A Flow Importance Measure with Application to an Italian Transmission Power System

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Abstract: Potential catastrophic events have been historically assessed through the techniques of power system security analysis. A correct assessment must consider not only events associated with “credible outages” but also the possibility of attacks and/or “rare” events. This paper combines concepts of reliability engineering and operations research to develop a new importance measure, \( D(\alpha) \), that identifies the most vulnerable group of \( \alpha \) links in a network. Since the computation of the new importance measure, \( D(\alpha) \), becomes computationally prohibitive for large systems, an approximation technique is provided based on the optimization of a multi-objective deterministic network interdiction problem. The efficiency of the algorithm is then illustrated by analyzing the Italian high-voltage electrical transmission network. In summary, \( D(\alpha) \) can be useful for an initial screening of the critical elements of the network.

Keywords: power system security, interdiction, multi-objective evolutionary optimization, contingency screening.

1. Introduction

As described by Moteff et al [1] critical infrastructures are those infrastructures so vital to a nation that their disruption or destruction entails a debilitating impact on its defense or economic security. Given that it sustains the economic and societal-well-being of a nation, the electric power grid is one of those infrastructures. To exemplify its importance consider that during the New York City (NYC) blackout on August 14th 2003 activity related to emergency medical services and hospital activity dramatically intensified due to unexpected increases of respiratory device failures in community-based patients [2]. Moreover, as suggested by Prezant et al [2], “…current capacity to respond to public health emergencies could be easily overwhelmed by widespread/prolonged power failure(s).” Events such as the NYC blackout show that critical infrastructures -and in particular the energy infrastructure- can be sensitive to partial or complete incapacitation, due to internal or external sources of failures or attacks. For internal failure sources, reliability engineering and risk analysis have provided tools and procedures for estimating, preventing and handling undesired failure events that occur at random in complex systems.

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As discussed by Chena et al [3] in spite of the analyses done with these tools and procedures, and of the investments in system reliability and security that followed, the US power transmission grid suffered more than 400 major blackouts in the 16 years from 1984 to 1999.

On the other hand, external sources of failures constitute a new challenge when considering the rare but potentially catastrophic involvement of “…malevolent intelligence directed towards maximum social disruption” [4].

Traditionally, for power and infrastructure systems, researchers have been interested in performance indicators or figures-of-merit (FOM) relating the effects of component outages on overall system rates [5-7, 21]. Potential catastrophic events have been historically assessed through the techniques of power system security analysis. Among these, contingency analyses evaluate “single or multiple equipment failure events one after another until all credible outages have been studied” [5]. However, under both external and internal failure scenarios, a correct assessment must consider not only contingencies associated with “credible outages” but also the possibility of intentional attacks and/or “rare” events. Since these attacks or rare events, along with the damage caused, are hard to quantify in a probabilistic manner, this new perspective adds an extra layer of complexity to classical studies that consider a pre-specified set of contingencies to be studied -for example, an “N-1” criterion- with their associated probability of occurrence.

To address this gap, the research presented in this paper combines concepts of reliability engineering and operations research. In particular, a new importance measure, $D(\alpha)$, is introduced to identify the most vulnerable group of $\alpha$ links in a network, i.e. that when incapacitated or interdicted network flow is reduced the most.

In safety and reliability engineering, component importance measures (IM) quantify the criticality of a particular component in the design and operation of a system [8, 9]. IM are widely used for identifying system weaknesses and prioritizing reliability improvement activities. Moreover, IM can also provide valuable information for the safety and operation of a system.

Since the actual computation of the new IM, $D(\alpha)$, is computationally prohibitive for large systems, an approximation technique is provided based on the optimization of a multi-objective (MO) deterministic network interdiction problem (DNIP). The optimization provides the network links that need to be interdicted so as to minimize the maximum flow between two specified nodes in a network –usually called source and sink- while keeping the total interdiction cost within a specified budget. The DNIP is here implemented as a surrogate approach to model the interaction between link incapacitation (due to rare events or external attacks) and the maximum flow in the network thus, in effect, computing $D(\alpha)$ for a given value of $\alpha$. Finally, a method of power systems vulnerability analysis by Rocco et al [10] is used to accurately approximate $D(\alpha)$ for any value of $\alpha$.

