Hybrid Poisson Processes with Fuzzy Rate

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Abstract: Poisson processes, particularly the time-dependent extension, play important roles in reliability and risk analysis. It should be fully aware that the Poisson modeling in the current reliability engineering and risk analysis literature is merely an ideology under which the random uncertainty governs the phenomena. In other words, current Poisson Models generate meaningful results if randomness assumptions hold. However, the real world phenomena are often facing the co-existence reality and thus the probabilistic Poisson modeling practices may be very doubtful. In this paper, we define the random fuzzy Poisson process, explore the related average chance distributions, and propose a scheme for the parameter estimation and a simulation scheme as well. It is expecting that a foundational work can be established for Poisson random fuzzy reliability and risk analysis.

Keywords: Poisson process, credibility measure, fuzzy variable, average chance distribution

1. Introduction

Vagueness is an intrinsic feature in today’s diversified business environments, just as Carvalho and Machado [1] commented, “In a global market, companies must deal with a high rate of changes in business environment. … The parameters, variables and restrictions of the production system are inherently vagueness.” Therefore the co-existence of random uncertainty and fuzzy uncertainty is inevitable reality of safety and reliability analysis and modeling.

It is a well-established fact that Poisson processes and particularly the non-stationary Poisson processes play important roles in safety and reliability modeling. Many researchers contributed to the probabilistic developments, see [2], [4], [5], [6], [7], [9], [10]. However, if fuzziness and randomness both appear then the traditional probabilistic modeling may be questionable. Therefore, developing the appropriate models for modeling fuzziness and randomness co-existence is necessary.

In this paper, we are trying to offer a systematic treatment for the random fuzzy Poisson processes not only in the mathematical sense (building models based on postulates and definitions) but also in the statistical sense (estimation and hypothesis testing based on sample data).
2. Foundation of Random Fuzzy Variable

Random fuzzy variable theory is established on the axiomatic credibility measure and probability measure foundations. Let us review the credibilistic fuzzy variable theory first.

Let $\Theta$ be a nonempty set, and $\mathcal{P}(\Theta)$ the power set on $\Theta$. Each element, let us say, $A \subseteq \Theta$, $A \in \mathcal{P}(\Theta)$ is called a fuzzy event. A number denoted as $\text{Cr}\{A\}$, $0 \leq \text{Cr}\{A\} \leq 1$, is assigned to event $A \in \mathcal{P}(\Theta)$, which indicates the credibility grade with which event $A \in \mathcal{P}(\Theta)$ occurs. $\text{Cr}\{A\}$ satisfies following axioms given by Liu (2004, 2007):

Axiom 1: $\text{Cr}\{\Theta\} = 1$.

Axiom 2: $\text{Cr}\{\} \leq \text{Cr}\{A\} \leq \text{Cr}\{B\}$, whenever $A \subseteq B$.

Axiom 3: $\text{Cr}\{\} = \text{Cr}\{A\} = 1$, whenever $A \in 2^\Theta$.

Axiom 4: $\text{Cr}\{\bigcup_{i} A_i\} = \sup_i \text{Cr}\{A_i\}$ for any $\{A_i\}$ with $\text{Cr}\{A_i\} \leq 0.5$.

Definition 1: (Liu [11], [12]) Any set function $\text{Cr}: \mathcal{P}(\Theta) \rightarrow [0,1]$ satisfies Axioms 1-4 is called a credibility measure. The triple $(\Theta, \mathcal{P}(\Theta), \text{Cr})$ is called the credibility measure space.

Definition 2: A fuzzy variable $\xi$ is a measurable mapping, i.e., $\xi: (\Theta, \mathcal{P}(\Theta)) \rightarrow (\mathbb{R}, \mathcal{B}(\mathbb{R}))$.

Definition 3: (Liu [11], [12]) The credibility distribution $\Lambda: \mathbb{R} \rightarrow [0,1]$ of a fuzzy variable $\xi$ on $(\Theta, \mathcal{P}(\Theta), \text{Cr})$ is

$$\Lambda(x) = \text{Cr}\{\theta \in \Theta \mid \xi(\theta) \leq x\}$$

(1)

It is necessary to stress that the fuzzy variable is not a fuzzy set in the sense of Zadeh’s fuzzy theory [13], [14], in which a fuzzy set is defined by a membership function.

