A Multi-objective Genetic Algorithm for Reliability Optimization Problem

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Abstract: This paper considers the allocation of maximum reliability to a complex system, while minimizing the cost of the system, a type of multi-objective optimization problem (MOOP). Multi-objective Evolutionary Algorithms (MOEAs) have been shown in the last few years as powerful techniques to solve MOOP. This paper successfully applies a Nondominated sorting genetic algorithm (NSGA-II) technique to obtain the Pareto optimal solution of a complex system reliability optimization problem under fuzzy environment in which the statements might be vague or imprecise. Decision-maker (DM) could choose, in a “posteriori” decision environment, the most convenient optimal solution according to his/her level of satisfaction. The efficiency of NSGA-II in solving this problem is demonstrated by comparing its results with those of simulated annealing (SA) and nonequilibrium simulated annealing (NESA).

Keywords: Reliability optimization, fuzzy optimization, multi-objective optimization problem (MOOP), genetic algorithm (GA), nondominated sorting genetic algorithm (NSGA-II)

1. Introduction

In the broadest sense, reliability can be defined as measure of performance of systems. As systems have grown more complex, the consequences of their unreliable behavior have become severe in terms of cost, effort, lives etc. and the interest in assessing system reliability along with its improvement has become very important. Unlike conventional optimization methods where it is assumed that all design data are precisely known and objectives are well defined and easy to formulate, many practical optimization problems e.g., reliability optimization, there are incompleteness and unreliability of input information. The reason for unreliability can be many such as; uncertainty in judgments, lack of evidence, etc. Further more, a DM often has vague desires such as, “this objective function should be less than or greater than or equal to this certain value”. Fuzzy set theory [1] is effectual in handling these cases. In reliability optimization problems, it is often required to minimize or maximize several objectives subject to several constraints. Such problem is formulated as multi-objective optimization problem (MOOP).

A MOOP can be solved in two ways; first one is to solve it by transforming the MOOP into single objective problem using positive weights (for objectives) and penalties (for constraints), and the other one which is also better is to obtain a Pareto-optimal solution which gives a DM suitable range of choice to adjust trade off between different
objectives. Sakawa [2] used the surrogate worth trade off method to a multi-objective formulation of a reliability allocation problem to maximize the system reliability while minimizing the system cost. Huang [3] tackled fuzzy multi-objective optimization decision-making problem on the series reliability system with two objectives. Ravi et al. [4] and [5] implemented simulated annealing (SA) algorithm for several reliability optimization problem. Mahapatra et al. [6] proposed a new fuzzy multi-objective optimization method to solve reliability optimization problem having several conflicting objectives. Genetic algorithms (GAs) are well-known stochastic methods of global optimization based on the evolution theory of Darwin and have successfully been applied in different real-world applications including reliability optimization. Since GAs work with a population of points, a number of Pareto-optimal solutions may be captured using GAs, making it a very powerful tool also for MOOPs. The Non-dominated Sorting Genetic Algorithm (NSGA-II) [7] is a well known and extensively used algorithm based on its predecessor NSGA [8]. It is a fast and very efficient Multi-objective evolutionary algorithm (MOEA), which incorporates the features an elitist archive and a rule for adaptation assignment that takes into account both the rank and the distance of each solution regarding others. Daniel et al. [9] has applied and compared the efficiency of NSGA-II with existing methods for different reliability optimization problems without including fuzzy environment.

This paper uses a fuzzy satisficing method projected and applied by Huang [3], Ravi et al. [4-5] to transform the multi-objective reliability optimization problem into fuzzy optimization problem using linear membership functions. Then NSGA-II is employed to the resulting fuzzified problem to obtain the Pareto solutions because NSGA-II is one of the most popular multi-objective optimization algorithm applied successfully for various real world problems. This paper suggests use of NSGA-II to obtain the Pareto-optimal solutions unlike Ravi et al. [5] who solved the problem using NESA. The DM can choose appropriate solution from the group of Pareto-optimal solutions obtained. If the DM is not satisfied with any of them, then he/she can seek a final solution by modifying parameters interactively according to his/her preferences.

