An Empirical Expression for Reliability Index of Flanged RC Beams in Limit State of Deflection

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Abstract: Reliability indices of reinforced concrete flange beams with respect to limit state of deflection designed as per the provisions of IS 456:2000 are found to be non-uniform. Through this paper, an attempt has been made to propose an empirical expression for reliability index of a flanged RC beam in limit state of deflection. This equation will be useful in deciding depth of member for a given span, load with target reliability index in limit state of deflection and also to obtain an estimate of reliability index of the designed RC beam.

Keywords: Serviceability, AFOSM, Reliability index, shrinkage and creep.

1. Introduction

Use of reliability theory and probabilistic concepts is well established as a basis for defining design criteria for the safety of concrete structures against collapse. The values of various safety factors have been developed through probabilistic analysis and code calibration procedures with an objective of producing approximately uniform levels of safety across a wide range of design conditions. On the contrary, the design for deflection control has for many years been based on either minimum thickness criteria or deflection calculations and prescribed limits on calculated deflections based on previous experience. While current design practice appears to produce designs that would perform satisfactorily for most of the cases, it can be expected that reliability with respect to serviceability is not uniform and that these criteria may not be adequate as the current procedures are getting refined and modern construction materials are being used, leading to slender members. Preliminary study of performance of various beams with respect to limit state of deflection when designed for limit state of safety as per the provisions of IS 456:2000 are not often consistent (uniform) [17]. Reliability analysis of beams in serviceability limit state with respect to deflection using Advanced First Order Second Moment (AFOSM) method also leads to non uniform (inconsistent) performance of beams.

In this paper an attempt has been made to establish an empirical expression for reliability index of flanged RC beam in limit state of deflection in terms of span, load and effective depth. This can be utilized in deciding the depth of a member for target reliability index. Also to determine the reliability index of the designed beam of a given span and loading conditions to get uniform performance in limit state of deflection.

Literature survey on probabilistic analysis have always focused on ultimate strength rather than serviceability limit states [1,2,3,4,5].
In fact, very few probability deflection design provisions have been developed and primarily these have been available for structures with steel and timber \cite{6,7} and reinforced concrete \cite{8,9,10,12}.

Notation

- $L$: Span of the beam
- $g()$: Failure surface function
- $\Delta_i$: Instantaneous deflection
- $\Delta_{\text{creep}}$: Creep deflection
- $\Delta_{\text{shrinkage}}$: Shrinkage deflection
- $W_i$: Total load
- $W_p$: Permanent load
- $W_d$: Dead load
- $W_l$: Live load
- $E_e$: Short term Young’s modulus
- $E_{et}$: Long term Young’s modulus
- $I_e$: Effective short term Moment of Inertia
- $I_{et}$: Effective long term Moment of Inertia
- $\varepsilon_{cs}$: Ultimate shrinkage strain of concrete

2. Preliminary estimate of reliability index

The serviceability limit state equation for maximum deflection with the resistance ($R$) and load effect ($S$) is written as \cite{11, 13, 14}

$$Z = R - S = \Delta_{\text{lim}} - \Delta_{\text{cal}}$$  \hspace{1cm} (1)

Where,

- $\Delta_{\text{lim}}$: Limiting deflection specified in code
- $\Delta_{\text{cal}}$: Max. Calculated deflection

The IS 456:2000\cite{15} Clause 23.2 specifies two limiting deflection states namely

1. The final deflection of a structure due to all the effects of temperature, creep and shrinkage should not exceed span/250
2. The deflection including the effects of temperature, shrinkage and creep occurring after erection of partitions shall not exceed span/350 or 20mm whichever is less.

