Redundancy Optimization using Intuitionistic Fuzzy Multi-Objective Programming

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Abstract: In this paper, redundancy allocation problem for system reliability has been presented with entropy as objective function. This model using entropy has been used to account for the lack of expert’s information. The goal of entropy redundancy allocation problem is to find out optimal level of redundancies at each subsystem in a way that maximizes reliability and entropy of the system subject to specified system cost. Intuitionistic fuzzy optimization technique is used to analyze an entropy based reliability redundancy optimization problem. Numerical examples have been provided to illustrate the model.

Keywords: Reliability optimization, intuitionistic fuzzy optimization, redundancy, entropy.

1. Introduction

Reliability of a multi-stage series-parallel system can be improved by adding similar components as redundancy to each sub-system [1]. The goal of redundancy allocation in such a system is to maximize system reliability under certain specified constraints on system resources and Misra et al [2-9] were the one of the earliest researchers to have provided several formulations and solution techniques to solve system reliability design problem. Several other authors [10-24] investigated redundancy optimization problem as a multi-objective optimization problem subject to several resource constraints using various techniques under different environments. Kuo and Wan [25] provides a broad overview of recent research on multi-objective reliability optimization problems and their solution methodologies.

In reliability redundancy allocation problem, entropy can represent the lack of the information about the state of the each sub-system. Few authors have discussed reliability analysis with entropy consideration. Musto and Saridis [26] presented entropy-based reliability assessment technique. The technique was demonstrated in a case study of a robotic system. Rocchi [27] introduced the entropy function in order to study the reliability and repairability of systems. Ridder [28] investigated the application and usability of the cross-entropy method for rare event simulation in Markovian reliability models. Rocchi [29] discussed and calculated the reliability function during system again through the stochastic entropy. Kroese et al. [30] introduced a cross-entropy method for optimization of network reliability.
Intuitionistic fuzzy set (IFS) was first introduced by Atanassov [31-33] and has been found to be well suited for dealing with problems concerning vagueness. The concept of IFS can be viewed as an alternative approach to define a fuzzy set (Zadeh [34]) in a situation where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. In fuzzy sets, the degree of acceptance is considered only but IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one (Atanassov [31]). IFS can be used to simulate human decision-making and any activity requiring human expertise and knowledge, which are inevitably imprecise. Angelov [35] proposed a framework of the optimization problem under uncertainty in intuitionistic fuzzy environment.

Mahapatra and Roy [36] applied multi-objective intuitionistic fuzzy non linear programming in reliability optimization model. However, there has been little research on multi-objective optimization using IFS, which is indeed in one of the most important areas in decision making as most real world decision problems involve multi-objective optimization problem. Use of intuitionistic fuzzy optimization (IFO) technique can be an appropriate approach in reliability optimization problems.

Usually, if the available data is not sufficient to assess system reliability accurately, entropy function can be used to catch the vagueness and subjectivity of expert’s information. In this paper, a multi-objective entropy reliability redundancy problem is considered, which not only maximizes the system reliability but also the entropy subject to available system cost. The redundancy optimization problem consisting of two objective functions, i.e., system reliability as well as entropy, is considered with a restriction on system cost. An optimal redundancy apportionment of multistage, multi-component system is considered using intuitionistic fuzzy multi-objective nonlinear programming (IFMONLP) technique.

2. Reliability Redundancy Allocation Model

Reliability redundancy allocation model is developed using the following notation:

- $R_i$: Reliability of each component in the $i^{th}$ stage
- $C_i$: Cost of each component in the $i^{th}$ stage
- $C$: Available system cost of the reliability model
- $x_i$: Number of redundancy components in the $i^{th}$ stage (decision variables)
- $R(x_1, x_2, ..., x_n)$: System reliability function
- $S(x_1, x_2, ..., x_n)$: Entropy function
- $C_s(x_1, x_2, ..., x_n)$: System cost function of the reliability model

2.1 Reliability Redundancy Allocation Problem

It is assumed that the system consists of an $n$ subsystem in series (as shown in Figure 1) and at each subsystem ($x_i-1$) redundant components in parallel are added to improve the system reliability. Then the objective is to determine the optimum number of redundant components at each subsystem such that the system reliability will be maximized subject to a specified cost constraint.

