Assessing Resource Requirements for Maritime Domain Awareness and Protection (Security)

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Abstract: An allied (Blue) maritime domain contains a number \( w (w>>1) \) of non-hostile neutral W (White) vessels. A hostile R (Red) vessel enters the domain. R is traveling through the domain towards a target. Overhead allied (Blue) sensors: manned aircraft, helicopters, and/or unmanned aerial sensor vehicles (UAVs), patrol the domain and classify (perhaps incorrectly) detected vessels as R or W. The misclassification of a W as an R is a false positive. An overhead sensor follows (or tracks) any vessel classified as R until it is relieved by another platform, e.g., a destroyer pair (DD). The overhead sensor is unable to detect and classify additional vessels while it is following a suspicious vessel. This may well be a somewhat pessimistic assumption. Models are formulated and studied to evaluate the probability that R is successfully neutralized before reaching its destination. The model results quantify the effect of the resources and time needed to prosecute misclassified neutral vessels (false positives) on the probability of successfully neutralizing R. The probability of neutralizing R depends on the area of the domain being patrolled, the number of sensor platforms, the sensor platform velocity, the time to classify a vessel of interest, the ability to correctly classify vessels of interest, the time until a sensor platform following a suspicious vessel is relieved, and the false positive rate. The results indicate that the probability of neutralizing an R vessel is very sensitive to the false positive rate. Technologies, processes, and procedures that can decrease the false positive rate will increase the effectiveness of the Maritime Intercept Operation (MIO). The same is true also of false negatives: classifying the R as a W. Note that we do not investigate the effect of tagging or labeling a detected entity; this has a down side if tagging is too error-prone. This important and interesting investigation is postponed.

Keywords: Maritime domain awareness and protection; port security; terminating and alternating renewal processes

1. Introduction

The term Maritime Domain Awareness (MDA) includes a broad range of initiatives to enhance both the security of ports and approaches to the United States and the force protection of U.S. and/or allied maritime assets (e.g., those of Japan, Singapore, Bahrain, etc.) in ports and choke points throughout the world. An essential requirement is to furnish adequate Blue surveillance force size and composition, and resource-assignment-effective concept of operations (CONOPS) to maintain useful knowledge of hostile (Red) elements. Consider this scenario. A maritime domain or region contains a number \( w \) of non-hostile White (W) vessels. At time 0 a hostile R (Red) vessel enters the domain. An overhead (OH) friendly (Blue) sensor platform \((S)\) patrols the domain and classifies...
(perhaps incorrectly) detected vessels as R or W. If the sensor misclassifies a W as an R, a false positive occurs. The false-positive rate is a function of the probability of correct classification, which may well be environmentally dependent (see [3]) and the number of vessels of interest that are Ws.

Each vessel classification takes a specified mean time $\tau$. The $S$ follows (or tracks) any vessel it classifies as R until it is relieved by escort vessels (e.g. a destroyer pair (DD)). The $S$ is assumed unable to detect and classify additional vessels while it is following a vessel. Assume an unescorted R will remain in the domain for a positive random time having a distribution function $F$; the sensor platform’s following times of vessels classified as R are independent and identically distributed nonnegative random variables; successive times $S$ spends searching for and traveling to vessels are independent and identically distributed.

In Section 2 appears the Laplace transform of the time to detect R. Sections 3 and 4 present approximations to the probability R is detected and correctly classified before leaving the domain, and the probability R is detected, correctly classified, and escorted before leaving the domain. One approximation uses a terminating renewal process argument. A second approximation uses an alternating renewal process. The accuracy of the approximations is assessed by comparing their results to those of a more detailed simulation model in Section 5; they give reasonable results for the cases studied. In Section 6 the approximations are used to explore the sensitivity of the probability of neutralizing the R to model parameters. The paper concludes with Section 7.

We assume the domain is a rectangle. The parameters of the model appear in Table 1.