Of the above two limiting states of deflection first limiting state is used for reliability analysis. So failure surface equation becomes

$$g(\ ) = \frac{L}{250} - \left( \Delta_i + \Delta_{\text{creep}} + \Delta_{\text{shrinkage}} \right)$$  \hspace{1cm} (2)

Substituting the values of $\Delta_i$, $\Delta_{\text{creep}}$, $\Delta_{\text{shrinkage}}$ in the above equation, we obtain

$$g(\ ) = \frac{L}{250} - \frac{5L^4}{384} \left( \frac{W_i}{E_e I_e} + \frac{W_p}{E_{et} I_{et}} - \frac{W_d}{E_{et} I_{et}} \right) - k_3 * k_4 * \varepsilon_{cs} + \frac{L^2}{D}$$  \hspace{1cm} (3)

Failure surface equation contains the variables like $I_e$, $I_{et}$, $W_p$ the statistical parameters of which are dependent on statistical parameters of basic variables. Therefore, reliability analysis is carried out in two phases

Phase 1: By knowing the statistical parameters of the basic variables, the statistical parameters of dependent variables are estimated by Monte Carlo simulation technique \cite{11, 14}. (Basic variables are those variables statistical parameters of which are available in
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For example by knowing the statistical parameters of \( W_i \) and \( W_d \) the statistical parameters of \( W_p \) can be obtained by the relation \( W_p = W_d + 0.3W_i \). The summary of details of available statistical parameters of the basic variables is shown in Table 1.

**Table 1:** Available details of statistical parameters of basic variables.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient of variation</th>
<th>Type of distribution</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width ((bw))</td>
<td>0.03</td>
<td>N</td>
<td>[11], Darwin et al. (1998)</td>
</tr>
<tr>
<td>Depth ((D))</td>
<td>0.05</td>
<td>N</td>
<td>Mirza and Macgregor</td>
</tr>
<tr>
<td>Concrete strength</td>
<td>0.15</td>
<td>N</td>
<td>Mirza et al. (1979)</td>
</tr>
<tr>
<td>Reinforcement cover</td>
<td>0.10</td>
<td>N</td>
<td>[11], Mirza et al. (1979)</td>
</tr>
<tr>
<td>Amount of steel reinforcement</td>
<td>0.10</td>
<td>N</td>
<td>[11]</td>
</tr>
<tr>
<td>Length ((L))</td>
<td>0.03</td>
<td>N</td>
<td>[11]</td>
</tr>
<tr>
<td>Live load ((W_l))</td>
<td>0.20</td>
<td>N, LN Type I</td>
<td>Ellingwood et al. (1980), [11].</td>
</tr>
<tr>
<td>Dead load ((W_d))</td>
<td>0.10</td>
<td>N</td>
<td>Darwin et al. (1978).</td>
</tr>
<tr>
<td>Width of flange</td>
<td>0.05</td>
<td>N</td>
<td>[11]</td>
</tr>
<tr>
<td>Depth of flange</td>
<td>0.03</td>
<td>N</td>
<td>[11]</td>
</tr>
</tbody>
</table>

**Phase 2:** Once by knowing the failure surface equation and deciding the statistical parameters of various variables involved in failure surface equation, reliability index can be obtained as follows;

**Step 1:** Formulation of failure surface equation. Failure surface equation is already formulated and is given by equation #3.

**Step 2:** Statistical parameters of variables involved in the failure surface equation. In all eight variables are considered namely \( W_p, W_l, E_t, E_c, D, I_t, I_c, L \).

**Step 3:** Obtain the initial design point; Substitute all the mean values of parameters in equation for limit state surface except \( W_t \) and equate it to zero to get \( W_t \). This ensures that design point is on failure boundary.

**Step 4:** For each design point values \( X_i \) corresponding to non-normal distribution determine the equivalent normal mean and standard deviations. In our case equivalent normal mean and standard deviation are determined for \( I_t, I_c \).