![Figure 1: A Schematic Diagram of an $n$-stage System](image-url)
Therefore the problem is to maximize system reliability (Rs) subject to a specified system cost C. Mathematically, the problem can be defined as:

\[
\text{Maximize } R_s(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} \left[ 1 - (1 - R_i)^{x_i} \right]
\]

subject to \( C_s(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} C_i x_i \leq C \)

\( x_i \geq 1 \) for \( i = 1, 2, \ldots, n \). All \( x_i \) being integer numbers.

2.2 Concept of Entropy for Redundancy Allocation

Entropy has an important physical implication as it signifies the amount of “disorder” of a system. A more abstract definition is used in mathematics.

The Shannon entropy of a variable \( X \) is defined as:

\[
S_s(X) = -\sum_{x} p(x) \ln p(x)
\]

where \( p(x) \) is the probability that \( X \) has the state \( x \), and \( p(x) \log p(x) \) is defined as 0 if \( p(x) = 0 \).

It is the redundancy distribution of each stage of a system. To determine a suitable measure of allocation, let us consider a \( n \)-stage system with \( x_i \) (\( i = 1, 2, \ldots, n \)) number of redundant component of each \( i^{th} \) stage of the system. It is known that \( x_i \) are positive integer and total number of components is \( \sum x_i \). Redundancy allocation of components share of \( i^{th} \) stage is the share of the total number of redundant component is \( p_i = \frac{x_i}{\sum x_i} \), which by normalizing the redundancy numbers \( x_i \) by dividing it by the total number of redundant components \( \sum x_i \), the probability distribution \( p_i = \frac{x_i}{\sum x_i} \) can be known.

The measure of allocation shall be defined as the expected information of the message which transforms the system shares into the share of each stage.

\[
S_s(x_1, x_2, \ldots, x_n) = -\sum_{i=1}^{n} p_i \ln p_i \quad \text{where } p_i = \frac{x_i}{\sum x_i}
\]

The each stage share \( p_i = \frac{x_i}{\sum x_i} \) satisfying the condition \( p_i \geq 0 \) (\( i = 1, 2, \ldots, n \)) and \( \sum p_i = 1 \) defines a probability distribution and the Shannon-entropy measure the diversity of the probability distribution \( \{p_1, p_2, \ldots, p_n\} \). Maximum is reached when \( p_1 = p_2 = \ldots = p_n = 1/n \) \( i.e., \) when allocation of all stage have the same number of redundant components. Since increasing of \( x_i \) maximizing \( \ln p_i \) is equivalent to maximizing entropy as defined above. This is one of the reasons why the entropy optimization model is particularly suitable for the redundancy allocation problem. In redundancy allocation problem, entropy acts as a measure of dispersal of allocation between stages. So it will be helpful, if we have maximum system reliability as well as maximum entropy as a measure.

2.3 Multi-objective Entropy Redundancy Allocation Problem

In reliability redundancy optimization problem, we have to allocate the redundancy level at each stage such that the entropy of the system is maximized. Therefore, the problem (1) can be reformulated as a multi-objective reliability redundancy optimization problem with maximization of system reliability and entropy of the system as follows:
Maximize \( R_i(x_1, x_2, ..., x_n) = \prod_i (1 - (1 - R_i))^\frac{1}{2} \) \( \) \( \) (4)  
Maximize \( S_i(x_1, x_2, ..., x_n) = -\sum_{i=1}^{n} \left( \sum_{j=1}^{m} \frac{1}{x_{ij}} \right) \) \log \left( \sum_{j=1}^{m} \frac{1}{x_{ij}} \right) \) \( \) Subject to same constraint and restriction as in (1) 

3. Intuitionistic Fuzzy Non-Linear Programming (NLP) Technique to solve Multi-objective Non-Linear Programming problem

A multi-objective non-linear programming (MONLP) takes the following form:
Maximize \( f(x) \equiv [f_1(x), f_2(x), ..., f_k(x)]^T \) \( \) (5) subject to \( x \in X = \{ x \in \mathbb{R}^n : g_j(x) \leq a \Rightarrow b_j \text{ for } j = 1, 2, ..., m; l_i \leq x_i \leq u_i \text{ for } i = 1, 2, ..., n \} \).

