Evaluating Network Robustness based on Failure Event Possibility

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Abstract: The robustness of a network is the ability to maintain a satisfactory performance level when there may be system endogenous random failures plus possible failures caused by external attacks. A new approach for determining network robustness is presented based on the difference between the possibilistic and probabilistic network dependability estimates. Both the probabilistic and possibilistic estimates are derived here using a simple approximation method proposed by von Collani [9], but with different operations for the possibility estimate in some system structures. The proposed robustness estimation method is demonstrated for a sample of network architectures.

Keywords: Network, robustness, dependability, reliability, possibilistic event.

1. Introduction

This paper presents a new approach for estimating network robustness using estimates of network dependability based on feasible or possible events that could occur. Network dependability is considered here to be the composite of intrinsic network reliability and its resistance to dysfunctions caused by possible external attack. Possibilistic dependability estimates, as used in the proposed robustness evaluation method, are especially relevant when there is a limited history of network failure events, or when subjective estimates are used, perhaps in condensed simplifications of massive non-homogeneous networks. Zadeh [11] in 1978 laid the foundations of possibility theory and some applications of it to reliability theory are described by Cappelle and Kerre [3] and Onisawa and Kacprzyk [7].

For the purpose of this paper, network or system robustness will be defined to be high when there is little difference between the dependability estimates from what will likely happen and from what could happen. Robustness will be considered to be low when there is a significant deviation between the dependability estimate based on what could occur and the dependability estimate based on likely random events. The robustness of three example networks is then estimated from the possibilistic and probabilistic dependability estimates derived using the simple approximation method of von Collani [9], which is applicable to networks of any degree of complexity.

2. Some Approximation Methods

Although a wide variety of analytical techniques exists for network reliability analysis [4,5,8] the analysis of network reliability for large networks is generally a NP-hard problem. For this reason simulation approaches are often applied to complex network architectures and the ‘exact” probability estimates for the following examples were thus derived using DSTO developed network analysis software [6].
Von Collani [9] has proposed that the probabilistic reliability of any set of nodes in a network of any complexity can be determined by using some very simple approximations, in most cases apparently with small deviation from more rigorous solutions. In essence, the approximations simply consider the simultaneous failure of all links at the specified nodes (which may be the whole network). The objective of applying these algorithms in this paper is simply to demonstrate the deviations between the probability and possibility dependency estimates.

2.1 Approximation Method Overview

Von Collani presented three different approximation methods for probabilistic network reliability computation: for any 2 nodes (2-terminal), for any set of nodes greater than 2 (k or all-terminal), and for any set of nodes in networks which can be obviously partitioned into sub-networks. The approximation equations can also be understood from a cut-set viewpoint. For the set of all nodes, or subsets greater than 2, if all the links out of each of the nodes in the set are down, communication is then impossible. Hence, equations relating to those joint events are used. However, when only 2 node reliability is required (in a non-partitionable network) additional subsystems must be considered associated with each node. For 2 node communication dependability, all links from the ends of each path out of each terminal node must then be added to the joint failure of all links at the terminal nodes themselves. The same reasoning applies for both possibilistic and probabilistic event dysfunctions enabling the same approximation equation to be applied for both types of estimate.

And as identified in [10] the primary computational difference when using event possibilities in reliability analysis of complex systems is with the disjunctive OR operator. For disjunctive operations failure event possibilities (as for series systems) are simply summed, as for mutually exclusive probability OR estimates. However, it should be noted that the summed event possibilities are limited to a maximum value of one. On the other hand, the conjunctive AND with possibility measures is still multiplicative as with probabilities. For these reasons, the same approximation equations can be used for both possibilistic and probabilistic dependency estimation. In the approximation equations below the power terms then represent AND for concurrent failure events for a set of links, and addition represents OR for alternative sets of failures, neglecting the product terms that would be included for non-mutually exclusive sets of failures.

Before applying the approximation methods, singular link dependability measures must be derived. In this paper we will derive each link dependability measure from the composite of its components as demonstrated below for communication between two distributed computers, for example. For this single link dependability calculation, the possibility value differs from the probability value because of the aforementioned difference in possibilistic disjunctive series system analysis.