Liu [11], [12] defined a random fuzzy variable as a mapping from the credibility space $(\Theta, \mathcal{P}(\Theta), \text{Cr})$ to a set of random variables. Our intuitive definition is similar to that of stochastic process in probability theory.

Definition 4: (Guo et al., [7]) A random fuzzy variable, denoted as $\xi = \{X_{\beta(\theta)}, \theta \in \Theta\}$, is a collection of random variables $X_{\beta}$ defined on the common probability space $(\Omega, \mathcal{A}, \text{Pr})$ and indexed by a fuzzy variable $\beta(\theta)$ defined on the credibility space $(\Theta, \mathcal{P}(\Theta), \text{Cr})$.

In random fuzzy variable theory, there are different types of chance measures proposed for characterizing random fuzzy events. What we are going to use is the average chance measure.
Definition 5: (Liu [11], [12]) Let $\xi$ be a random fuzzy variable, then the average chance measure denoted by $\text{ch}\{\cdot\}$, of a random fuzzy event $\{\xi \leq x\}$, is

$$
\text{ch}\{\xi \leq x\} = \int_0^1 \text{Cr}\left\{\theta \in \Theta | \text{Pr}\{\xi(\theta) \leq x\} \geq \alpha\right\}d\alpha
$$

Then function $\Psi(\cdot)$ is called as average chance distribution if and only if

$$
\Psi(x) = \text{ch}\{\xi \leq x\}
$$

Liu [11], [12] presented that an exponential density, $\beta e^{-\beta}$ having a fuzzy parameter $\beta$, then it characterizes a random fuzzy variable. We state Liu’s ideas formally as a theorem.

Theorem 1: Let $\zeta$ be a fuzzy variable defined on the credibility space $(\Theta, \mathcal{F}(\Theta), \text{Cr})$ and $\tau$ be a random variable defined on the probability space $(\Omega, \mathcal{A}(\Omega), P)$, then

1. Let $\oplus$ be an arithmetic operator, which can be “+”, “-”, “$\times$” or “$\div$” operation, such that $\zeta \oplus \tau$ maps from $(\Theta, \mathcal{F}(\Theta), \text{Cr})$ to a collection of random variables on $(\Omega, \mathcal{A}(\Omega), P)$, denoted by $\xi$. Then $\xi$ is a random fuzzy variable defined on hybrid product space $(\Theta, \mathcal{F}(\Theta), \text{Cr}) \times (\Omega, \mathcal{A}(\Omega), P)$.

2. Let $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be a continuous, such that $f(\zeta, \tau)$ maps from $(\Theta, \mathcal{F}(\Theta), \text{Cr})$ to a collection of random variables on $(\Omega, \mathcal{A}(\Omega), P)$, denoted by $\xi$. Then $\xi = f(\zeta, \tau)$ is a random fuzzy variable defined on hybrid product space $(\Theta, \mathcal{F}(\Theta), \text{Cr}) \times (\Omega, \mathcal{A}(\Omega), P)$.

3. Let $F(x; \theta)$ be the probability distribution of random variable $\tau$ with parameter $\theta$ (possible vector-valued), then $F(x; \zeta)$ defines a random fuzzy variable $\xi$ on the hybrid product space $(\Theta, \mathcal{F}(\Theta), \text{Cr}) \times (\Omega, \mathcal{A}(\Omega), P)$.

3. Homogeneous Random Fuzzy Poisson Processes

Similar to Poisson process definition Grimmett and Stirzaker [3], an intuitive formation of a random fuzzy Poisson process is to assume the intensity $\lambda$ to be a credibilistic fuzzy variable defined on credibility space $(\Theta, \mathcal{F}(\Theta), \text{Cr})$ with a credibility distribution function $\Lambda$.

Definition 6: A random fuzzy Poisson process with a fuzzy intensity $\lambda$ on credibility space $(\Theta, \mathcal{F}(\Theta), \text{Cr})$ is a process $N = \{N(t), t \geq 0\}$ taking values in $S = \{0, 1, 2, \cdots\}$ such that:

(a) $N(0) = 0$; if $s < t$, then $N(s) \leq N(t)$;

(b) $\text{Pr}\{N(t+h) = n + m | N(t) = n\}$
\[
\begin{align*}
\lambda(h + o(h)) & \quad \text{if } m = 1 \\
o(h) & \quad \text{if } m > 1 \\
1 - \lambda(h + o(h)) & \quad \text{if } m = 0
\end{align*}
\]

(c) if \( s < t \) then number \( N(t) - N(s) \) of an emission in the interval \( [s, t] \) is independent of the times of emissions during \( [0, s) \).