To solve the MONLP problem of (5), following Zimmermann [37] and Angelov [35] approach, a solution procedure is presented to solve the MONLP problem by IFO technique, the following steps are used:

**Step 1:** Solve the MONLP (5) as a single objective non-linear problem \( k \) times for each problem by taking one of the objective at a time and ignoring the others. These solutions are known as ideal solutions. Let \( x^i \) be the respective optimal solution for the \( i^{th} \) different objective and evaluate each objective values for all these \( i^{th} \) optimal solution. It is assumed that at least two of these solutions are different for which the \( i^{th} \) objective function has different bounded values.

**Step 2:** From the result of Step 1, determine the corresponding values for every objective for each derived solution. With the values of all objectives at each ideal solution, pay-off matrix can be formulated as follows:

\[
\begin{pmatrix}
 f_1(x) & f_2(x) & ... & f_k(x) \\
 f_1(x^1) & f_2(x^1) & ... & f_k(x^1) \\
 f_1(x^2) & f_2(x^2) & ... & f_k(x^2) \\
 \vdots & \vdots & \ddots & \vdots \\
 f_1(x^k) & f_2(x^k) & ... & f_k(x^k)
\end{pmatrix}
\]

Here \( x^1, x^2, ..., x^k \) are the ideal solutions of the objectives \( f_1(x), f_2(x), ..., f_k(x) \) respectively. For each objective \( f_i(x) \), find lower bound (minimum) \( L_i^m \) and the upper bound (maximum) \( U_i^{acc} \). But in IFO the degree of non-membership (rejection) and degree of membership (acceptance) are considered so that the sum of both values is less than one (Atanassov [31]). To define the non-membership function of MONLP problem, let \( L_i^{rej} \) and \( U_i^{acc} \) be the lower and upper bound of the objective function \( f_i(x) \) where \( L_i^{rej} \leq L_i^m \leq U_i^{rej} \leq U_i^{acc} \). Find the worst (\( U_i^{rej} \)) and the best (\( L_i^{acc} \)) values of each objective for the degree of acceptance and rejection corresponding to the set of solutions as \( U_i^{acc} = \max \{ f_i(x^i) \} \) and \( L_i^{rej} = \min \{ f_i(x^i) \} \) for \( i = 1, 2, ..., k \) for degree of acceptance of objectives.

**Theorem:** For objective function of maximization problem, the upper bound for non-membership functions (rejection) is always less than that the upper bound of membership functions (acceptance). For Proof of this theorem, see Appendix-A.
From the above theorem, we can determine the lower and upper bound for the non-membership function as follows:

\[ \mu(r_{i \text{acc}}^{j}) = \frac{U_{i \text{acc}}^{j} - L_{i \text{acc}}^{j}}{U_{i \text{acc}}^{j} - L_{i \text{acc}}^{j}} \]

Based on the decision maker choice and \( L_{i \text{acc}}^{j} = L_{i \text{acc}}^{j} \) for \( i = 1,2,\ldots,k \).

**Step 3:** The initial intuitionistic fuzzy model with aspiration levels of objectives becomes

Find \( \{x_{i}\}_{i=1}^{k} \) \( (6) \)

So as to satisfy

\[ f_{i}(x) \leq L_{i \text{acc}}^{j} \] with tolerance \( (U_{i \text{acc}}^{j} - L_{i \text{acc}}^{j}) \) for the degree of acceptance for \( i = 1,2,\ldots,k \)

\[ f_{i}(x) \geq U_{i \text{acc}}^{j} \] with tolerance \( (U_{i \text{acc}}^{j} - L_{i \text{acc}}^{j}) \) for the degree of rejection for \( i = 1,2,\ldots,k \)

Define the membership (acceptance) and non-membership (rejection) functions of above uncertain objectives as follow:

For the \( i^{th} \) \( (i=1,2,\ldots,k) \) objectives functions the membership function \( \mu(f_{i}(x)) \) and non-membership \( \nu(f_{i}(x)) \) are taken as following linear functions:

\[
\begin{align*}
\mu(f_{i}(x)) &= \begin{cases} 
0 & \text{if } f_{i}(x) \leq L_{i \text{acc}}^{j} \\
\frac{f_{i}(x) - L_{i \text{acc}}^{j}}{U_{i \text{acc}}^{j} - L_{i \text{acc}}^{j}} & \text{if } L_{i \text{acc}}^{j} < f_{i}(x) \leq U_{i \text{acc}}^{j} \\
1 & \text{if } f_{i}(x) \geq U_{i \text{acc}}^{j}
\end{cases} \\
\nu(f_{i}(x)) &= \begin{cases} 
0 & \text{if } f_{i}(x) \geq U_{i \text{acc}}^{j} \\
\frac{U_{i \text{acc}}^{j} - f_{i}(x)}{U_{i \text{acc}}^{j} - L_{i \text{acc}}^{j}} & \text{if } U_{i \text{acc}}^{j} < f_{i}(x) \leq L_{i \text{acc}}^{j} \\
1 & \text{if } f_{i}(x) \leq L_{i \text{acc}}^{j}
\end{cases}
\end{align*}
\]

Rough sketch of the membership function and non-membership function for maximization type objective function are shown in Figure 2.

