Common Cause Failure Model of System Reliability Based on Bayesian Networks

YIN XIAOWEI*
Shenyang Institute of Engineering, Shenyang, 110136, P. R. China

(Submitted on Oct. 15, 2008 and made available by Guest Editor on Aug. 25, 2009)

Abstract: Common cause failure is an important phenomenon for a system with failure dependent parts. In this paper, several common cause failure models are analyzed and compared. A new common cause failure model for system reliability estimation is presented based on Bayesian Networks. Examples of series system, parallel system and series-parallel system are given to explain how to use the model to evaluate the reliability of system, through which the weak parts of the system can be identified. Also, Monte-Carlo simulation method is used to estimate the reliability based on Bayesian Networks, the results of which are compared with the system reliability under failure independence assumptions. The simulation results show that the reliability model based on Bayesian Network is consistent with the traditional qualitative analysis which proves that the Bayesian Networks model is accurate and valid.

Keywords: System reliability model, Bayesian networks, common cause failure, failure independence, numerical simulation.

1. Introduction

With the advancement of science and technology, more complicated systems, such as power plants, have appeared and require a high reliability, i.e., a very low probability of failure. There are statistical techniques that can predict the reliability of a complex system based on its composition and the reliability of each component. However, some traditional techniques for reliability analysis have several important limitations, including the assumption that all the failures are independent.

In order to improve system reliability, many kinds of redundancy technology are proposed. Under complicated conditions the failures of components in a system are not independent.
The dependence of the failures influences the system reliability significantly. Dependence is a common feature of failures for complex systems. Common cause failure (CCF) is an important mechanism for the dependent failure of the parts of a system. Calculating system reliability under the assumptions of independent failure will introduce serious error.

Conventional reliability estimation methods, such as Fault Tree Analysis (FTA) and Reliability Block Diagram (RBD), have been widely used. However, these methods do not consider the dependence of the failures between components. Some assumptions need to be made when applying these methods, leading to inaccurate results. It is also difficult to estimate the change of system reliability when one or more component fails. Therefore, it is necessary to develop new methods to estimate the system reliability [1-3].

Bayesian Networks (BN) is widely used now in many fields such as economics, artificial intelligence etc. It is easy for BN to represent the uncertainty of variables. BN has the inherent ability to gain the conditional failure probability of one component or several components of a system [4-6]. This paper aims to develop a computational method that can incorporate explicitly dependencies between failures in the reliability analysis of complex systems in operation. A Bayesian network is used to represent the system reliability structure, and obtain its reliability via failure propagation. With this representation, the limitations of other techniques are avoided, so it is possible to manage dependencies.

2. CCF Model

Though there is no widely accepted definition at present, CCF includes the basic meaning that it is a multi-failure event caused by a single exterior source. Since the 1970s, many reliability models have been proposed to solve the problems of CCF, such as \( \beta \)-factor model, \( \alpha \)-factor model, Multi-Greece Letter Model (MGLM), Binomial Failure Rate Model (BFRM) etc.

The \( \beta \)-factor model is the first parameter model to be applied to the risk assessment and reliability analysis [7]. In this model, independent failure and CCF are all included. Independent failure means that one component’s failure does not affect the other components. CCF means that one component’s failure affects the other components. \( \beta \)-factor model is applicable to the redundancy system which consists of only two components. Because the failure rates of middle components are zero when there are more than two components in the system.

Multi-Greece Letter Model (MGLM) and \( \alpha \)-factor model were proposed by Fleeting and Mosleh [8], respectively. These models both consider an arbitrary quantity of components failure. With more components in system, the parameter estimation becomes
more complex.

Binomial Failure Rate model (BFRM) considers two kinds of failures: one is the independent failure under the normal loads and the other is dependent failure caused by impact loads. BFRM is better than β-factor model but it cannot apply to the probabilistic risk assessment of the Nuclear Industry because of the limitation of assumption and complexity of parameter estimation.