The remainder of the paper is organized as follows. Section 2 introduces the importance measures for flow networks. Section 3 reviews the implementation of the DNIP in the context of this work and its transformation to a MO problem. Section 4 presents the implementation of the IM to the Italian high-voltage (380 kV) electrical transmission network. Finally, Section 5 presents the conclusions of the work.
Acronyms
NYC New York City
FOM Figures of Merit
IM Importance Measures
MO Multi-Objective
DNIP Deterministic Network Interdiction Problem
HVET High Voltage Electrical Transmission
PSDA Probabilistic Solution Discovery Algorithm
MOEA Multi-Objective Evolutionary Algorithm
DC Direct Current

2. Flow Importance Measures

Let \( G(N, A) \) represent a capacitated network with known source node \( s \) and target node \( t \). \( N \) represents the set of nodes and \( A=A_1 \cup A_2 \) where, \( A_1=\{(s,i), (j,t) \mid i \neq j, \neq s, \neq t \} \) and \( A_2=\{(i,j) \mid i \neq j, \neq s, \neq t \} \) represents the set of all links connecting the source, target and the other network nodes. Associated with each link \((i,j)\) is a capacity defined by \( k_{ij} \). The network state vector \( x=(x_1, x_2, \ldots, x_w, \ldots, x_{|A|}) \) denotes the state of all the links in the network where \( x_w=0 \) if link \((i,j)\) is down (no flow is transmitted), \( x_w=1 \) otherwise. Without loss of generality it has been assumed that every link in set \( A \) is indexed by \( w=1,2,\ldots,|A| \). The function \( F: \mathbb{Z}^{|A|} \rightarrow \mathbb{Z}^+ \) maps a network state vector \( x \), into a network flow (load) between \( s \) and \( t \). In this paper, the maximum flow reduction between \( s \) and \( t \) after incapacitation of \( \alpha \) components is taken as an indicator of network vulnerability.

Based on this description, Equation 1 defines the Flow Reduction importance measure of link \( w \), \( FR(w) \), as the maximum flow transmitted in the network from source node \( s \) to target node \( t \), when \( w \) is incapacitated or interdicted.

\[
FR(w) = F(x) - F(x \mid x_w = 0)
\]  

where, \( FR(w) \) can take values in the interval \([0, F(x)]\).

Any numerical algorithm can be used to determine the maximum flow in the network for a given configuration \( x \). For example, an implementation of the Ford-Fulkerson algorithm can be used [11], but other methods can be implemented depending on the network/application characteristics.

The importance measure \( FR \) is analogous to the Birnbaum IM used to characterize the role of components in engineered systems and plants [12]. However, as defined in (1), it accounts only for the failure of single components. A generalization to multiple failures of \( \alpha \) components is:

\[
FR(w_1, w_2, \ldots, w_j, \ldots, w_{\alpha}) = F(x) - F(x \mid x_{w_1} = x_{w_2} = \ldots = x_{w_j} = \ldots = x_{w_{\alpha}})
\]  

(2)

In (2), \( w_j \) indexes the components in the network where \( w_j \in \{1, 2, \ldots, |A|\} \) and \( w_1 \neq w_2 \neq \ldots \neq w_j \neq \ldots \neq w_{\alpha} \). Also, \( \alpha \) describes the number of incapacitated components to be considered.

Based on (2), the vulnerability of the network \( G(N, A) \), when \( \alpha \) components are incapacitated is described by the vulnerability set:

\[
V(\alpha) = \{FR(w_1, w_2, \ldots, w_j, \ldots, w_{\alpha}) \mid w_1 \neq w_2 \neq \ldots \neq w_j \neq \ldots \neq w_{\alpha}\}
\]  

(3)

The set presented in (3) describes for every different combination of component failures of size \( \alpha \), the corresponding distribution of the network reduction in flow. Note
that, for example, for a network with \(|A|\) components considering \(\alpha\) failures, \(V(\alpha)\) could have a maximum cardinality of \(\frac{|A|!}{\alpha!(|A|-\alpha)!}\).

The group of \(\alpha\) links that reduce the network flow from source node \(s\) to target node \(t\) the most, can then be identified by:

\[
D(\alpha) = \arg\max_{w_1 \neq w_2 \neq \ldots \neq w_\alpha} \{V(\alpha)\}
\]  

\(4\)

Finally, FR(\(D(\alpha)\)) identifies the maximal reduced flow for a group of \(\alpha\) links. From the computational perspective, two aspects must be considered in the calculation of \(D(\alpha)\): 1) the maximum flow in \(G(N,A)\) between \(s\) and \(t\) must be calculated for every group of incapacitated links of cardinality \(\alpha\) and, 2) the calculations must be performed for every value of \(\alpha\), which entails \(C_\alpha = \frac{|A|!}{\alpha!(|A|-\alpha)!}\) number of computations, where \(C_\alpha\) is the number of computations.