2. The Mathematical Model of Problem

Let $R_j$ and $C_j$ be the reliability and cost of $j^{th}$ component of the system and $R_s$ and $C_s$ denote the total reliability and cost of the system. It is often required to consider, in addition to maximization of system reliability, the minimization of the cost. Mathematically, this problem can be expressed as

Maximize: $R_s(R_1, R_2, R_3, ..., R_n) = \begin{cases} \prod_{j=1}^{n} R_j \text{ for series system} \\ 1 - \prod_{j=1}^{n} (1 - R_j) \text{ for parallel system} \end{cases}$

or combination of series and parallel system

Minimize: $C_s(R_1, R_2, R_3, ..., R_n) = \sum_{j=1}^{n} C_j(R_j)$

subject to: $R_{j, min} \leq R_j \leq 1, \quad R_{s, min} \leq R_s \leq 1$ for $j=1,2,\ldots, n$
where $n$ represent total number of components in the system while $R_{j,\text{min}}$ and $R_{s,\text{min}}$ are minimum values for the $j^{th}$ component and system respectively. Here, we consider a complex system which represents a block diagram of reliability of a life-support system in a space capsule. Fig. 1 shows the system.

![Block Diagram](image)

**Fig. 1:** Life-support system in a space capsule

The mathematical model for the life support system in a space capsule can be formulated using the block diagram (see Fig. 1) as follows:

Maximize $R(R) = 1 - R_j (1 - R_j) (1 - R_j) (1 - R_j)$

Minimize $C_j(R) = 2 \sum_{j=1}^{4} K_j R_j$

subject to: $0.5 \leq R_{j,\text{min}} \leq R_j \leq 1 \quad 0.5 \leq R_{s,\text{min}} \leq R_j \leq 1 \quad \text{for } j = 1, 2, 3, 4.$

where different parameters values are $K_j$'s as 100, 100, 200, 150 respectively and all $\alpha_j$'s equal to 0.6.

3. Genetic Algorithm

Genetic algorithms have been successfully applied as an optimization technique. GA introduced by Holland [10] and further described by Goldberg [11] and Deb [12], mimics natural selection or Darwinian Theory of ‘survival of the fittest’. The basic GA methodology can be presented in the following form:

1. Set population size, tournament size, crossover rate, mutation rate, mutation exponent and elitism size. Set the parameters of the stopping criterion.
2. Initialize the population with random numbers.
3. Compute the fitness function values. Perform selection, crossover, mutation and elitism in order to create a new population.
4. If the stopping criterion is not satisfied, return to step 3. Otherwise, choose the best individual found as the final solution.

4. Nondominated Sorting Genetic Algorithm (NSGA-II)

There were some major drawbacks in NSGA such as
• High computational complexity of non-dominated sorting.
• Lack of elitism.
• Lack of specification of sharing parameter.

Deb et al. [7] proposed an improved version of NSGA [8], called NSGA-II which dealt all the drawbacks of original NSGA. NSGA-II incorporates an archive and a rule for adaptation assignment that takes into account both the rank and the distance of each solution.

Let $P_t$ represents the current population during any generation $t$, and $P_A^t$ the population which consist of non-dominated solutions archive. The Pseudo code for NSGA-II can be stated as follows

**Input:**
- $N$ (Population size)
- $M$ (Archive size)
- $t_{max}$ (maximum number of generation)

**Begin:**
- Randomly initialize $P_A^1$, set $P^0 = \emptyset$, $t=0$
- while $t < t_{max}$
  - $P^t = P^t + P_A^t$
  - Assign adaptation to $P^t$
  - $P_A^{t+1} = \{ M$ best individuals from $P^t \}$
  - Mating Pool = $\{ N$ individuals randomly selected from $P_A^{t+1}$ using a binary tournament $\}$
  - $P^{t+1} = \{ N$ new individuals generated by applying recombination (crossover and mutation) on Mating Pool $\}$
- $t = t + 1$

**Output:**
- Non-dominated solution from $P_A^t$

5. **Methodology**

Present methodology to solve a multi-objective reliability optimization problem consists of following steps

Step 1) Fuzzification:
- Fuzzification of the problem with the help of a linear membership functions.

Step 2) Problem Reformulation:
- Conversion of multi-objective crisp reliability optimization problem into fuzzified MOOP of membership functions.

Step 3) Solution:
- Finding the solution of the resulting MOOP using NSGA-II described in section 4.

Now present methodology is applied to solve the multi-objective reliability optimization problem of life-support system in a space capsule.