Another kind of models, known as physical models include Common Load Model (CLM) [9], Distributing Failure Probability Model (DFPM) which emphasize the influence of circumstance stress on dependent failure. In Stochastic Reliability Analysis Model (SRAM), the diversity in components performance causes the failure dependence. The knowledge Based on Multidimensional Discrete Model (KBMDM) was proposed by Xie which considers both circumstance loads and components performance [10]. Physical models are better than other ones because the causes of system failure can be embodied in the models so that defense measures can be formulated.

Xie suggested a CCF model in [11] and proposed two methods to develop reliability models: explicit modeling method and implicit modeling method. Both methods can analyze the system reliability when considering the CCF. The explicit modeling method needs to calculate the minimize cut set which will increase the amount of calculation for big and complex systems. The implicit modeling method requires that the components in a common cause group have the same failure probability which may be correct for electronic components but not for mechanical systems. Wang Xuemin introduced exponential power into building a CCF model of system reliability which is more complex with more components [12].

3. Bayesian Networks

BN, also called Bayesian Belief Network (BBN), are graphical method, sometimes known as a directed acyclic graph (DAG). The random variables are denoted by nodes and the directed arcs represent the conditional dependencies among the nodes. Each node has a probability density function associated with it. The arc emanates from a parent node to a sub-node. A node without any arcs linking into it is known as a root node, and a node with arrows linking into it is known as a sub-node. A sub-node without any arcs leading out is a leaf node. Each sub-node thus carries a conditional probability density function, given the value of the parent node. The entire BN can be represented using a joint probability density function [13-16].

According to Bayesian Formula:
Suppose there are $n$ states $a_1, a_2, \ldots, a_n$, and from the total probability formula we can get:

\[
P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}
\]

So we can get the posterior probability from Bayesian Formula.

BN can not only do forward reasoning, which means to obtain the posterior probability through the prior probability, but also can do backward reasoning, obtaining prior probability through the posterior probability. BN can deduce the causes from the results. The reasoning methods of BN comprise accurate and approximate methods.

A simple decision of BN is shown in Fig.1. The relations between the $C$ (Cloudy), $S$ (Sprinkler), $R$ (Rain) and $W$ (Wet Grass) are expressed using BN. The additional Conditional Probability Table (CPT) shows the causal relationship among the nodes. For the example of the additional CPT for node $R$, when $C$ is false, namely no cloudy weather, the probability of no rain (-$R$) is 0.8 the probability of raining $R$ is 0.2. When $C$ is true, namely cloudy weather, the probability of no rain -$R$ is 0.2 and the probability of raining $R$ is 0.8. The interpretation of CPT for other nodes are the same.

Figure 1: A simple BN

The unique structure of BN includes a powerful conditional independence. As the parent node of node $A$ is given, which is independent of other nodes except its sub-nodes. By utilizing the conditional independence the joint probability distribution can be simplified.

According to the chain rule, the joint probabilities of all nodes in Fig.1 can be determined.

\[
P(C,S,R,W) = P(C)P(S \mid C)P(R \mid C,S)P(W \mid C,S,R)
\]

Utilizing the characteristics of conditional independency, all joint probabilities can be simplified to:

\[
P(C,S,R,W) = P(C)P(S \mid C)P(R \mid C)P(W \mid S,R)
\]
A joint probability distribution can be expressed using BN with different structures. Pearl found that one structure corresponds to peoples’ thinking habits mostly, namely the parent nodes of a node can be thought to be a direct reason to the node (what does it mean?). The BN which is composed by cause and effect relation is very similar to the thinking procedure of people. According to this relationship, a more meaningful explanation can be provided. As shown in Fig.1, the dependence between C and R can be explained as that C is the direct cause for R.

### 4 System Reliability Model Based on BN

The FTA method is often used to assess system reliability. When we assess a system using FTA, a minimized cut set must be determined first, but it is a baldness and complex job. Especially when we assess the reliability of a complex system, it is unnecessary to obtain minimized cut sets of the system. In fact, the structure of fault tree corresponds to that of BN one-by-one. If the fault trees of a system have been determined, it is easy to convert fault trees to BN. There are three principles to follow. Firstly, the root nodes of a BN correspond to the basic events of a fault tree; secondly the middle nodes correspond to the logic gates of a fault tree; and lastly, the conditional probability distributions represent the relations of the gates.