**Figure 2:** Membership and Non-membership function of Objective Functions.

**Step 4:** IFO technique (Angelov, [35]) for MONLP problem with the membership and non-membership functions can be written as:

Maximize \( \mu(f_{i}(x)) \) \( (7) \)

Minimize \( \nu(f_{i}(x)) \)

Subject to \( \nu(f_{i}(x)) \geq 0 \)

\[ \mu(f_{i}(x)) \geq \nu(f_{i}(x)); \mu(f_{i}(x)) + \nu(f_{i}(x)) < 1 \]

\[ g_{j}(x) \leq b_{j}; \quad x \geq 0 \]

for \( i=1,2,\ldots,k; j=1,2,\ldots,m \).

If the decision-maker selects the additive operator, the problem to be solved is an equivalent crisp model by using the membership and non-membership functions of objectives by IFO as follows:

Maximize \( \sum_{i=1}^{k} \{\mu(f_{i}(x)) - \nu(f_{i}(x))\} \) \( (8) \)

Subject to same constraint and restriction as in (7)
Step 5: Solve the above (8) crisp model by an appropriate mathematical programming algorithm to get optimal solution.

4. Intuitionistic Fuzzy NLP technique to solve Multi-Objective Entropy Reliability Redundancy Allocation Problem

Maximum system reliability \( R_s(x) \) and maximum entropy \( S_s(x) \) has to be found, having subject to the system cost goal \( C \). So the problem is a multi-objective entropy based reliability redundancy allocation problem as follows:

\[
\text{Maximize } R_s(x) \\
\text{Maximize } S_s(x) \\
\text{Subject to } C_s(x) \leq C \\
x_i > 1 \text{ for } i=1,2,...,n.
\]

To solve the above MOROP (9), step 1 of section 3 is used. After that according to step 2 of section 3 pay-off matrix is formulated as follows:

\[
R(x) \\
S(x)
\]

\[
x^1 \begin{bmatrix} R(x) & S(x) \\ R(x') & S(x') \end{bmatrix} \\
x^2 \begin{bmatrix} R(x) & S(x) \\ R(x) & S(x') \end{bmatrix}
\]

Now \( U_1^{rev}, L_1^{rev}, U_2^{rev}, L_2^{rev} \) \( ( \text{where } L_1^{rev} \leq R_s(x)^{rev} \leq U_1^{rev} \text{ and } L_2^{rev} \leq S_s(x)^{rev} \leq U_2^{rev} ) \) and \( L_1^{acc}, U_1^{acc}, L_2^{acc}, U_2^{acc} \),

\[
( \text{where } L_1^{acc} = U_1^{acc} \text{ and } L_2^{acc} = U_2^{acc} \text{ and } L_1^{acc} = S_s(x)^{acc} \leq U_1^{acc} ) U_2^{acc} = U^{acc} - \varepsilon, \text{ for } i = 1,2; \\
\text{where } 0 < \varepsilon < (U^{acc} - U^{rev}) \)

are identified. Here for simplicity linear membership functions \( \mu_R(R_s(x)), \mu_S(S_s(x)) \) and linear non-membership functions \( \nu_R(R_s(x)), \nu_S(S_s(x)) \) for the objective functions \( R_s(x) \) and \( S_s(x) \) respectively, are defined as follows:

\[
\mu_R(R_s(x)) = \begin{cases} 
0 & \text{if } R_s(x) \leq L_2^{acc} \\
\frac{R_s(x) - L_1^{acc}}{U_1^{acc} - L_1^{acc}} & \text{if } L_1^{acc} \leq R_s(x) \leq U_1^{acc} \\
1 & \text{if } R_s(x) \geq U_1^{acc}
\end{cases}
\]