It is obvious that Definition 6 defines a stationary random fuzzy Poisson process.

**Theorem 2:** The successive inter-arrival (sojourn) times in a random fuzzy Poisson process \( N = \{N(t), t \geq 0\} \) with credibilistic fuzzy intensity \( \lambda \) having a piecewise linear credibility distribution

\[
\Lambda(x) = \begin{cases} 
0 & x \leq a \\
\frac{x - a}{2(b - a)} & a < x \leq b \\
\frac{1}{2} & b < x \leq c \\
\frac{x + d - 2c}{2(d - c)} & c < x \leq d \\
1 & x > d
\end{cases}
\]  

(4)

are i.i.d. random fuzzy variables with common average chance density:

\[
\psi(t) = e^{-at} - e^{-bt} + \frac{be^{-at} - ae^{-bt}}{2(b - a)t} + \frac{e^{-ct} - e^{-dt}}{2(c - d)t} + \frac{ce^{-ct} - de^{-dt}}{2(d - c)t}
\]  

(5)

**Proof:** Note that

\[
\Pr\{T(\lambda) \leq t\} = 1 - e^{-\lambda t}
\]  

(6)

Therefore event \( \{\theta : \Pr\{T(\lambda(\theta)) \leq t\} \geq \alpha\} \) is a fuzzy event and is equivalent to the fuzzy event \( \{\theta : \lambda(\theta) \geq -\ln(1 - \alpha)/t\} \). As a critical toward the derivation of the average chance distribution, it is necessary to calculate the credibility measure for fuzzy event \( \{\theta : \lambda(\theta) \geq -\ln(1 - \alpha)/t\} \), i.e., obtain the expression for

\[
\Cr\{\theta : \lambda(\theta) \geq -\ln(1 - \alpha)/t\}
\]  

(7)

Recall that for the credibilistic fuzzy variable, \( \lambda \), the credibility measure \( \Cr\{\theta : \lambda(\theta) \leq x\} = \Lambda(x) \) where \( \Lambda(\cdot) \) defined in Equation (4), accordingly, the range for integration with \( \alpha \) can be determined as shown in Table 1. Recall that the expression of \( x = -\ln(1 - \alpha)/t \) appears in Equations (7), which facilitates the link between intermediate variable \( \alpha \) and average chance measure.

The average chance distribution for the exponentially distributed random fuzzy lifetime is then derived by splitting the integration into five terms according to the range of \( \alpha \) and the corresponding mathematical expression for the credibility measure.
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Then the exponential random fuzzy lifetime has an average chance distribution function:

\[
\Psi(t) = \int_0^1 \text{Cr}\{\theta : \lambda(\theta) \geq -\ln(1-\alpha)/t\} d\alpha = 1 + \frac{e^{bt} - e^{-at}}{2(b-a)t} + \frac{e^{ct} - e^{-dt}}{2(d-c)t}
\]  
\[
(8)
\]

and the average chance density is

\[
\psi(t) = \frac{e^{at} - e^{bt}}{2(b-a)t^2} + \frac{be^{bt} - ae^{at}}{2(b-a)t} + \frac{ce^{ct} - de^{dt}}{2(d-c)t^2}.
\]  
\[
(9)
\]

This concludes the proof.