These computational challenges are addressed based on a transformed multi-objective DNIP.

3. Computation of \(D(\alpha)\) via DNIP Optimization

The DNIP considers a flow network with known source-sink configuration and known nominal flow transmitted between any two nodes. A known cost \(c_w\) is associated with the interdiction of each link \(w_j \in \{1, 2, \ldots, |A|\}\) and a specified interdiction budget is available [13]. For simplicity of illustration, it is assumed that \(c_w = 1\) for every \(w\).

A first optimization scheme to obtain \(D(\alpha)\) considers as objective function the network \(s-t\) flow, \(F(x)\), and constraints on the expenditures for interdiction, \(C(x)\), and the physical flow conservation:

**Optimization Scheme 1:**

\[
\begin{align*}
\text{Min } & F(x) \\
\text{s.t. } & C(x) = \alpha, \text{ where } C(x) = \sum_{w=1}^{|A|} c_w (1 - x_w) \\
\text{Flow Conservation Equations (as dictated by the implementation)} \\
x_w \in \text{Bin (0,1)}
\end{align*}
\]

To obtain \(D(\alpha)\) for every possible \(\alpha\), optimization scheme 1 must be solved \(|A|\) times. To bypass this computational burden, evolutionary algorithms for the solution of MO optimization problems can be used. MO optimization has been proposed as an approach to solve the problem of finding solutions for mathematical models that have multiple objective functions to be optimized. Unlike optimization models with a single objective function, the interest is on finding a set of solutions that describe how the improvement of a single objective function value impacts the value of the other objectives. This set is commonly known as the Pareto-optimal set and each of its elements as a Pareto optimal solution.

In this respect, a second optimization scheme can be devised, for a general formulation of the multi-objective problem of network interdiction:

**Optimization Scheme 2:**

\[
\begin{align*}
\text{Max } & f(x) = \{f_1(x), f_2(x), \ldots, f_L(x)\}
\end{align*}
\]
\[ \text{Min } g(x) = (g_1(x), g_2(x), ..., g_O(x)) \]
\[ \text{s.t. } \]
\[ f_l(x) \geq F_l \forall l = 1, ..., L' \]
\[ g_o(x) \leq G_o \forall o = 1, ..., O' \]
\[ x_i \in \text{Bin}(0,1), x_i \text{ is an element of } x. \]

In this new scheme, vectors \( f(x) \) and \( g(x) \) describe objective functions to be maximized and minimized, respectively. Similarly, the first and second sets of constraints describe possible constraints on network performance and resources; finally, the last constraint dictates the decision variables to be binary.

A solution \( x' \) that satisfies the constraints, is called Pareto optimal if:

\[ (f(x') > f(x') \lor g(x') < g(x')) : \text{for some } l \]
\[ \text{or } o) \land \text{does not } \exists x'' \forall (f(x'' \geq f(x') \lor g(x'') \leq g(x')) : \forall l \text{ and } o) \tag{5} \]

To obtain \( D(\alpha) \) and its corresponding \( \text{FR(D(\alpha))} \), the surrogate MO model should consider minimizing the network’s maximum flow and minimizing the total expenditures while satisfying necessary constraints (for example, flow conservation constraints). A solution to such MO would provide for very possible failure the minimum flow from source \( s \) to sink \( t \), a flow that can easily be used to obtain \( \text{FR(D(\alpha))} \). Optimization scheme 3 describes such a model:

**Optimization Scheme 3:**

\[ \text{Min } F(x) \text{ and Min } C(x) \]
\[ \text{s.t. } \]
\[ \text{Flow Conservation Equations} \text{ (as dictated by the implementation)} \]
\[ x_i \in \text{Bin}(0,1) \]

To solve optimization scheme 3, a Multiple-Objective Evolutionary Algorithm (MOEA) is used. MOEA is a term employed in the evolutionary multi-criteria optimization field to refer to a family of evolutionary algorithms formulated to deal with MO. MOEA are able to deal with non-continuous, non-convex and/or non-linear objectives/constraints, and objective functions possibly not explicitly known (e.g. the output of Monte Carlo simulation). The development of these algorithms has successfully evolved, producing efficient algorithms like M-PAES [14] or PESA-II [15] among others.

The algorithm used in this paper (known as MO-PSDA) [10] offers a simple, intuitive and efficient approach for the solution of the MO-DNIP, with a minimum number of tuning parameters. The flow of the algorithm is presented in Figure 1 and for a complete description of it the interested reader is referred to [10].

