay in accessing the network due to an infrastructure failure in the BS. To analyze the network survivability, we compute \( EDF \) before and after an occurrence of a natural disaster, respectively denoted as \( EDF_0 \) and \( EDF_a(t) \). To obtain \( EDF \), we first define expected excess delay (EED) as \[ 11 \]:

\[
EED \big|_{T-F(t)} = \int_0^\infty t dF(t),
\]

where \( T \) is a random variable for system downtime with CDF \( F(t) \). In this proposed model, time to infrastructure failure follows exponential distribution with rate \( \beta \). Therefore, we have \( T \sim \text{Exp}(\beta) \). By assigning \( EED \) as reward rate to each down state shown in Figure 5, the expression for \( EDF_0 \) is given as \[ 11 \]:

\[
EDF_0 = \sum_{i=1}^{N} \sum_{j=0}^{i} \alpha(EED \big|_{T-\text{Exp}(\beta)}) \pi_{i,j,0} = \sum_{i=1}^{N} \sum_{j=0}^{i} \frac{\alpha}{\beta} \pi_{i,j,0}
\]

The time-dependent expression \( EDF_a(t) \), is given as

\[
EDF_a(t) = \sum_{i=1}^{n} \sum_{j=0}^{i} \alpha(EED \big|_{T-\text{Exp}(\beta)}) p_{i,j,0}(t) = \frac{\alpha}{\beta} \sum_{i=1}^{n} \sum_{j=0}^{i} p_{i,j,0}(t)
\]

4. Stochastic Reward Net Model

In this section, we develop SRN model for concise specification and the automated generation of the underlying CTMC. For a detailed study on modeling with SRN, readers are referred to \[ 12 \].

![Figure 6: Stochastic Reward Net Models](image-url)
Figures 6 (a) and (b) respectively show the SRN models for performability and survivability. Place \( P_{\text{chpool}} \) is a channel pool which initially holds \( N \) tokens, one token for each non-failed, idle channel available in the BS. Transitions \( T_{\text{RTS}_{\text{arr}}} \) (rate \( \lambda_{RT} \)) and \( T_{\text{NRTS}_{\text{arr}}} \) (rate \( \lambda_{NT} \)) represent arrival of RTS and NRTS calls respectively. Transition \( T_{\text{RTS}_{\text{arr}}} \) is enabled only if there is at least one token in place \( P_{\text{chpool}} \). Transition \( T_{\text{NRTS}_{\text{arr}}} \) is blocked if the number of tokens in place \( P_{\text{chpool}} \) is less than \( (g+1) \) because \( g \) channels are reserved for high priority RTS calls. The places \( P_{\text{RTS}} \) and \( P_{\text{NRTS}} \) hold a token for each ongoing RTS and NRTS calls, respectively. Transitions \( T_{\text{RTS}_{\text{ser}}} \) (rate \( \mu_{RT} \)) and \( T_{\text{NRTS}_{\text{ser}}} \) (rate \( \mu_{NT} \)) represent the channel holding times of RTS and NRTS calls, respectively. A token is deposited from the place \( P_{\text{chpool}} \) through transition \( T_{\text{RTS}_{\text{ser}}} \) (i.e., \( T_{\text{NRTS}_{\text{ser}}} \)). Recall that in Section 3, we assumed that \( \mu_{RT} = \mu_{NT} = \mu \). Now this assumption is relaxed, i.e., we consider that \( \mu_{RT} \neq \mu_{NT} \) for constructing SRN models. Transition \( T_{\text{idle}} \) (rate \( \gamma(P_{\text{chpool}}) \)) represents the failure of an idle channel. On its firing, a token is moved from the place \( P_{\text{chpool}} \) to the place \( P_{\text{fail}} \). The tokens in the place \( P_{\text{fail}} \) depict the number of failed channels in the BS. The transitions \( T_{\text{RTS}_{\text{fail}}} \) (rate \( \gamma(P_{\text{RTS}}) \)) and \( T_{\text{NRTS}_{\text{fail}}} \) (rate \( \gamma(P_{\text{NRTS}}) \)) represent the failure of a busy channel carrying an RTS and an NRTS call, respectively. The transitions with marking dependent firing rates are indicated by the \( # \) sign placed next to them. On recovery of a failed channel, a token moves from the place \( P_{\text{fail}} \) to \( P_{\text{chpool}} \) through transition \( T_{\text{chrep}} \) (rate \( \delta \)).

Next, we model the occurrence of an infrastructure failure and its recovery. The presence of a token in the place \( P_{\text{no inf}} \) denotes that there is no infrastructure failure in the system. The transition \( T_{\text{inf}} \) (rate \( \alpha \)) represents occurrence of an infrastructure failure. On its firing, a token is removed from \( P_{\text{no inf}} \) and is deposited in the place \( P_{\text{inf}} \). This indicates the state of an infrastructure failure in a BS. In this state, channel failures and repair, arrival of RTS and NRTS calls and call completion cannot occur until this infrastructure failure has been recovered. Therefore, guard function \( #(P_{\text{inf}})<1 \) is assigned to the transitions \( T_{\text{RTS}_{\text{arr}}} \), \( T_{\text{RTS}_{\text{ser}}} \), \( T_{\text{RTS}_{\text{fail}}} \) and \( T_{\text{NRTS}_{\text{ser}}} \), \( T_{\text{RTS}_{\text{fail}}} \) and \( T_{\text{NRTS}_{\text{fail}}} \). Here \( #(P_{\text{inf}}) \) denotes the number of tokens in the place \( P_{\text{inf}} \).