Table 1: Parameter Values for Table 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity of Red vessel in knots (kts)</td>
<td>$v_r$</td>
</tr>
<tr>
<td>Velocity of OH sensor platform $S$ in kts</td>
<td>$v_s$</td>
</tr>
<tr>
<td>Velocity of escort platforms in kts</td>
<td>$v_d$</td>
</tr>
<tr>
<td>Number of White vessels</td>
<td>$w$</td>
</tr>
<tr>
<td>y-direction length of the domain rectangle in nautical miles (nm)</td>
<td>$M_y$</td>
</tr>
<tr>
<td>x-direction length of the domain rectangle in nm</td>
<td>$M_x$</td>
</tr>
<tr>
<td>Side of square of OH sensor footprint in nm</td>
<td>$f$</td>
</tr>
<tr>
<td>Time to classify a detected vessel in hours (hrs.)</td>
<td>$\tau$</td>
</tr>
<tr>
<td>Probability of correctly classifying a R</td>
<td>$c_{rr}(\tau)$</td>
</tr>
<tr>
<td>Probability of correctly classifying a W</td>
<td>$c_{ww}(\tau)$</td>
</tr>
<tr>
<td>Probability a vessel in the footprint of the OH sensor is detected</td>
<td>$p_d$</td>
</tr>
</tbody>
</table>

2. A Stochastic Model for a Simple MDA Scenario

The assumptions of the model are as follows.

1. At $t=0$ a single hostile/lethal R enters the rectangular domain $\mathcal{D}$ at its upper boundary $\mathcal{D}$ and travels down to the lower boundary $\mathcal{D}$. R is attempting to invade the homeland at the lower boundary. R’s travel time through the domain is a random variable
independent of actions to detect and neutralize it; see [5] for a game theoretic model. R is attempting to blend in with the other vessels in the domain.

2. There is a constant number, \( w \), of White vessels (Ws) in the domain; the constant number may be maintained by replacement if any W is misidentified as R and followed and escorted to quarantine.

3. There is one overhead sensor platform, \( S \). The sensor platform alternates between searching for suspicious vessels and following vessels it has classified as R until relieved by escort platforms. The successive times for \( S \) to detect and travel to a vessel are independent identically distributed (iid) random variables (rvs).

4. The type of vessel detected by \( S \) is drawn at random with replacement from the \( w+1 \) vessels in the domain; the probability a detected vessel is a W is \( \frac{w}{w+1} \), independent of the past. If the vessel is W, it is classified correctly with probability \( c_{\text{ww}}(\tau) \), and released. Note that another case not considered here is one in which such a discovery might be followed by labeling or tagging the particular platform correctly as a W, in which case it is ignored on future encounters; the Automatic Identification System (AIS) is a possible way of tagging/identifying a (subset of) Ws, but is not infallible, being subject to interference, system failure, and deliberate jamming or deception; c.f. [6]. Thus our results tend to be pessimistic. The detected vessel is the R with probability \( \frac{1}{w+1} \) independent of the past; it is classified incorrectly with probability \( 1-c_{\text{rr}}(\tau) \) and released. Another case not considered here is if the R is encountered and misclassified as a W then the R may be ignored in future searches, and should make it to the lower boundary \( \mathcal{D} \) and be able to invade the homeland \( \mathcal{D}(H) \).

5. \( S \) follows (tracks) any vessel classified as R for an independent random time until escort platforms (e.g. a DD pair) approach and take over from \( S \), and \( S \) returns to searching; successive following times are independent and identically distributed. We assume that escort platforms are always available to respond; see [1] for models in which this assumption is relaxed.

### 2.1. Backward Equation for Time of Capture and Probability of Capture of Single Hostile R for Quarantine Before Homeland \( \mathcal{D}(H) \) is entered

Let \( T_{fq}(w) \) denote the time an R survives being taken to quarantine. Assume that \( X_i (i=1, 2, \ldots) \) are successive times for \( S \) to travel to and detect vessels. A conventional example is that the distribution of the time to detection is exponential with rate proportional to the number of targets present:

\[
P\{X_i(w)>t\}=\exp(-\delta(w+1,\tau)t).
\]

This need not be assumed, but is convenient. In fact, any sequence of independent identically distributed nonnegative random variables is possible, and analytically tractable in the present situation. Let \( D_i \) be the ith follow (or tracking) period for any vessel selected to be followed (that has been classified correctly or incorrectly, as R). At the end of follow periods, the vessel is released to the escort platforms (Diverters).
Note: We only study the stages involved in Detection and Follow to the Diverters (D) at present. We assume that a correct classification is made (almost) as soon as the D reaches the vessel.