As an example to illustrate the steps to build system reliability model [17-19], a series-parallel system of three valves $V_1$, $V_2$ and $V_3$ is considered. Fig.2a is the Reliability Block Diagram (RBD) of the system. Every valve is good when it works and is bad when it does not. Usually we can build the fault tree of the system as in Fig.2b, where T represents the top event of the fault tree, $X_i$ ($i=1,2,3$) represents the basic event and M is a middle event of the fault tree.

In the meantime, a BN model can be built as in Fig.2c, where $X_i$ ($i=1,2,3$) is a root node, $m$ is the middle node and $t$ is a leaf node of BN. Each node of BN has a CPT and the variables in CPT represent the actual state. The state “1” means the component is in good condition and state “0” means the component failed. For this system, $m$ and $x_3$ can be thought as sub-systems.

When the BN model of the system is built, the reliability can be calculated by several methods. Here the method of bucket propagation is used as follows.

$$P(t=1) = \sum_{X_1, X_2, X_3, m} P(X_1, X_2, X_3, m, t) =$$

$$\sum_{X_1, X_2, X_3} P(t=1|m, X_3) \sum_{X_1, X_2} P(m|X_1, X_2) P(X_1) P(X_2) =$$

$$\sum_{X_1, X_2, X_3} P(t=1|m, X_3) P(X_1 = 1) P(X_2 = 1) =$$

$$1 - (1 - P(X_1 = 1) P(X_2 = 1)) P(X_3 = 0) \right). \quad (3)$$

If the failure probability of every valve is known, it is easy to calculate the system reliability.
reliability through above formula. It is thus clear that BN model complies with the basic principle of mechanical system reliability.

Figure 2: Modeling procedure of a Series-Parallel system
a) RBD of the system         b) Fault tree of the system         c) BN model of the system

5. Procedure for System Reliability CCF Modeling

BN has been applied to the fault diagnosis field for its bidirectional reasoning function and conditional independence. BN has the advantages that other reliability analysis methods haven’t in studying dependent system reliability.

The key of using BN to construct a CCF model of system reliability is to divide the failure rate $\lambda$ of common cause component into independent failure rate $\lambda_1$ and CCF rate $\lambda_2$. That is to divide the common cause component into independent failure part and CCF part, which are in series. The logical relationship between common cause component and other parts is analyzed according to the RBD of the system, such as in series or parallel et al. In the following, the procedure to construct a CCF model of system reliability based on BN will be showed by some typical CCF systems in mechanical system.
5.1 Series system

When a system fails if one component of the system fails, the system is called series system. Series system is the commonest and simplest model of systems. RBD of series system is shown in Fig.3 and the mathematical model is

\[ R_s(t) = \prod_{i=1}^{n} R_i(t) = \prod_{i=1}^{n} e^{-\lambda_i(t) t} \]  

(4)

Where, \( R_s(t) \rightarrow \) reliability of the system, \( R_i(t) \rightarrow \) reliability of component \( i \), \( \lambda_i(t) \rightarrow \) failure rate of component \( i \), \( n \rightarrow \) number of components in the system.

![Figure 3: RBD of series system](image)

When considering CCF the reliability model of two-component system based on BN is shown in Fig.4. \( C \) is the second order failure gene and \( S_1 \) and \( S_2 \) are the first order failure genes respectively. \( S_i \) and \( C \) are in series. \( A_i \) is middle nodes. They are the new state of components when considering CCF. \( A_1 \) and \( A_2 \) are in series.