2.2 Preliminary Link Dependability Estimation

Consider a distributed system composed of disparate local PC networks (Figure 1). A link between any two distributed PC nodes \( \{1,2\} \) in the system consists of five communication stages, each of which may fail or be degraded. For demonstration purposes, simply consider the probabilistic and possibilistic likelihood of failure to be the same (0.07) for all five stages of the series system, although in general the component failure possibilities would be expected to be greater than the equivalent probability estimates because they relate to all feasible events that could occur for the link components.
For probabilistic dependability analysis:
Link dependability = \((1 - 0.07)^5\) = 0.6957
Link undependability = 1 - 0.6957 = 0.3043
\(\approx 0.30\)

For possibilistic dependability analysis:
Link undependability = \(5 \times 0.07\) = 0.35
Link dependability = 1 - 0.35 = 0.65

3. Some Network Examples

For simplicity, all links are assumed to be identical in the following three examples with the above failure probability value of 0.30 and the failure possibility value of 0.35. The method would be unchanged, and little more complicated, if the simplifying assumptions were relaxed and the possibility and probability link dependencies were not equal, and different links had different dependabilities.

Table 1 summarizes the dependability (D) results and their deviations from the "exact" solutions as found in Beichelt [1], Blechschmidt [2] and von Collani [9]; and validated using simulation techniques [6].

Example 1: A Hexagonal Network (Figure 2). All node communication dependability is computed from the joint failure of all links at each node as previously described.

Approximate Probabilistic Dependability
\[
D\{\text{All Nodes}\} = 1 - \left\{6 (0.3)^3\right\} \\
= 1 - 0.1458 \\
= 0.8542
\]

Possibilistic Dependability
\[
D\{\text{All Nodes}\} = 1 - \left\{6 (0.35)^3\right\} = 1 - 0.03151 = 0.96849
\]

Example 2: ART1 Network, as in Blechschmidt [2].
Node set \{ All Nodes \}:

Approximate Probabilistic Dependability

\[
D\text{\{All Nodes\}} = 1 - \left\{ 4(0.3)^3 + (0.3)^4 + 4(0.3)^4 + 2(0.3)^6 \right\} = 1 - 0.1273 = 0.8727
\]

Possibilistic Dependability

\[
D\text{\{All Nodes\}} = 1 - \left\{ 4(0.35)^3 + (0.35)^4 + 4(0.35)^4 + 2(0.35)^6 \right\} = 1 - 0.21119 = 0.7888
\]

Node set \{ 1, 11 \}:

For 2 nodes dependability, joint failure of all links at the ends of each path out of each of the 2 nodes are added to the joint link failures at the two nodes themselves. For this network let \(E_i\) be number of links from Node i and \(E_{i,j}\) be the sum of the links out of both i and j.

For computing joint failures at each node:

\(E_1 = 3, \quad E_{11} = 3\).

For joint failures at ends of each path out of each of the two nodes:

\(E_{1-2} = 6, \quad E_{1-5} = 7, \quad E_{1-3} = 6, \quad E_{6-11} = 6, \quad E_{9-11} = 6, \quad E_{10-11} = 4\).

Approximate Probabilistic Dependability

\[
D\{1,11\} = 1 - \left\{ 2(0.3)^3 + (0.3)^4 + 4(0.3)^4 + (0.3)^5 \right\} = 1 - 0.0652 = 0.9348
\]

Possibilistic Dependability

\[
D\{1,11\} = 1 - \left\{ 2(0.35)^3 + (0.35)^4 + 4(0.35)^4 + (0.35)^5 \right\} = 1 - 0.108752 = 0.8912
\]

Example 3: Network as in Beichelt’s [1] Figure 3.4 (14 nodes, 35 arcs), shown here in Figure 4.

Node set \{ All Nodes \}:

Approximate Probabilistic Dependability

\[
D\text{\{All Nodes\}} = 1 - \left\{ 2(0.3)^3 + 3(0.3)^3 + 6(0.3)^5 + (0.3)^7 \right\} = 1 - 0.08208 = 0.91792
\]

Possibilistic Dependability

\[
D\text{\{All Nodes\}} = 1 - \left\{ 2(0.35)^3 + 3(0.35)^3 + 6(0.35)^5 + (0.35)^7 \right\} = 1 - 0.14408 = 0.85592
\]

Node set \{ 1, 13 \}:

For computing joint failures at each node:

\(E_1 = 3, \quad E_{13} = 5\).

For joint failures at ends of each path out of each of the two nodes:

\(E_{1-2} = 5, \quad E_{1-3} = 5, \quad E_{1-4} = 7, \quad E_{9-13} = 10, \quad E_{10-13} = 9, \quad E_{11-13} = 9, \quad E_{12-13} = 8, \quad E_{14-13} = 6\).