Consider the time until first detection, and these subsequent possibilities:

(a) If the R is detected first, and correctly classified as an R, it is followed to a signaled D; this requires a relatively long random time $D$ (a strong assumption that can be relaxed). In this case the R is detected first and is correctly followed to D in time $X+D$ with probability $\frac{1}{w+1}c_{rr}(\tau)$.

(b) If the R is detected first and misidentified as a W it is released. The time this event takes place is at $t=X$, and its probability is $\frac{1}{w+1}c_{rw}(\tau)$. The search for R begins again from scratch.

(c) If a W is detected first and identified as a W, then it is released and $S$ immediately begins search again from scratch. In this case the first time duration is $X$ with probability $\frac{w}{w+1}c_{ww}(\tau)$.

(d) If a W is detected first and misclassified as an R with probability $c_{wr}(\tau)$, then a D is summoned. In this case the time taken by a complete first step/event is $X+D$ with probability $\frac{w}{w+1}c_{wr}(\tau)$; at the end of this step $S$ begins search again.

Then for the model

\begin{align*}
(a') \quad T_{rq}(w) &= X+D \quad \text{with probability} \quad \frac{1}{w+1}c_{rr}(\tau) \quad ; \\
(b') \quad T_{rq}(w) &= X'+T_{rq}'(w) \quad \text{with probability} \quad \frac{1}{w+1}c_{rw}(\tau) \quad ; \\
(c') \quad T_{rq}(w) &= X''+T_{rq}''(w) \quad \text{with probability} \quad \frac{w}{w+1}c_{ww}(\tau) \quad ; \\
(d') \quad T_{rq}(w) &= X''+D'+T_{rq}'(w) \quad \text{with probability} \quad \frac{w}{w+1}c_{wr}(\tau) \quad ;
\end{align*}

the various apostrophes indicate that the rvs are iid replicas of the basic $X,D$ components, and those on $T_{rq}$ on the right side indicate that the search process starts over “from scratch”.

Now compute the Laplace-Stieltjes transform of $T_{rq}(w)$: for transform variable $s \geq 0$ and using independence where needed, conditioning gives

\begin{align*}
E\left[e^{-sT_{rq}(w)}\right] &= E\left[e^{-s(X+D)}\right] \frac{1}{w+1}c_{rr}(\tau) \\
&= 1 - E\left[e^{-sX}\right] \frac{1}{w+1}c_{rw}(\tau) + \frac{w}{w+1}c_{ww}(\tau) + E\left[e^{-sD}\right] \frac{w}{w+1}c_{wr}(\tau) \quad (2)
\end{align*}
**Special Tractable Case: Exponential Red Transit Time**

Let the probability distribution of the time for R to reach the lower boundary \( D \) starting at any point in the domain \( D \) if it has not yet been linked to \( D \) be \( \exp(\mu_r) \), i.e. exponential with mean \( 1/\mu_r \). Then it can be seen that, conditional on \( T_r(w) \), the probability of R capture before leakage is just \( E\left[e^{-\mu_r T_r(w)}\right] \); the result of setting \( s=\mu_r \) in (2).

**2.2. A Strip-Search Approximation for Gamma (Erlang) Red Transit Times**

An initial example is that a Red’s unopposed transit time of \( D \) is exponentially distributed with mean \( 1/\mu_r \). This assumption can conveniently be made more physically plausible by dividing \( D \) into parallel strips and assuming that the times to pass through consecutive strips are independently exponential; if there are \( I \geq 1 \) strips, say \( I = 12 \), then the mean transit time is \( 1/\mu_r \), with variance \( 1/\mu_r^2 \) and coefficient of variation \( 1/I \), the square root, \( 1/\sqrt{I} \), giving an assessment of the variability of transit time, expressed as a fraction of its mean. Approximations for the probability R is detected, correctly classified and escorted in this case are presented in Sections 3 and 4. The results of [1] and [4] suggest that the probability of R being detected and correctly classified before transiting the domain is sensitive to distribution of the time an unopposed R spends in the domain.