5.1 **Fuzzification**

Let $f_1$ and $f_2$ be the fuzzy region of satisfaction of system reliability ($R_s$) and system cost ($C_s$) respectively and $\mu_{R_s}$ and $\mu_{C_s}$ be their corresponding membership functions where $Rs$ and $Cs$ are defined in (1) and (2). Then $\mu_{R_s}$ is defined as
Parameters for linear membership function used in equation (3 and 4) are taken from Ravi et al. [4-5]. The above formulation represents fuzzy representation for a complex system (fig. 1) in terms of reliability and cost. DM wants to optimize the system reliability within 0.9 and 0.99 while maintaining the cost between 641 to 700 units thus making it a multiobjective problem having two objective functions namely reliability ($R_s$) and cost ($C_s$).

### 5.2 Problem Reformulation

Optimal decision is made by selecting the best alternative from the fuzzy decision space $D$ characterized by the membership function $\mu_D$. In other words, the problem is to find the optimum $R^*$ which maximizes $\mu_D$ where $\mu_D \in [0,1]$ where $R^*$ is the decision variable, $R^* = (R_1, R_2, \ldots, R_4)$. Within the frame work of Bellman and Zadeh [13] model, and following Zimmermann [14] the optimal solution is obtained by maximizing $\mu_D$ where $\mu_D = \mu_{\mu_j}(R_j) \times \mu_{\mu_j}(C_j)$ and $*$ represents the operator used in the decision. In the present paper this problem is solved in purely multiobjective manner. So the reformulated fuzzy problem can be mathematically expressed as

Maximize $\left( \mu_{\mu_j}(R_j), \mu_{\mu_j}(C_j) \right)$

Subject to: $0.5 \leq R_j \leq 1 \quad j = 1,2,3,4$

### 5.3 Solution of MOOP

A NSGA-II is employed to solve the resulting MOOP maximization problem of membership functions. Varying the different parameters (crossover probability, mutation probability, population size, crossover and mutation index, etc.) of NSGA-II different Pareto-optimal fronts can be obtained. Based on rigorous experimentation and tuning of the parameters some better optimal front have been reported here.

### 6. Results and Discussion

To solve the transformed problem (5) NSGA-II has been applied. Parameters of NSGA-II such as population size $N$, maximum number of generation $t_{\text{max}}$, crossover probability $p_c$, and mutation probability $p_m$. Different Pareto-optimal fronts can be obtained varying these parameters. This paper discusses few better ones out of them after a number of experimental runs.

Case-1 If population size $N$ is being fixed at 100 and number of generations $t_{\text{max}}$ equal to 300 while crossover probability ($p_c$) equal to 0.9 and mutation probability ($p_m$) = 0.02 then resulting Pareto solution is given in Fig. 2.
Case-2 For Pareto-optimal solution obtained in Fig. 3, NSGA-II parameters such as $p_c$ is taken 0.9 again while $p_m$ is 0.03 and population size $N$ of 200 for 500 ($t_{max}$) generations. Similarly many Pareto optimal solutions of this problem can be achieved. More points on Pareto curves require bigger population size. Table 1 shows the top 15 coordinates of the Fig 2.


Present results are better than those obtained by Ravi et al. [4], as it provides the DM more choices. Also present methodology provides Pareto optimal front in a single run of algorithm (NSGA-II) unlike [5] which needs different values of membership functions of the fuzzy set “decision” for different runs. It also improves the accuracy of results obtained compared to [5] as it provides the results up to 6 decimal places (see Table 1) unlike [5] which gave results up to 5 decimal places only. Hence this approach is much more efficient and flexible, and thus provides a better range of choice out of which DM can choose the preferred solution interactively.

7. Conclusion

A real life reliability optimization problem of a life-support system in a space capsule under fuzzy environment has been discussed here. Ravi et al.[5] solved the same problem using NESA. This paper solves the problem using NSGA-II which is highly efficient for continuous multi-objective optimization problems as it provides a decision maker more flexibility in obtaining Pareto-optimal front. Also present methodology provides Pareto optimal front in a single run of algorithm (NSGA-II) unlike [5] which needs different values of membership functions of the fuzzy set “decision” for different runs. Also accuracy of the results has been increased. Therefore, the paper proves the effectiveness of NSGA-II for continuous multi-objective optimization problems.
A Multi-objective Genetic Algorithm for Reliability Optimization Problem

Fig. 3: Pareto-optimal front by NSGA-II for Case-2

Table 1: Coordinates of last 15 points

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<tr>
<th>i</th>
<th>$R_s(i)$</th>
<th>$C_s(i)$</th>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$R_3$</th>
<th>$R_4$</th>
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References


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