**Step 5:** Determine the reduced variables corresponding to the design point using

\[
Z_{\text{LM}} = \frac{(x_i - mx_i)}{sx_i} \quad \text{(4)}
\]

**Step 6:** Write limit state equation in terms of reduced variables

\[
g(\hat{X}) = \frac{(z_8^8sL + mL)^4}{250} - \frac{5(z_8^8sL + mL)^4}{384} \left( \frac{z_2^8sW + kW_j}{p} \right) \left( \frac{z_1^8sW + kW_j}{p} \right) + \left( \frac{z_4^8sE + mE_j}{p} \right) \left( \frac{z_6^8sI + mI_j}{p} \right) + \frac{5(z_8^8sL + mL)^4}{384} \left( \frac{z_1^8sW + kW_j}{p} \right) \left( \frac{z_4^8sE + mE_j}{p} \right) \left( \frac{z_6^8sI + mI_j}{p} \right) - \frac{128k_4^8sE + mE_j}{p} \left( \frac{z_8^8sL + mL)^2}{p} \right) \left( \frac{z_6^8sI + mI_j}{p} \right) (5)
\]

**Step 7:** Determine the partial derivatives of limit state function with respect to the reduced variants

\[
g_i(\hat{X}) = -\frac{\partial g(\hat{X})}{\partial \xi_i} \quad \text{(6)}
\]
Step 8: Calculate an estimate of $\beta$ using formula

$$\beta = \left( \frac{(G^T \gamma^* + \xi)}{\sqrt{(G^T \gamma^* + G)}} \right)$$  \hspace{1cm} (7)

Step 9: Calculate the vector containing sensitivity factor using

$$\alpha = \left( \frac{(G)}{\sqrt{(G^T \gamma^* + G)}} \right)$$  \hspace{1cm} (8)

Step 10: Determine the new design point values in reduced variants using

$$Z_i = \alpha_i \cdot \beta$$  \hspace{1cm} (9)

Step 11: Repeat steps 3-10 till $\beta$ and design point converge.

A MATLAB program was developed to perform the above calculation. Reliability index is determined for various beams with span ranging from 4m-10m. Variations of reliability indices for different span to depth ratio is studied and is observed that different beams will provide target reliability index of 3 when span to depth ratio is in between 11 to 13 for different loads acting on the beams. Hence data base of reliability index for each beam with span/depth ratio ranging from 11 to 13 and loading ranging from 15KN/m to 50KN/m is generated. Cross sectional details of the flanged beam are shown in Figure 1. Specimen details of parameters of the beam for a span of 4m and span/depth ratio of 11 are given in Table 2.

**Table 2:** Details of parameters of the beam for a span of 4m and span/depth ratio of 11.

<table>
<thead>
<tr>
<th>bw</th>
<th>D</th>
<th>ds</th>
<th>As</th>
<th>wd</th>
<th>wl</th>
<th>Bf</th>
<th>Df</th>
<th>RI</th>
<th>RI*</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>365</td>
<td>40</td>
<td>477.4125</td>
<td>12.1</td>
<td>6</td>
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<td>110</td>
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<td>365</td>
<td>40</td>
<td>559.5934</td>
<td>12.1</td>
<td>9</td>
<td>1000</td>
<td>110</td>
<td>3.2024</td>
<td>3.1514</td>
</tr>
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<td>40</td>
<td>726.8301</td>
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<td>110</td>
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<td>40</td>
<td>810.5279</td>
<td>24.05</td>
<td>6</td>
<td>1000</td>
<td>110</td>
<td>2.751</td>
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<tr>
<td>250</td>
<td>365</td>
<td>40</td>
<td>896.6891</td>
<td>24.05</td>
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<td>1000</td>
<td>110</td>
<td>2.783</td>
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<tr>
<td>250</td>
<td>365</td>
<td>40</td>
<td>1072.3349</td>
<td>24.05</td>
<td>15</td>
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<td>110</td>
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<tr>
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<td>365</td>
<td>40</td>
<td>587.1958</td>
<td>15.1</td>
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<td>1000</td>
<td>125</td>
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<td>3.1254</td>
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<td>670.647</td>
<td>15.1</td>
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<td>1000</td>
<td>125</td>
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<td>3.0474</td>
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<td>883.6962</td>
<td>15.1</td>
<td>17.5</td>
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<td>125</td>
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<td>40</td>
<td>826.2441</td>
<td>18.6</td>
<td>12</td>
<td>1000</td>
<td>140</td>
<td>2.9122</td>
<td>2.9044</td>
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<tr>
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<td>365</td>
<td>40</td>
<td>1059.0021</td>
<td>18.6</td>
<td>20</td>
<td>1000</td>
<td>140</td>
<td>2.6406</td>
<td>2.6964</td>
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<tr>
<td>250</td>
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<td>40</td>
<td>1050.2464</td>
<td>30.304</td>
<td>8</td>
<td>1000</td>
<td>140</td>
<td>2.6125</td>
<td>2.7041</td>
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<td>365</td>
<td>40</td>
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<td>30.304</td>
<td>12</td>
<td>1000</td>
<td>140</td>
<td>2.5746</td>
<td>2.6001</td>
</tr>
<tr>
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<td>365</td>
<td>40</td>
<td>1414.8642</td>
<td>30.304</td>
<td>20</td>
<td>1000</td>
<td>140</td>
<td>2.4105</td>
<td>2.3921</td>
</tr>
</tbody>
</table>