**3. A Terminating Renewal Process Approximation**

The detection rate of vessels in the entire domain is assumed to be

\[
\delta_0(\tau) = p_d \left[ \frac{M_x M_y}{f_{rs}} + \frac{\tau[w+1]}{\text{Mean time to classify vessels in domain}} \right]^{-1}
\]

(3)

The sensor platform \( S \) alternates between looking for suspicious vessels and following vessels it classifies as R. We assume the successive times spent searching until detection are iid having an exponential distribution with mean \( 1/\delta_0(\tau) \). Let \( T_f \) be the first time a vessel is classified as R; the classified vessel could be White (false positive) or Red.
Let $T_r$ be the time until $R$ is correctly classified as $R$, and $T_w$ be the first time a $W$ is classified as $R$.

$$P{T_w < T_r} = \frac{w}{w+1} c_{wr}(\tau) + \frac{1}{w+1} c_{rr}(\tau) = p$$

In what follows we assume $p > 0$.

When a vessel is classified as $R$, $S$ follows (tracks) the vessel until relieved by escort vessels. Assume $S$'s following time, $D$, has a distribution with mean $1/\phi$ and Laplace transform $\mathcal{L}(\zeta(s)D(s))$ where the mean time the $S$ follows a vessel classified as $R$ is assumed to be

$$1/\phi = \frac{M_y}{2} \frac{1}{v_r + v_d}$$

### 3.1. An approximation to the time until $R$ is detected and correctly classified

The probability $R$ is detected and correctly classified before time $t$ satisfies the equation

$$P\{T_r \leq t\} = 1 - (1-p)P\{T_f \leq t\} + \int_0^t P\{T_r \leq s\} pP\{T_f + D \in ds\}$$

where $p$ is given by (5). Thus the probability $R$ is not detected and correctly classified before time $t$ satisfies

$$P\{T_r > t\} = 1 - (1-p)P\{T_f \leq t\} - \int_0^t P\{T_r \leq s\} pP\{T_f + D \in ds\}$$

$$= pP\{T_f \leq t\} + \int_0^t P\{T_f \leq t\} pP\{T_f + D \in ds\}$$

Following [2] p. 322, assume there is a $\kappa_0 > 0$ that satisfies

$$pE[e^{\kappa_0(T_f + D)}] = 1$$

Let

$$\mu^\#(\kappa_0) = pE[(T_f + D)e^{\kappa_0(T_f + D)}]$$
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\[ e^{K_0 t} P\{T_r > t\} = e^{K_0 t} \left[ pP\{T_f \leq t\} + pP\{T_f > t\} - pP\{T_f + D \leq t\} \right] \tag{11} \]

\[ + \int_0^t e^{K_0 (t-s)} P\{T_r > t-s\} e^{K_0 s} pP\{T_f + D \in ds\} \]

The key renewal theorem implies

\[ \lim_{t \to \infty} e^{K_0 t} P\{T_r > t\} = \frac{c_R(K_0)}{\mu^\#(K_0)} \tag{12} \]

where using (4) and (9)

\[ c_R(K_0) = (1-p) \frac{1}{K_0} \left[ \delta_0(\tau) \left[ \frac{-w}{w+1} c_{wr}(\tau) + \frac{1}{w+1} c_{rr}(\tau) \right] \right] \tag{13} \]

Thus

\[ P\{T_r > t\} \sim \frac{c_R(K_0)}{\mu^\#(K_0)} e^{-K_0 t} \tag{14} \]

Assume an unescorted R is in the domain for a time having a distribution \( F \) with Laplace transform \( \zeta_R(s) \). An approximation to the probability R is not detected and correctly classified while in the domain is

\[ \frac{c_R(K_0)}{\mu^\#(K_0)} \zeta_R(K_0) \tag{15} \]

If \( F \) is gamma with mean \( \beta / \sigma \) and shape parameter \( \beta \) then an approximation to the probability R is not detected and correctly classified while in the domain is

\[ \frac{c_R(K_0)}{\mu^\#(K_0)} \left[ \frac{\sigma}{\sigma + \kappa_0} \right]^\beta \tag{16} \]

3.2. An approximation to the probability R is detected, correctly classified, and escorted before it leaves the domain

Let \( T_e \) be the time until R is escorted.

\[ P\{T_e \leq t\} = (1-p) P\{T_f + D \leq t\} + \int_0^t P\{T_e \leq t-s\} pP\{T_f + D \in ds\} \tag{17} \]