### Table 1: Range analysis for $\alpha$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\alpha$ and credibility measure expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-\infty &lt; x \leq a$</td>
<td>Range for $\alpha$</td>
</tr>
<tr>
<td></td>
<td>$0 \leq \alpha \leq 1 - e^{-at}$</td>
</tr>
<tr>
<td></td>
<td>$\text{Cr}{\lambda(\theta) \geq -\ln(1-\alpha)/t}$</td>
</tr>
<tr>
<td>$a &lt; x \leq b$</td>
<td>Range for $\alpha$</td>
</tr>
<tr>
<td></td>
<td>$1 - e^{-at} &lt; \alpha \leq 1 - e^{-bt}$</td>
</tr>
<tr>
<td></td>
<td>$\text{Cr}{\lambda(\theta) \geq -\ln(1-\alpha)/t}$</td>
</tr>
<tr>
<td>$b &lt; x \leq c$</td>
<td>Range for $\alpha$</td>
</tr>
<tr>
<td></td>
<td>$1 - e^{-bt} &lt; \alpha \leq 1 - e^{-ct}$</td>
</tr>
<tr>
<td></td>
<td>$\text{Cr}{\lambda(\theta) \geq -\ln(1-\alpha)/t}$</td>
</tr>
<tr>
<td>$c &lt; x \leq d$</td>
<td>Range for $\alpha$</td>
</tr>
<tr>
<td></td>
<td>$1 - e^{-ct} &lt; \alpha \leq 1 - e^{-dt}$</td>
</tr>
<tr>
<td></td>
<td>$\text{Cr}{\lambda(\theta) \geq -\ln(1-\alpha)/t}$</td>
</tr>
<tr>
<td>$d &lt; x &lt; +\infty$</td>
<td>Range for $\alpha$</td>
</tr>
<tr>
<td></td>
<td>$1 - e^{-dt} &lt; \alpha \leq 1$</td>
</tr>
<tr>
<td></td>
<td>$\text{Cr}{\lambda(\theta) \geq -\ln(1-\alpha)/t}$</td>
</tr>
</tbody>
</table>

Similar to the probabilistic reliability theory, we define a reliability function or survival function for a random fuzzy lifetime and accordingly name it as the average chance reliability function, which is defined accordingly. Then, for exponential random fuzzy lifetime, its average chance reliability function is

\[
\Psi(t) = 1 - \Psi(t) = \frac{e^{at} - e^{bt}}{2(b-a)t} + \frac{e^{ct} - e^{dt}}{2(d-c)t}.
\]  
\[
(10)
\]

### 4. Time-dependent Random Fuzzy Poisson Processes

In reliability engineering and risk analysis, the non-stationary Poisson processes enjoy wide applications because the intensity function is time-dependent. It is expected that the mathematical treatments may be much more complicated since the fuzzy functional nature of intensity when the parameters are credibilistic fuzzy variables. For a concrete discussion purpose, we narrow our attention to a linear intensity function:

\[
\lambda(t) = \beta_0 + \beta_1 t, \quad \beta_0 > 0, \quad \beta_1 > 0
\]  
\[
(11)
\]

Further, we assume that $\beta_0$ and $\beta_1$ both have piecewise linear credibility distribution:
Then the integrated intensity function (mean measure):

\[ m(t) = \beta_0 t + \beta_1 t^2 \]  

will have a credibility distribution:

\[ \Lambda_{m(t)}(y) = \begin{cases} 0 & y < a \\ \frac{y-a}{2(b-a)} & a \leq y < b \\ \frac{y+c-2b}{2(c-b)} & b \leq y < c \\ 1 & y \geq c \end{cases} \]  

(14)

where

\[ a = a_0 t + a_1 t^2 \]
\[ b = b_0 t + b_1 t^2 \]
\[ c = c_0 t + c_1 t^2 \]  

(15)

In general, the credibility distribution of the integrated intensity function \( m(t) \), it is necessary to apply Zadeh’s [14] extension principle, denoted as \( \Lambda_{m(t)} \), but for the piecewise linear credibility distribution case, the mathematical arguments are simpler. Now let us derive the average chance distribution for the inter-arrival times. Note that the average chance distribution for \( T_1 \), the first inter-arrival time, is

\[ \Psi_{T_1}(t) = \int_0^1 \text{Cr}\{\theta: m(t) \geq -\ln (1-\alpha)\} d\alpha \]  

(16)

where

\[ \text{Cr}\{m(t) > y\} = \begin{cases} 1 & y < a \\ \frac{2b-2-y}{2(b-a)} & a \leq y < b \\ \frac{c-y}{2(c-b)} & b \leq y < c \\ 0 & y \geq c \end{cases} \]  

(17)

Then
\[ \Psi_i(t) \] = \int_0^1 \text{Cr}( \theta : m(t) \geq -\ln(1 - \alpha)) d\alpha \\
= 1 - e^{-m(a)} + \frac{2b-a-1}{2(b-a)}(e^{-m(c)} - e^{-m(b)}) + \frac{1}{2(b-a)}(-m(b)e^{-m(b)} + m(a)e^{-m(a)}) \\
+ \frac{e-1}{2(c-b)}(e^{-m(b)} - e^{-m(c)}) + \frac{1}{2(c-b)}(-m(c)e^{-m(c)} + m(b)e^{-m(b)}) \\
(18)