RI  Calculated reliability by AFORM method
RI*  Calculated reliability index by proposed expression
3. Data base of reliability indices

- Approach consists of developing sufficient data for the reliability indices of flanged RC beam
- In this study, flanged beams with spans ranging from 4m to 10m and spacing of beams ranging from 3 to 4m are considered
- For each span of the beam four cases of span/depth ratios namely 11,11.5,12,12.5 are considered
- For each span and span/Depth ratio 18 load cases ranging from 15 to 50KN/m are considered. Reliability indices are determined for above designed beams.

A specimen plot of Load verses reliability index for span of 5m, span to depth ratio of 11.5 and range of loads is shown in Figure 2.

**Figure 2:** Plot of load V/S reliability index for span of 5m and span to depth ratio of 11.5

This procedure is repeated for different spans, span to depth ratios and loading cases and the results along with the corresponding equation of the regression fit are shown in the Figures 3 to 9.
From the plots of reliability indices for various spans following inferences can be made

- Reliability index increases with increase in span
- Reliability index decreases with increase in load
- Changes in reliability index with changes in span/depth ratio is consistent (same pattern) as can observed in above figures

Based on various regression fit equations connecting the reliability indices with load level and span/depth ratio for each span as written in above mentioned figures and further manipulation an equation can be proposed for reliability index of a flanged reinforced concrete beam for a given span and loading as follows

$$ RI = C + 0.6 \left( 11 - \frac{L}{D} \right) - \left( 0.026 + 0.005 \times \left( \frac{L}{D} - 11 \right) \right) \times W, $$

where,

$ RI = $ Reliability index in limit state of deflection

$ C = $ Parameter depends on span length.

Values of $ C $ are given in table below:

<table>
<thead>
<tr>
<th>L</th>
<th>4m</th>
<th>5m</th>
<th>6m</th>
<th>7m</th>
<th>8m</th>
<th>9m</th>
<th>10m</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>3.7</td>
<td>4.0</td>
<td>4.2</td>
<td>4.3</td>
<td>4.5</td>
<td>4.5</td>
<td>4.5</td>
</tr>
</tbody>
</table>

5. Conclusion

Reliability index of flanged reinforced concrete beams in limit state of deflection is determined by using advanced first order second moment method. It is found that reliability index of beams designed as per IS code provisions for safety is non uniform with respect to limit state of deflection. Based on data base of reliability index of various beams an empirical expression is developed for the reliability index of flanged reinforced beams. This equation will be useful in estimating the reliability index of a designed beam and also in estimating the span/depth ratio for target reliability index.

References


**K. Manjunath** has a Ph.D. in Engineering. After a brief professional experience for two years in Karnataka Power Corporation Ltd., as design engineer, he joined Malnad College of Engineering, Hassan and has been serving the institute for the past 26 years. Apart from teaching UG and PG students, he is actively engaged in research and consultancy. His area of interest in research is in the field of reliability analysis and reliability based design of structures, performability and durability of structures, time dependent reliability analysis of RC elements.

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