\[
\mu_S(S_s(x)) = \begin{cases} 
0 & \text{if } S_s(x) \leq L_2^{acc} \\
\frac{S_s(x) - L_1^{acc}}{U_2^{acc} - L_2^{acc}} & \text{if } L_1^{acc} \leq S_s(x) \leq U_2^{acc} \\
1 & \text{if } S_s(x) \geq U_2^{acc}
\end{cases}
\]

\[
\nu_R(R_s(x)) = \begin{cases} 
0 & \text{if } R_s(x) \geq U_1^{acc} \\
\frac{U_2^{acc} - R_s(x)}{U_2^{acc} - L_2^{acc}} & \text{if } U_1^{acc} \leq R_s(x) \leq U_2^{acc} \\
1 & \text{if } R_s(x) \leq L_1^{acc}
\end{cases}
\]

\[
\nu_S(S_s(x)) = \begin{cases} 
0 & \text{if } S_s(x) \geq U_2^{acc} \\
\frac{U_2^{acc} - S_s(x)}{U_2^{acc} - L_2^{acc}} & \text{if } U_1^{acc} \leq S_s(x) \leq U_2^{acc} \\
1 & \text{if } S_s(x) \leq L_1^{acc}
\end{cases}
\]

According to IFO technique step 6 of section 3, having elicited the above membership and non-membership function for MOROP (9) crisp NLP problem is formulated as follows:

\[
\text{Maximize } \{ \mu_R(R_s(x)) + \mu_S(S_s(x)) - \nu_R(R_s(x)) - \nu_S(S_s(x)) \} \\
\text{Subject to } \nu_R(R_s(x)), \nu_S(S_s(x)) \geq 0, \\
\mu_R(R_s(x)) \geq \nu_R(R_s(x)), \mu_S(S_s(x)) \geq \nu_S(S_s(x)), \\
\mu_R(R_s(x)) + \nu_R(R_s(x)) < 1, \mu_S(S_s(x)) + \nu_S(S_s(x)) < 1, \\
C_s(x) \leq C, \\
x_i > 1 \text{ for } i=1,2,...,n.
\]
Solve the above crisp model by an appropriate mathematical programming algorithm to get optimal solution of system reliability and entropy of the system.

5. Illustrative Example

A four stage entropy based reliability redundancy allocation problem with cost constraint is considered for numerical exposure. The problem becomes as follows:

\[
\begin{align*}
\text{Maximize } & \quad R_i(x_1, x_2, x_3, x_4) = \prod_{i=1}^{4} \left[1 - (1 - R_i) x_i \right] \\
\text{Maximize } & \quad S_i(x_1, x_2, x_3, x_4) = -\sum_{i=1}^{4} \left( S_i x_i \right) \log \left( S_i x_i \right) \\
\text{subject to } & \quad C_i(x_1, x_2, x_3, x_4) = \sum_{i=1}^{4} C_i x_i \leq C, \\
\text{and } & \quad x_i > 1 \text{ for } i=1,2,3,4.
\end{align*}
\]

Input parameters of the problem (11) are given in table 1.

<table>
<thead>
<tr>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$R_3$</th>
<th>$R_4$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85</td>
<td>0.9</td>
<td>0.8</td>
<td>0.95</td>
<td>7</td>
<td>7</td>
<td>5</td>
<td>9</td>
<td>120</td>
</tr>
</tbody>
</table>

Solution: According to step 2 of section 3 pay-off matrix is formulated as follows:

\[
\begin{bmatrix}
R_1(x) \\
S_1(x)
\end{bmatrix} =
\begin{bmatrix}
0.9996525 & 1.36163 \\
0.7163109 & 1.386294
\end{bmatrix}
\]

\[
\begin{bmatrix}
R_2(x) \\
S_2(x)
\end{bmatrix} =
\begin{bmatrix}
0.7163109 & 1.36163 \\
0.9996525 & 1.386294
\end{bmatrix}
\]

Here, $U_1^{\text{acc}} = 0.9996525$, $L_1^{\text{acc}} = 0.7163109$, $U_2^{\text{acc}} = 1.386294$, $L_2^{\text{acc}} = 1.36163$. $U_1^{\text{rej}} = 0.9996525 - \epsilon_1$, and $U_2^{\text{rej}} = 1.386294 - \epsilon_2$.