![Figure 4: The CCF model of 2-component series system](image)

The mathematical expression of system reliability is

\[ p(x = 0) = \sum_{c_i} p(s_i, c, c_i, c, c_i) p(c_i = 0 | s_i, c) p(c_i) \sum_{c_i} p(c_i = 0 | s_i, c) p(c_i) p(s_i) p(c) ] \]

\[ = p(s_1 = 0) p(s_2 = 0) p(c = 0) \]

Since \( S_1 = S_2 \), equation (5) can be simplified to,

\[ p(x = 0) = p^2 (s_1 = 0) p(c = 0) \]

When considering CCF the reliability model of three-component system based on BN is shown in Fig.5. \( C_{ij} \) is the second order failure gene of components \( i \) and \( j \). \( C_{ijk} \) is the...
third order failure gene of components \( i, j \) and \( k, S_1, S_2 \) and \( S_3 \) are the first order failure gene of components 1, 2 and 3. The middle nodes \( A_i \) are the new state of components when considering CCF. They are in series.

\[ \begin{align*}
S_1, S_2 \text{ and } S_3 & \text{are the first order failure gene of components } 1, 2 \text{ and } 3. \\
\text{The middle nodes } A_i & \text{are the new state of components when considering CCF. They are in series.}
\end{align*} \]

Since \( S_1 = S_2 = S_3 \), \( C_{12} = C_{13} = C_{23} \), equation (7) can be simplified to,

\[ \begin{align*}
p(x = 0) = p(x = 0 | a_1, a_2, x) & = \sum_{s_1, s_2} p(s_1 = 0 \mid s_2 = 0, x) \sum_{c_{12}, c_{13}, c_{23}} p(c_{12} = 0 \mid s_1, c_{12}, c_{13}, c_{23}) \cdot p(s_2 = 0 \mid s_1 = 0, c_{12}, c_{13}) \cdot p(c_{23} = 0 \mid c_{12}, c_{13}) \\
& = p(s_1 = 0) \cdot p(s_2 = 0) \cdot p(s_3 = 0) \cdot p(c_{12} = 0) \cdot p(c_{13} = 0) \cdot p(c_{23} = 0) \cdot p(c_{123} = 0)
\end{align*} \]

Similarly, the mathematical expression of system reliability is

\[ \begin{align*}
p(x = 0) & = \sum_{c_{12}, c_{13}, c_{23}} p(s_1 = 0 \mid c_{12}, c_{13}, c_{23}) \cdot p(s_2 = 0 \mid c_{12}, c_{13}) \cdot p(s_3 = 0 \mid c_{23}) \\
& = \sum_{c_{12}, c_{13}, c_{23}} [p(s_1 = 0 \mid c_{12}, c_{13}) \cdot p(s_2 = 0 \mid c_{12}, c_{13}) \cdot p(s_3 = 0 \mid c_{23})]
\end{align*} \]

Since \( S_1 = S_2 = S_3 \), \( C_{12} = C_{13} = C_{23} \), equation (7) can be simplified to,

\[ \begin{align*}
p(x = 0) & = p^3(s_1 = 0) \cdot p^3(c_{12} = 0) \cdot p(c_{13} = 0)
\end{align*} \]

\[ \begin{align*}
5.2 \text{ Parallel system}
\end{align*} \]

When a system fails if and only if all components of the system fail, the system is called parallel system. Parallel system is the simplest desired model. RBD of parallel system is shown in Fig.6 and the mathematical model is

\[ \begin{align*}
R_p(t) & = 1 - \prod_{i=1}^{n} [1 - R_i(t)]
\end{align*} \]

The meanings of the variables are the same as those of the series system. When considering CCF the reliability model of two-component system based on BN is shown in Fig.7. \( C \) is the second order failure gene and \( S_1 \) and \( S_2 \) are the first order failure genes.
respectively. $S_i$ and $C$ are in series. $A_i$ are middle nodes. They are the new state of components as considering CCF. $A_1$ and $A_2$ are in parallel.