Hence,

\[ P\{T_e > t\} = 1 - (1-p) P\{T_f + D \leq t\} - \int_0^t P\{T_e \leq t-s\} pP\{T_f + D \in ds\} \tag{18} \]

\[ = P\{T_f + D > t\} + \int_0^t P\{T_e > t-s\} pP\{T_f + D \in ds\} \]
Assume (9) holds. The key renewal theorem implies that
\[ P\{T < t\} = c_E(\kappa_0) \mu^\#(\kappa_0) e^{-\kappa_0 t} \] (19)
where \( \mu^\#(\kappa_0) \) is given by (10) and
\[ c_E(\kappa_0) = \frac{1}{\kappa_0} \left( \frac{1-p}{p} \right) \] (20)

The approximation corresponding to (15) is \( \frac{c_E(\kappa_0)}{\mu^\#(\kappa_0)} S_R(\kappa_0) \). The approximation corresponding to (16) is
\[ \frac{c_E(\kappa_0)}{\mu^\#(\kappa_0)} \left[ \frac{\sigma}{\sigma + \kappa_0} \right]^\beta \] (21)

4. An Alternating Renewal Process Approximation

Assume only Whites are in the domain. Let
\[ \delta = p_d \left[ \frac{M_x M_y}{\int y \phi(y)} \right] \left[ \begin{array}{c} 1 \\ \text{Mean time for } \mathcal{S} \text{ to cover domain} \end{array} \right] \] (22)

\( \mathcal{S} \) alternates between looking for suspicious vessels and classifying/following vessels. The expected time \( \mathcal{S} \) is busy when it detects a vessel (busy period) is
\[ E[B] = \tau + E[D] c_{WF}(\tau) \] (23)
The expected time between busy periods is \( 1/\delta_w \) where \( w \) is the number of White vessels. The long-run unavailability of \( \mathcal{S} \) is
\[ 1 - \alpha = \bar{\alpha} = \frac{E[B]}{1 + E[B]} \] (24)

Assume \( R \) enters the domain \( \mathcal{D} \) when the system is in steady state. Let
\[ \delta^* = \delta \alpha \] (25)
Approximate the distribution of the time until \( R \) is detected and correctly classified with an exponential distribution with mean \( 1/\delta^* c_{RR}(\tau) \).
4.1. An approximation to the probability $R$ is not detected and correctly classified before it travels through the domain

Assume an unescorted and unfollowed $R$ is in the domain for a random time having a distribution $F$ with Laplace transform $\zeta_R(s)$. An approximation to the probability $R$ is not detected and correctly classified while in the domain is $\zeta_R(\delta^* c_{RR}(\tau))$. If the time $R$ travels through the domain has a gamma distribution with shape parameter $\beta$ and mean $\beta/\sigma$, then this approximation becomes

$$\left[ \frac{\sigma}{\sigma + \delta^* c_{RR}(\tau)} \right]^\beta$$  \hspace{1cm} (26)

4.2. An approximation to the probability $R$ is not detected, correctly classified and escorted before traveling through the domain

The probability $R$ is not escorted before it leaves the domain is approximated thus: assume the successive times $S$ follows a vessel classified as $R$ are independently exponentially distributed with mean $(6)$. The probability that $R$ is not escorted before time $t$ is approximately

$$P(T_e > t) = e^{-\delta^* c_{RR}(\tau)t} + \int_0^t \delta^* c_{RR}(\tau)e^{-\delta^* c_{RR}(\tau)s}e^{-\phi(t-s)}ds$$

$$= e^{-\delta^* c_{RR}(\tau)t} + \frac{\delta^* c_{RR}(\tau)}{\delta^* c_{RR}(\tau) - \phi} \left[ e^{-\phi t} - e^{-\delta^* c_{RR}(\tau)t} \right]$$  \hspace{1cm} (27)

An approximation to the probability $R$ is not detected, correctly classified and escorted while in the domain is

$$\zeta_R(\delta^* c_{RR}(\tau)) + \frac{\delta^* c_{RR}(\tau)}{\delta^* c_{RR}(\tau) - \phi} \left[ \zeta_R(\phi) - \zeta_R(\delta^* c_{RR}(\tau)) \right]$$  \hspace{1cm} (28)

The approximation to the probability $R$ is not escorted before it leaves the domain corresponding to (26) is

$$\left[ \frac{\sigma}{\sigma + \delta^* c_{RR}(\tau)} \right]^\beta + \frac{\delta^* c_{RR}(\tau)}{\delta^* c_{RR}(\tau) - \phi} \left[ \left( \frac{\sigma}{\sigma + \phi} \right)^\beta - \left( \frac{\sigma}{\sigma + \delta^* c_{RR}(\tau)} \right)^\beta \right]$$  \hspace{1cm} (29)

5. A Spatial Simulation

Results from a more detailed simulation that includes representation of the spatial movement of the overhead sensor platform and $R$ are used to explore the robustness of the renewal process approximations. A description of the simulation is as follows.