Firing of Transition \( T_{\text{rep}} \) (rate \( p\beta \)), \( p \) is the probability of efficient recovery and \( \beta \) is the rate at which repair facility recovers the failure) deposits a token in the place \( P_{\text{no inf}} \) from \( P_{\text{inf}} \). Then, we consider three transitions \( T_{\text{degrep1}} \), \( T_{\text{degrep2}} \) and \( T_{\text{degrep3}} \) (all having rate \( (1-p)\beta \)). \( T_{\text{degrep1}} \) represents partial recovery of an infrastructure failure with probability \( (1-p) \), and also involves failure of an idle channel. On its firing, a token moves from the place \( P_{\text{inf}} \) to the place \( P_{\text{no inf}} \) and also a token move from the \( P_{\text{chpool}} \) to \( P_{\text{fail}} \) to represent the failure of an idle channel. If all channels are busy, transition \( T_{\text{degrep1}} \) is disabled and the transition \( T_{\text{degrep2}} \), which represents partial recovery of an infrastructure failure along with a failure of a busy channel carrying an NRTS call, is enabled. On firing of \( T_{\text{degrep2}} \), a token moves from \( P_{\text{inf}} \) to \( P_{\text{no inf}} \) representing the recovery of an infrastructure failure. Also, a token is deposited from \( P_{\text{NRTS}} \) to \( P_{\text{fail}} \). If all ongoing calls are RTS, then \( T_{\text{degrep2}} \) is disabled, then \( T_{\text{degrep3}} \) is enabled. It represents recovery of an infrastructure failure along with failure of channel carrying a RTS call. Through \( T_{\text{degrep3}} \), a token moves from \( P_{\text{inf}} \) to \( P_{\text{no inf}} \). Also, a token is transferred from the place \( P_{\text{RTS}} \) to \( P_{\text{fail}} \) to indicate the failure of a busy channel.
carrying an RTS call. The guard functions for $T_{degrep2}$ and $T_{degrep3}$ are respectively given as

\[ \#(P_{chpool}) < 1 \quad \text{and} \quad \#(P_{chpool}) < 1 \quad \text{and} \quad P_{NRTS} < 1. \]

Let $\Omega$ denote the set of markings of the SRN model. A marking $j \in \Omega$, is a 6-tuple given as $j = (\#(P_{chpool}), \#(P_{RTS}), \#(P_{NRTS}), \#(P_{fail}), \#(P_{no-inf}), \#(P_{inf}))$, where $\#(P_i)$ represent the number of tokens in a place $P_i$. To obtain $M_{\text{BR}}$, we assign the reward rate $r_{j}^{\text{BR}}$ to a marking $j$ as:

\[
r_{j}^{\text{BR}} = \begin{cases} 
1, & \text{if } \#(P_{chpool}) = 0 \text{ or } \#(P_{inf}) = 1 \\
0, & \text{otherwise}
\end{cases}
\]

(8)

Here, a reward rate 1 is assigned to the markings in which the $P_{chpool}$ is empty or the BS is in state of infrastructure failure, and reward rate 0 is assigned to the other states. Then, $M_{\text{BR}}$ is given as $M_{0}^{\text{BR}} = \sum_{j \in \Omega} r_{j}^{\text{BR}} \pi_j$. Similarly, the reward rate $r_{j}^{\text{BN}}$ to a marking $j$ is given as:

\[
r_{j}^{\text{BN}} = \begin{cases} 
1, & \text{if } \#(P_{chpool}) < (g + 1) \text{ or } \#(P_{inf}) = 1 \\
0, & \text{otherwise}
\end{cases}
\]

(9)

and the blocking probability of NRTS calls is given as $M_{0}^{\text{BN}} = \sum_{j \in \Omega} r_{j}^{\text{BN}} \pi_j$ where $\pi_j$ is steady state probability of marking $j$.

To compute the excess delay due to the failures, the reward rate $r_{j}^{\text{EDF}}$ assigned to the marking $j$ is given as:

\[
r_{j}^{\text{EDF}} = \begin{cases} 
0, & \text{if } \#(P_{chpool}) < 1 \text{ or } \#(P_{inf}) = 1 \\
1, & \text{otherwise}
\end{cases}
\]

(10)

Then, the excess delay due to the failures, before the natural disaster occurs, denoted by $EDF_{0}$ is given as

\[
EDF_{0} = \sum_{j \in \Omega} r_{j}^{\text{EDF}} \pi_j
\]