$$p(x = 0) = \sum_{c_1,c_2} p(s_1,c_1,s_2,c_2,x)$$

$$= \sum_{c_1,c_2} \left[ p(x = 0|c_1,s_2,c_2) \sum_{c_1} p(c_1 = 0|s_1,c) p(s_1) \right] \sum_{c_2} [p(c_2 = 0|s_2,c) p(s_2) p(c)]$$

$$= 2p(s_1 = 0)p(c = 0) - p^2(s_1 = 0)p(c = 0)$$

When considering CCF the reliability model of three-component system based on BN is shown in Fig.8. The middle nodes $A_i$ are the new state of components when considering CCF. They are in parallel.
\[ p(x = 0) = \sum_{c_1, c_2, c_3} p(s_1, c_{12}, s_2, c_{13}, c_{23}, s_3, a_1, a_2, a_3, x) \]
\[ = \sum_{a_1, a_2, a_3} \left\{ p(x = 0 | a_1, a_2, a_3) \sum_{s_1, s_{12}, s_{13}, s_2, s_{23}, s_{13}} \left[ p(a_1 = 0 | s_1, c_{12}, c_{13}, c_{23}, s_1, s_{12}, s_{13}) p(c_{12}) p(c_{13}) p(c_{23}) \right] \right\} \]
\[ = 3 p(s_1 = 0) p(c_{12} = 0) p(c_{13} = 0) p(c_{23} = 0) p(s_2 = 0) p(c_{12} = 0) - 3 p(s_1 = 0) p(s_2 = 0) p(c_{12} = 0) p(c_{13} = 0) p(c_{23} = 0) p(s_3 = 0) p(c_{12} = 0) \]
\[ p(c_{13} = 0) p(c_{23} = 0) p(c_{12} = 0) p(c_{23} = 0) + p(s_3 = 0) p(s_2 = 0) p(s_3 = 0) p(c_{12} = 0) \]
\[ p(c_{13} = 0) p(c_{23} = 0) p(c_{12} = 0) p(c_{23} = 0) = 0 \] (11)

Since \( S_i = S_1 = S_2 = S_3 \)
\( C_{ij} = C_{12} = C_{13} = C_{23} \),
\[ p(x = 0) = 3 p(s_i = 0) p^3(c_{ij} = 0) p(c_{ijk} = 0) - 3 p^3(s_i = 0) p^3(c_{ij} = 0) p(c_{ijk} = 0) + \]
\[ p^3(s_i = 0) p^3(c_{ij} = 0) p(c_{ijk} = 0) \] (12)

### 5.3 Series-Parallel System

Without considering the CCF, the reliabilities of the five-component of series-parallel system in Fig.9 are represented by \( P_1, P_2, P_3, P_4 \) and \( P_5 \) respectively which all equal \( P(t) \). Components 1 and 2 constitute a common cause group and the failure rate of each component is \( \lambda \).

**Figure 9: RBD of series-parallel system**

1. Without considering the CCF
\[ R_s = P_1 (P_1 + P_2 - P_1 P_2) (P_3 + P_4 - P_3 P_4) = (P_1 P_3 + P_2 P_4 - P_1 P_3 P_2 P_4) (P_1 + P_2 - P_1 P_2) = P_1 P_3 P_4 + P_1 P_3 P_5 + P_1 P_4 P_5 + P_2 P_3 P_4 + P_2 P_3 P_5 - P_1 P_3 P_2 P_4 - P_1 P_4 P_2 P_3 - P_1 P_2 P_3 P_4 - P_1 P_2 P_4 P_5 + P_1 P_2 P_3 P_4 P_5 = 4 P^3(t) - 3 P^4(t) \] (13)

2. When considering the CCF the reliability model based on BN is shown in Fig.10. Nodes \( A_1 \) and \( A_2 \) are new state nodes after considering the common cause gene \( C \).
Figure 10: BN model of series-parallel system

5.4 Applications to Reliability Analysis

5.4.1 Two-Component Series system

A series system consists of two components. The reliabilities of components are $P_1$ and $P_2$ respectively. $P_1$ equals $P_2$ which equals $P$ under the assumption of independent failure. The failure rate of system $\lambda_1$ equals 0.002 when one component fails and the failure rate of system $\lambda_2$ equals 0.0002 when two components fail. Now let $R_s(t)$ represents the system reliability without considering the CCF and $R_c(t)$ represents the system reliability when considering the CCF. With the given information $R_s(t)$ and $R_c(t)$ can be determined when $t=100h$, as follows,