The search domain

The domain is rectangular: $M_x$ nautical miles (nm) along the x-axis and $M_y$ nm along the y-axis. The footprint of the sensor is a square with side $f$ nm. The domain is tiled
with squares having sides $f$ nm; we assume both $M_x$ and $M_y$ are multiples of $f$.

**Initializing the simulation**

1. One Red vessel (R) enters the upper row at time 0. The column it enters is chosen at random; each column is equally likely to be chosen. The R travels down the column at a constant velocity $v_R$.
2. The initial position of the overhead sensor platform $S$ is chosen randomly from the rectangle; each position is equally likely to be chosen. We assume a raster scan $S$ path over the tiles. The initial direction of travel is chosen at random; each direction is equally likely.

**Detection of R**

Each time $S$ and R are in the same grid square there is a probability $p_d c_{rr}(\tau)$ that R will be detected and correctly classified as R where $p_d$ is the probability R is detected and $c_{rr}(\tau)$ is the probability a detected R is correctly classified as R.

**Number of White (Neutral) vessels**

Let $w$ be the mean total number of White vessels in the domain. The mean number of White vessels in each square, $m_W$, is the mean total number divided by the number of grid squares. Each time the sensor enters a square a Poisson random variable having mean $m_W$ is drawn; the resulting value is the number of White vessels in that square when it is searched. Each vessel in the square takes a time $\tau$ to be investigated. A White vessel is misclassified as R with probability $p_d c_{wr}(\tau) = p_d (1 - c_{ww}(\tau))$. If a White vessel is misclassified as R in grid square with center $(y,x)$, $S$ follows it for a time $(G_y - y)/(v_d + v)$ where $v_d$ is the velocity of the escort vessels, $v = v_R = v_W$ is the velocity of all the vessels, and $G_y$ is the number of grid squares in the $y$-direction. After this following time is completed $S$ proceeds instantaneously to the next square in its original search pattern. The total time spent in the grid square is the sum of the travel time through the square plus the time spent investigating all vessels in the square plus any following time if applicable. If R is detected and correctly classified, it will be escorted.

**5.1. Comparing Results from the Spatial Simulation to Those of the Renewal Process Approximations**

The parameter values appear in Table 1. The results of the spatial simulation with 1000 replications and two approximations (16) and (26) are displayed in Table 2. In both approximations, the sensor’s following time of a vessel classified as Red is exponentially distributed with mean $M_y/(2(v_d + v_R))$; the exponential distribution is not the following time distribution in the simulation. In the simulation the undetected R spends in the domain is constant and equal to $M_y/v_R$. In the approximations the time R spends in the domain is assumed to have a gamma distribution with mean $M_y/v_R$ and shape parameter equal to the number of grid squares in the $y$-direction, $G_y = 8$. In the approximations the number of Ws is constant; in the simulation the number of white
vessels has a Poisson distribution with mean equal to the constant number of Ws in the approximations.

Table 2: Approximation and Simulation Results for the Probability R is not Correctly Classified Before Transiting the Domain