SRN model for survivability of a BS is shown in Figure 6(b). The notations for places and transitions are same as discussed in SRN model of performability. Now, the place $P_{chpool}$ contains $n$ tokens as it is assumed that, $n \ (0 < n \leq N)$ channels are available just after a natural disaster has occurred. Also, a token is deposited in the place $P_{inf}$ to represent that the system is in state of an infrastructure failure. $M_{a}^{\text{BR}}(t)$, $M_{a}^{\text{BN}}(t)$, and $EDF_{0}(t)$ are then obtained by appropriately assigning reward rates to the markings of this SRN model. The transient blocking probability of RTS and NRTS calls, are given as $M_{a}^{\text{BR}}(t) = \sum_{j \in \Omega} r_{j}^{\text{BR}} p_{j}(t)$, and $M_{a}^{\text{BN}}(t) = \sum_{j \in \Omega} r_{j}^{\text{BN}} p_{j}(t)$, where $r_{j}^{\text{BR}}$ and $r_{j}^{\text{BN}}$ are given by Equations (8) and (9) respectively. Excess delay due to failures after the occurrence of natural disasters, $EDF_{0}(t)$, is given by
5. Numerical Results

One can compute the failure and repair/restoration rates by reviewing operator shift reports, monitoring system logs, using vendor supplied downtimes and availabilities for the components [13,14]. Computing these rates is beyond the scope of this paper.

![Figure 7: Blocking probabilities for RTS calls](image)

![Figure 8: Blocking Probabilities for NRTS calls](image)

However, interested readers may refer to [15, 16]. For the purpose of numerical illustration, various parameter values are taken from [13], where system performance is studied through simulation models. We assume the rates as: \(\lambda_{NT} = 1\), \(\lambda_{RT} = 2\), \(\mu_T = 1.5\), \(\mu_{NT} = 1.0\), \(\gamma = 0.002\), \(\delta = 0.03\), \(\alpha = 0.00004\) and \(\beta = 15\). The probability \(\rho\) for efficient recovery of an infrastructure failure is assumed to be 0.98. We assume \(N = 12\) channels with \(p = 3\) guard channels. By assigning reward rates \(r^B_{BR}\) and \(r^B_{BN}\) given by Equations (8) and (9) respectively, we obtain numerical values for \(M^B_{BR}\) and \(M^B_{BN}\) as 0.00010 and 0.00084 respectively.

We next incorporate the aftermaths of natural disaster such as network congestion and frequent infrastructure failures, in the proposed model. Network congestion is exhibited by assuming a 50% increase in RTS and NRTS calls arriving at the BS. Note that this percentage of increment in the network traffic depends on the destruction caused in disaster struck area. Let new arrival rates be \(\hat{\lambda}_{RT} = 3\) and \(\hat{\lambda}_{NT} = 2\). The rate of occurrence of an infrastructure failure is now set as 0.0004 to depict rise in frequency of these failures. The deterioration of the repair facility is exhibited by reducing the value of the probability \(\rho\) of an efficient recovery from 0.98 to 0.90. With these incorporations, time-dependent call blocking probabilities \(M^B_{BR}(t)\) and \(M^B_{BN}(t)\) are computed and shown in Figures 7 and 8, respectively. The increasing time-dependent call blocking probabilities for both RTS and NRTS calls indicate performance degradation of a cellular network. This degradation is at a faster rate when fewer channels are available after the occurrence of a natural disaster. These results were comparable with the results as observed in [13]. To stabilize the network performance, it is desirable to shed off some network traffic load. It is therefore important to study the performance of the system for different offered traffic load expressed in ‘Erlangs’. Let the call arrival rate be \(\hat{\lambda}\) and call service time be \(\mu\). The
dimensionless quantity $\rho = \lambda / \mu$ is the offered traffic load in Erlangs. We assume that $n = 8$ channels are available at the BS just before the occurrence of a natural disaster.

Figure 9 depicts the system performance in terms of $M_a^{BR}(t)$. It is observed that the blocking probability increases rapidly when traffic load is high. In contrast, less traffic load in the network system results in the stabilized performance which is more important from the communication viewpoint in a disaster struck area. By assigning reward rate $r_{j_{EDF}}$ given by Equation (9), the numerical value for $EDF_0$ is obtained as 0.0000026. The numerical result for $EDF_a(t)$ is shown in Figure 10.

![Figure 9: Blocking Probabilities of RTS calls](image1)

![Figure 10: Excess Delay due to Failures](image2)

It is observed that $EDF_a(t)$ decreases when $n = 6$ and 7. This is explained from the fact that when fewer channels are available, most of the calls are blocked. In contrast, for $n = 8$ and 10, more calls are accepted and therefore, the delay experienced by the calls is more. From Figure 10, it is observed that the delay is expected to reduce with time because of the recovery of the failures.

6. Conclusions

This paper has dealt with survivability quantification of cellular networks in terms of the blocking probabilities of RTS and NRTS calls and excess delay due to failures. Assuming the exponential distributions for the failures and repairs, a survivability model has been presented as a SRN model. This SRN model allows for the time-dependent analysis of the network performance after the occurrence of a natural disaster. Numerical results show that blocking probability continues to increase gradually and this can be stabilized by shedding off the traffic load. It improves the connectivity for RTS calls. It was observed that the excess delay experienced by the arriving calls is reduced because fewer calls are allowed to access the network.

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