Here linear membership and non-membership functions for the objective functions $R_i(x)$ and $S_i(x)$ respectively, are define as follows:

\[
\mu_{R_i}(R_i) = \begin{cases}
0 & \text{for } R_i(x) \leq 0.7163109 \\
0.28346 & \text{for } 0.7163109 \leq R_i(x) \leq 0.9996525 \\
1 & \text{for } R_i(x) \geq 0.9996525
\end{cases}
\]

\[
\nu_{R_i}(R_i) = \begin{cases}
0 & \text{for } R_i(x) \leq 0.9996525 - \epsilon_1 \\
0.28346 - \epsilon_1 & \text{for } 0.9996525 - \epsilon_1 \leq R_i(x) \leq 0.9996525 \\
1 & \text{for } R_i(x) \geq 0.9996525 - \epsilon_1
\end{cases}
\]

and

\[
\mu_{S_i}(S_i) = \begin{cases}
0 & \text{for } S_i(x) \leq 1.36163 \\
0.02464 & \text{for } 1.36163 \leq S_i(x) \leq 1.386294 \\
1 & \text{for } S_i(x) \geq 1.386294
\end{cases}
\]

\[
\nu_{S_i}(S_i) = \begin{cases}
1 & \text{for } S_i(x) \leq 1.36163 \\
0.02464 - \epsilon_1 & \text{for } 1.36163 \leq S_i(x) \leq 1.386294 \\
0 & \text{for } S_i(x) \geq 1.386294 - \epsilon_1
\end{cases}
\]

Now IFO technique for MOROP (11) with this membership and non-membership functions can be solved for different value of $\epsilon_1$ and $\epsilon_2$. The optimal solution of the MONLP model (11) using IFMONLP technique is given in table 2. The optimal solution obtained by IFMONLP technique is compared with solution obtain by fuzzy multi-objective nonlinear programming (FMONLP) technique of the same MOROP (11) model.

<table>
<thead>
<tr>
<th>Method</th>
<th>$x_1^*$</th>
<th>$x_2^*$</th>
<th>$x_3^*$</th>
<th>$x_4^*$</th>
<th>$R_1^* (x^*)$</th>
<th>$S_1^* (x^*)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FMONLP</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>0.9977885</td>
<td>1.386294</td>
</tr>
<tr>
<td>IFMONLP</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>0.9989491</td>
<td>1.370502</td>
</tr>
</tbody>
</table>
Here, best solution is found for the tolerance $\varepsilon_1 = 0.02$ and $\varepsilon_2 = 0.0005$ for non-membership function of the objective functions. From the Table 2, it is shows that IFMOMLP technique gives better optimal result in the perspective of system reliability. It is noted that the model is a redundancy allocation problem, so out come must be integer; here results in table 2 are displayed after appropriate approximation of out come.

6. Conclusion

In this paper, IFO technique has been presented to solve the problem of multi-objective entropy based redundancy allocation problem. IFO technique can effectively deal with the vagueness and subjectivity of expert’s information. A decision-maker can obtain the optimal results according to his expectations of reliability costs. An illustrative example shows that when system reliability increases with the increase in redundancies level, also the entropy of the system increases, which is desirable.

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References

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Appendix A

Theorem: For objective function of maximization problem, the upper bound of non-membership function (rejection) is always less that the upper bound of membership function (acceptance).

Proof: Form the definition of IFS, sum of the degree of rejection and acceptance is less than unity.

\[ \mu_i(f(x)) + \nu_i(f(x)) \leq 1 \] for all \( i=1,2,\ldots,k \).

or \( \frac{f(x) - l_i}{u_i - l_i} \leq \frac{u_i - f(x)}{u_i - l_i} \leq 1 \)

Case I. If possible, let \( u_i = u_i^\mu \) then we have
\[ f(x) - L^x + \frac{U^x - f(x)}{U^x - L^x} < 1, \] this gives \( L^x < L^x \), which is contradicting the fact that lower bound of the membership and non-membership function is equal.

Hence \( U^x \neq U^x \)

Case II. Let us consider \( L^x = L^x \) then we have

\[ \frac{f(x) - L^x + U^x - f(x)}{U^x - L^x} < 1, \] which imply that \( U^x < U^x \)

Case III. Let us consider \( L^x = L^x + \varepsilon \) for all \( i=1,2,...,k \) then we have

\[ \frac{f(x) - L^x + U^x - f(x)}{U^x - L^x} < 1, \] which imply that \( U^x > U^x + \varepsilon \)

i.e., \( U^x > U^x \)

Hence \( U^x > U^x \) (QED).

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