Approximate Probabilistic Dependability

\[
D\{1,13\} = 1 - \left\{ 3(0.3)^3 + 3(0.3)^5 + (0.3)^6 + (0.3)^7 + 2(0.3)^8 + (0.3)^9 \right\} = 1 - 0.03534 = 0.96466
\]

Possibilistic Dependability

\[
D\{1,13\} = 1 - \left\{ 3(0.35)^3 + 3(0.35)^5 + (0.35)^6 + (0.35)^7 + 2(0.35)^8 + (0.35)^9 \right\} = 1 - 0.00649 = 0.99351
\]

Node set \{ 1, 5, 13 \}:

Dependability from joint failures of all links at each node.
Approximate Probabilistic Dependability

\[ D\{1, 5, 13\} = 1 - \left\{ (0.3)^3 + 2(0.3)^5 \right\} = 1 - 0.03186 = 0.96814 \]

Possibilistic Dependability

\[ D\{1, 5, 13\} = 1 - \left\{ (0.35)^3 + 2(0.35)^5 \right\} = 1 - 0.05337 = 0.94663 \]

4. Summary of Results

Table 1 lists the results for the three example networks where the last column shows the difference between the probability and possibility dependability estimates as robustness variations. Thus, example 2 (-9.1%) would be the most susceptible to communication failure between all nodes, example 3 (-6.4%) the next most susceptible, with example 1 (-2.0%) the least susceptible to all node communication failure.

The deviations between the probabilistic and possibilistic dependability estimates for the given data ranged from about 2 to 9%. And while this may not seem very significant in light of the uncertainty in the subjective inputs, these deviations would be greater if the link element failure possibilities were more reasonable, and were greater than the equivalent probability values. For example, with network example 3, if the computer link component dysfunction possibilities were changed to more realistic values like 0.15 for the end human/computer nodes, 0.04 for the long link, and 0.05 for the two local area networks; the link undependability possibility would be 0.44. And when this value is inserted into the equation for \( D\{\text{All Nodes}\} \) the result is 0.66 which is 27% lower than the probabilistic network dependability of 0.91. In this way, the possibility of human caused dysfunctions can be incorporated into network robustness analysis.

Table 1: Summary of Example Network Dependability Estimates

<table>
<thead>
<tr>
<th>Node Set</th>
<th>Probabilistic Network Dependability</th>
<th>Possibilistic Network Dependability</th>
<th>Possibilistic Dependability Difference From D Exact</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D Exact</td>
<td>D Approximate</td>
<td>Possibilistic Network Dependability</td>
</tr>
<tr>
<td>Example 1: Hexagonal network (6 Nodes, 15 Arcs)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>{all nodes}</td>
<td>.98497</td>
<td>.98542</td>
<td>.9685</td>
</tr>
<tr>
<td>Example 2: ART 1 (11 Nodes, 25 Arcs)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>{1, 11}</td>
<td>.93149</td>
<td>.93480</td>
<td>.8912</td>
</tr>
<tr>
<td>{all nodes}</td>
<td>.86550</td>
<td>.87272</td>
<td>.7888</td>
</tr>
<tr>
<td>Example 3: Beichelt’s Network (14 Nodes, 35 Arcs)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>{1, 13}</td>
<td>.96358</td>
<td>.96466</td>
<td>.9385</td>
</tr>
<tr>
<td>{1, 5, 13}</td>
<td>.96358</td>
<td>.96814</td>
<td>.9466</td>
</tr>
<tr>
<td>{all nodes}</td>
<td>.91461</td>
<td>.91792</td>
<td>.8559</td>
</tr>
</tbody>
</table>

The deviation between probabilistic and possibilistic dependability estimates is influenced by both the component dependability values and the structure of the network through the approximation equations. These examples are only intended to demonstrate that dependability is always less with possibilistic analysis and the more conservative estimate (i.e., lower) would result in fewer surprises from unexpected dysfunctions. Also of note, the deviations between the approximate probabilistic results and the more rigorous probabilistic results were shown to be small, and well within the error magnitude expected in any input subjective estimates of failure events for network elements (say ± 5%).
5. Conclusions

Since the input estimates of network elements’ dependabilities may often contain considerable uncertainty, the output network dependency values may only be indicative in many cases. Nevertheless, these indicative values can enable the robustness of different network architectures to be qualitatively compared, with rough differences of robustness indicated by this “back of the envelope” approximation method. For networks within conflict situations, the possibilistic estimate has special relevance because there may only be a small amount of factual evidence necessitating subjective estimates of one-off events, and “potentiality” evaluation focuses on feasible events without being limited to what has happened in the past, for example a plane crashing into a building. And although other approaches may be applied for determining the dependency of a network, the key notion that has been presented here is that robustness can be assessed by the difference between the probabilistic and possibilistic estimates.

References


Lewis Warren has a background in Operations Research and decision analysis theoretics. His educational qualifications are B. Industrial Eng. (University of Melbourne), Master of Systems Engineering (RMIT University), and a Ph.D. in complex systems modelling (Swinburne University of Technology). In recent years he has focused on soft OR, hybrid uncertainty representation, and robust model definition for strategic decision analysis and complex systems performance evaluation.