Next let us derive the \( i^{th} \) inter-arrival time. Recall that conditioning on the \((i-1)^{th}\) occurrence time \( w_{i-1} \), the mean measure is

\[ m(t | w_{i-1}) = \beta_i(t - w_{i-1}) + \beta_i(t^2 - w_{i-1}^2) \]
(19)

Accordingly, the credibility distribution for \( m(t | w_{i-1}) \) is

\[ \Lambda_{m(t | w_{i-1})}(y) = \begin{cases} 
0 & y < a \\
\frac{y-a}{2(b-a)} & a \leq y < b \\
\frac{y+c-2b}{2(c-b)} & b \leq y < c \\
1 & y \geq c 
\end{cases} \]
(20)

where

\[ a = a_i(t - w_{i-1}) + a_i(t^2 - w_{i-1}^2) \]
\[ b = b_i(t - w_{i-1}) + b_i(t^2 - w_{i-1}^2) \]
\[ c = c_i(t - w_{i-1}) + c_i(t^2 - w_{i-1}^2) \]
(21)

Thus, the average chance distribution for the \( i^{th} \) inter-arrival time is:

\[ \Psi_i(t) = 1 - e^{-m(\text{a}_i | w_{i-1})} + \frac{2b-a-1}{2(b-a)}(e^{-m(\text{c}_i | w_{i-1})} - e^{-m(\text{b}_i | w_{i-1})}) \\
+ \frac{1}{2(b-a)}(-m(\text{b}_i | w_{i-1})e^{-m(\text{b}_i | w_{i-1})} + m(\text{a}_i | w_{i-1})e^{-m(\text{a}_i | w_{i-1})}) \\
+ \frac{e-1}{2(c-b)}(e^{-m(\text{b}_i | w_{i-1})} - e^{-m(\text{c}_i | w_{i-1})}) + \frac{1}{2(c-b)}(-m(\text{c}_i | w_{i-1})e^{-m(\text{c}_i | w_{i-1})} + m(\text{b}_i | w_{i-1})e^{-m(\text{b}_i | w_{i-1})}) \]
(22)

5. A Parameter Estimation Scheme

The parameter estimation is in nature an estimation problem of credibility distribution from fuzzy observations. Guo and Guo [8] recently proposed a maximally compatible random variable to a fuzzy variable and thus the fuzzy estimation problem is converted into estimating the distribution function of the maximally compatible random variable.

Definition 8: Let \( X \) be a random variable defined in \((\mathbb{R}, \mathcal{B}(\mathbb{R}))\) such that

\[ \mu^\prime = \text{Cr} \circ \xi^{-1} = \mu = P \circ X^{-1} \]
(23)
Then $X$ is called a maximally compatible to fuzzy variable $\xi$.
In other words, random variable $X$ can take all the possible real-values the fuzzy variable $\xi$ may take with and the distribution of $X$, $F_X(r)$ equals the credibility distribution of $\xi$, $\Lambda_\xi(r)$ for all $r \in \mathbb{R}$. It is expected that
\[
\{0 \in \Theta : X(\theta) \leq r\} \subseteq \{0 \in \Theta : \xi(\theta) \leq r\},
\]
but
\[
\Pr\{0 \in \Theta : X(\theta) \leq r\} = \Cr\{0 \in \Theta : \xi(\theta) \leq r\}.
\]
The statistical estimation scheme for parameters $(a,b,c)$ of the credibility distribution based on fuzzy observations $\{x_1,x_2,\ldots,x_n\}$ can be stated as:

**Estimation Scheme:**

**Step 1:** Rank fuzzy observations $\{x_1,x_2,\ldots,x_n\}$ to obtain “order” statistics $\{x_{(1)},x_{(2)},\ldots,x_{(n)}\}$ in ascending order;

**Step 2:** Set $\hat{a} = x_{(1)}$ and $\hat{c} = x_{(n)}$;

**Step 3:** Set a tentative estimator for $b$,
\[
\hat{b} = \frac{4{x_n} - x_{(1)} - x_{(n)}}{2}
\]
where
\[
{x_n} = \frac{1}{n}\sum_{i=1}^{n} x_i
\]

**Step 4:** Identify $x_{(k)}$ from $\{x_{(1)},x_{(2)},\ldots,x_{(n)}\}$ such that $x_{(k)} \leq \hat{b} < x_{(k)}$ and $1 < k < n$, then we may see $\{x_{(1)},x_{(2)},\ldots,x_{(n)}\}$ as a set of order statistics from uniform $(a,b)$. Hence the “sufficient” statistic for parameter $b$ is $X_{(k)}$.