Without considering the CCF, the reliability of component $P$ is

$$P = P_1 = P_2 = \exp(- (\lambda_1 + \lambda_2)t) = 0.8025$$

The reliability of system $R_s(t)$ is

$$R_s(t) = P_1P_2 = P^2 = \exp(-2(\lambda_1 + \lambda_2)t) = 0.6440$$

Then construct the CCF model of series system reliability with BN as shown in Fig.4. So when considering the CCF the reliability of a component is

$$P_s = P_{s1} = P_{s2} = \exp(-\lambda_1t) = 0.8187$$

$$P_c = \exp(-\lambda_2t) = 0.9845$$

where, $P_s$ is the reliability of component when only one component fails and $P_c$ is the reliability of component as considering CCF.

According to the inference relation of BN the system reliability when considering the
CCF is \( R_s(100) = 0.6599 \).

### 5.4.2 Two-Component Parallel system

A parallel system consists of two components. The reliabilities of components are \( P_1 \) and \( P_2 \) respectively. \( P_1 \) equals \( P_2 \) which equals \( P \) under the assumption of independent failure. The failure rate of system \( \lambda_1 \) equals 0.002 when one component fails and the failure rate of system \( \lambda_2 \) equals 0.0005 when two components fails. Now let \( R_s(t) \) represents the system reliability without considering the CCF and \( R_c(t) \) represents the system reliability when considering the CCF. With this information \( R_s(t) \) and \( R_c(t) \) can be determined when \( t=100h \), as follows.

Without considering the CCF, the reliability of component is

\[
P = P_1 = P_2 = \exp(- (\lambda_1 + \lambda_2) t) = 0.7788
\]

The reliability of the system \( R_s(t) \) is

\[
R_s(t) = P_1 + P_2 - P_1 P_2 = 2P - P^2
\]

\[
= 2\exp(-\lambda_1 t) - \exp(-2\lambda_1 t)
\]

\[
= 0.9511
\]

Then construct the CCF model of parallel system reliability with BN as shown in Fig. 7. So when considering the CCF the reliability of a component is

\[
P_s = P_{s1} = P_{s2} = \exp(-\lambda_1 t) = 0.8187
\]

\[
P_c = \exp(-\lambda_2 t) = 0.9512
\]

where, \( P_s \) is reliability of component when one component fails only and \( P_c \) is the reliability of component as considering CCF.

According to the inference relation of BN, the system reliability when considering CCF is

\[
R_c(100) = 0.9199
\]

The results of above two examples are shown in Fig. 11, together with comparison with those from Monte Carlo simulation.

As can be seen from Fig.11, \( R_s(t) \) is greater than \( R_c(t) \) at the same time in series system but opposite is true in parallel system, which is traditionally in accordance with the qualitative analysis. The calculated results with BN model are of high precision which is in full agreement with the simulation results.
6 Conclusion and Future Research

In this paper, a general methodology for modeling the reliability of complex systems based on BN is presented. It has been shown in the paper that a RBD of a system can be transformed to a BN representation, so as the reliability of the system can be determined using the probability propagation technique. Also, the system reliability CCF model is presented and applied to estimate the dependent system reliability. Monte-Carlo simulation method is used to prove that the model is in accordance with the qualitative analysis of system reliability.

It is acknowledged that the CCF model is built under the assumption that events are binary states, i.e., working or not-working. In many industry applications, systems and components have several states from ideal state to complete failure with different degrees of failure in between. Therefore only considering the binary events can cause serious errors. This is indeed for further research, in which the multi-state events should be considered in reliability analysis.

Acknowledgment

This work was supported by Scientific Research Fund of Liaoning Provincial Education Department (Grant No. 2009A543).

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Yin Xiaowei is a lecturer at Shenyang Institute of Engineering, China. She is a senior member of Chinese Mechanical Engineering Society. She received her Ph.D. in Mechanical Engineering from the Northeastern University, China. Her research interests include reliability engineering, risk assessment and fault diagnosis systems.