4. The Italian High-voltage (380 kV) Electrical Transmission Network

The Italian high-voltage (380 kV) electrical transmission network (HVIET) can be represented by an undirected graph of 310 nodes and 361 lines. HVIET has been analyzed from a topological viewpoint [16] and in detail in [17]. The latter work illustrates the results of a vulnerability analysis based on a dynamical model of the network, reproducing the power flow conditions. The results obtained showed that “…only a small number of nodes having a "functional" relevance for the network can be discovered through the topology analysis of its graph".
Finally, the authors remarked “topological analysis and the simulation of "functional" models (such as the DC power flow model, for the case of electrical networks) provide complementary information”.

It is important to mention that there are numerous approaches to obtain the network flow in power systems. For example, in [18, 19] the authors utilized a DC load flow model to characterize the behavior of the power system, recognizing that this simplification may lead to optimistic results. Nevertheless, the simplification significantly reduces the computational burden associated with a complete power systems interdiction study, which must include analyzing the impact of stability, primary regulation, and reactive power. To overcome this burden, this paper implements optimization scheme 3 to the HVIET as a computationally manageable approach to be used as an initial screening technique where the procedure for computing the maximum flow is based on the Ford-Fulkerson algorithm [20]. Thus, while the network flow model used respects flow capacity limits on lines and nodal balances, as a computational simplification, it overlooks Ohm and Kirchhoff loop laws.

For the case study considered, the implementation of MO-PSDA considers 361 variables, the number of transmission lines in the system. Based on the data, the maximum power-flow without any interdiction strategy equals 23869 MW.

Figure 2 shows the Pareto set approximation found using the MO-PSDA implementation. This set was identified by analyzing 5000 solutions out of a total of $2^{361}$ potential solutions. Figure 2 illustrates the first extreme point located at (0,23869) representing the value associated with the solution where no interdiction is implemented. The point located at (1,22898) corresponds to the greatest flow reduction in the network when considering the interdiction of a single line -line between nodes 167 and 106. The value associated with $\text{FR}(D(1)) = 23869-22898=971$ MW, represents the maximum amount of electrical load that the system is unable to satisfy (about 4 % of the maximum system load) when considering a single failure, $\alpha=1$.

To complement Figure 2, Table 1 provides the lines of the Pareto-optimal groups that should be interdicted to obtain the highest reduction of the network flow, for groups of
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size $\alpha$ between 1 and 7. The optimization approach proposed is shown to be a successful surrogate screening technique for identifying a subset of system components that should be subject of a further detailed analysis as they might provide the sought strategy of interdiction without having to analyze in detail the interdiction effects of all system lines (361 in the current example). To support this claim, it is noted that the detailed approach presented by Rosato et al. [17] recognizes that critical lines for electrical transport are those connecting nodes 214, 184, 117, 190, 127, 130: these lines form a subset of the screening results obtained by MO-PSDA.

![Figure 2: Pareto Front](image)

Table 1: Interdiction Strategies for the first 8 Solutions in the Pareto – Optimal Set

<table>
<thead>
<tr>
<th>Flow Reduction</th>
<th>$\alpha$</th>
<th>$D(\alpha)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6273</td>
<td>7</td>
<td>117-190</td>
</tr>
<tr>
<td>5594</td>
<td>6</td>
<td>117-190</td>
</tr>
<tr>
<td>4904</td>
<td>5</td>
<td>117-190</td>
</tr>
<tr>
<td>3927</td>
<td>4</td>
<td>117-190</td>
</tr>
<tr>
<td>2927</td>
<td>3</td>
<td>127-103</td>
</tr>
<tr>
<td>1927</td>
<td>2</td>
<td>127-103</td>
</tr>
<tr>
<td>971</td>
<td>1</td>
<td>167-106</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
</tbody>
</table>

5. Conclusions

This work introduced a new importance measure, $D(\alpha)$, to characterize the groups of $\alpha$ network links which when interdicted reduce most the network flow. The related computation can be very time consuming in networks of realistic dimensions. To overcome this problem, a transformed multi-objective deterministic network interdiction problem has been formulated and solved using evolutionary algorithms.

The efficiency of the procedure proposed has been illustrated by analyzing the Italian high-voltage electrical transmission network. $D(\alpha)$ has been proved useful for an initial screening of the critical elements of the network.

As with any evolutionary optimization technique the Pareto set is in fact only an approximation of the true one; however, the comparison of the results obtained with those
obtained in the literature demonstrate that the MO-PSDA is capable of developing accurate Pareto sets without extensive computational effort or iterative solutions of single objective problems.

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References

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