Then $({\hat{a}}, {\hat{b}}, {\hat{c}}) = (x_{(1)}, x_{(k)}, x_{(n)})$ is the parameter estimator for the piecewise linear credibility distribution.

\[
\check{\lambda}(x) = \begin{cases} 
0 & x < \hat{a} \\
\frac{x - \hat{a}}{2(\hat{b} - \hat{a})} & \hat{a} \leq x < \hat{b} \\
\frac{\hat{c} - 2\hat{b}}{2(\hat{b} - \hat{c})} & \hat{b} \leq x < \hat{c} \\
1 & x \geq \hat{c}
\end{cases}
\]

The next issue is how to extract the information on intensity rate $\lambda$ in stationary random fuzzy Poisson process.
It is noticed that for probabilistic Poisson process case, the interpretation of intensity $\lambda$ is the occurrence rate in unit time. Based on such an observation, therefore, for any individual value $\lambda_0$ the fuzzy intensity may take, it results in a probabilistic Poisson process. Sample this Poisson process until $n_0$ events and record the total waiting time $w_{n_0}$, then $\hat{\lambda}_0 = n_0/w_{n_0}$ is an estimate of intensity $\lambda_0$. Repeat the sampling procedure from the random fuzzy Poisson process as many times as possible, say, $m$ times, then the intensity “observation” sequence is

$$\{\hat{\lambda}_1, \hat{\lambda}_2, \ldots, \hat{\lambda}_m\} = \left\{\frac{n_1^{(0)}}{w_0^{(0)}}, \frac{n_2^{(0)}}{w_0^{(0)}}, \ldots, \frac{n_m^{(0)}}{w_0^{(0)}}\right\}$$  \hspace{1cm} (29)

Apply the Estimation Scheme 1 to the estimated rate observations $\{\hat{\lambda}_1, \hat{\lambda}_2, \ldots, \hat{\lambda}_m\}$ the piecewise linear credibility distribution shown in Equation (28).

For the non-stationary random fuzzy Poisson process, the mean measure involves two linear piecewise credibility distributions for fuzzy parameters $\beta_0$ and $\beta_1$ respectively.

The scheme can state as follows:

**Step 1**: Sampling procedure from the random fuzzy Poisson process $N = \{N_i, t \geq 0\}$ $m$ times. Let the $i^{th}$ $n_i$ events the waiting time are $\{w_i^{(0)}, w_i^{(1)}, \ldots, w_i^{(m)}\}$.

**Step 2**: For the $i^{th}$ sample, perform the maximum likelihood estimation and obtain the parameter pair $\{\hat{\beta}_0^{(i)}, \hat{\beta}_1^{(i)}\}$ which is regarded as the fuzzy parameters taking values. Repeat the estimation process until all $m$ MLE pairs $\{\hat{\beta}_0^{(i)}, \hat{\beta}_1^{(i)}\}, i = 1, 2, \ldots, m$ are obtained.

**Step 3**: Applying the Estimation Scheme 1 to fuzzy sequences $\{\beta_0^{(0)}, \beta_0^{(1)}, \ldots, \beta_0^{(m)}\}$ and $\{\beta_1^{(0)}, \beta_1^{(1)}, \ldots, \beta_1^{(m)}\}$ respectively, the parameters $\hat{a}_0, \hat{b}_0, \hat{c}_0$ and $\hat{a}_1, \hat{b}_1, \hat{c}_1$ define the two piecewise linear credibility distributions for $\beta_0$ and $\beta_1$ respectively.

**6. Conclusion**

In this paper, we give a systematic treatment of random fuzzy Poisson processes not only from the stationary one and then non-stationary one, but also a parameter estimation scheme as well as a simulation scheme is proposed. In this way, the foundation for the random fuzzy Poisson processes is formed although in its infant stage, particularly, the time-dependent random fuzzy Poisson process. The applications to reliability engineering fields and the risk analysis now can extend from random uncertainty only cases to randomness and fuzziness co-existence cases. It is expecting that this development will help the reliability and risk analysis researchers as well as reliability analysts and engineers.

**References**


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