<table>
<thead>
<tr>
<th>$M_X$ (nm)</th>
<th>Number of Ws</th>
<th>$c_{rr}$</th>
<th>$c_{ww}$</th>
<th>Spatial simulation fraction of replications (std. error)</th>
<th>Approximate probability (16)</th>
<th>Approximate probability (26)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>200</td>
<td>0.90</td>
<td>0.90</td>
<td>0.82 (0.01)</td>
<td>0.80</td>
<td>0.81</td>
</tr>
<tr>
<td>200</td>
<td>200</td>
<td>0.90</td>
<td>0.99</td>
<td>0.54 (0.02)</td>
<td>0.51</td>
<td>0.52</td>
</tr>
<tr>
<td>200</td>
<td>200</td>
<td>0.99</td>
<td>0.99</td>
<td>0.49 (0.02)</td>
<td>0.48</td>
<td>0.49</td>
</tr>
<tr>
<td>200</td>
<td>200</td>
<td>0.999</td>
<td>0.999</td>
<td>0.43 (0.01)</td>
<td>0.39</td>
<td>0.39</td>
</tr>
<tr>
<td>200</td>
<td>100</td>
<td>0.90</td>
<td>0.90</td>
<td>0.72 (0.01)</td>
<td>0.67</td>
<td>0.69</td>
</tr>
<tr>
<td>200</td>
<td>100</td>
<td>0.99</td>
<td>0.99</td>
<td>0.39 (0.01)</td>
<td>0.35</td>
<td>0.36</td>
</tr>
<tr>
<td>200</td>
<td>100</td>
<td>0.999</td>
<td>0.999</td>
<td>0.33 (0.01)</td>
<td>0.29</td>
<td>0.29</td>
</tr>
<tr>
<td>200</td>
<td>50</td>
<td>0.90</td>
<td>0.90</td>
<td>0.61 (0.02)</td>
<td>0.53</td>
<td>0.55</td>
</tr>
<tr>
<td>200</td>
<td>50</td>
<td>0.90</td>
<td>0.99</td>
<td>0.34 (0.02)</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>200</td>
<td>50</td>
<td>0.99</td>
<td>0.99</td>
<td>0.30 (0.01)</td>
<td>0.27</td>
<td>0.27</td>
</tr>
<tr>
<td>200</td>
<td>50</td>
<td>0.999</td>
<td>0.999</td>
<td>0.22 (0.01)</td>
<td>0.23</td>
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<td>0.999</td>
<td>0.58 (0.02)</td>
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<td>0.58 (0.02)</td>
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<tr>
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<td>0.8</td>
<td>0.99</td>
<td>0.58 (0.02)</td>
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<td>0.9</td>
<td>0.65 (0.01)</td>
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<td>0.67</td>
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Discussion: The two approximations give about the same numerical results which compare well with those of the more general simulation. The alternating renewal process approximation (26) requires less computation.


The parameter values are those of Table 1. Figure 1 displays the alternating renewal process approximate probability R is detected and correctly classified for various values of the probability of correct classification and number of Ws in the domain whose x-direction length is 400 nm.

Discussion: Not surprisingly, the probability of detecting and correctly classifying R before it leaves the domain increases dramatically as the probability of correctly classifying a vessel increases; the increase is larger the more Ws there are in the domain. Being able to decrease the number of Ws from 200 to 100 or from 100 to 50 results in a larger increase in the probability of detecting and correctly classifying R before it leaves the domain than increasing the probability of correctly classifying a detected vessel from 0.90 to 0.95.

Suppose that if s overhead sensor platforms (P3s) are used to patrol a rectangular domain of 200 nm by 200 nm, then each P3 patrols a domain of length 200/s nm in the x-direction and 200 nm in the y-direction. Other CONOPS are certainly possible. If there are w Ws in the large domain, then we assume there are $\frac{x}{200} w$ Ws in a domain of size x nm in the x-direction by 200 nm in the y-direction. Assume 1 R enters the domain at time 0 and travels straight down the domain in the y-direction. The time the unescorted R spends in the domain has a gamma distribution with mean $\frac{200}{v_f}$ and shape parameter $\frac{200}{f}$. Thus the R will be in the domain patrolled by one sensor platform. All of the sensors can misclassify Ws as R. We assume there are a sufficient number of escort
vessels to escort all the vessels classified as R. Other parameter values are those of Table 1 except $f=10$ nm; $c_{RR}(\tau)=c_{WR}(\tau)=0.99$. Figure 2 displays the alternating renewal process approximation for the probability R is detected and correctly classified as a function of the $x$-distance of the rectangular domain patrolled by one sensor and the number of Ws in the total domain of size 200 nm by 200 nm, $w$.

Discussion: In order to have an approximate probability of detecting and correctly classifying R before it leaves the domain of about 0.9, 4 sensor platforms are needed. Each sensor platform patrols a rectangular domain 50 nm by 200 nm. For the parameter values considered, the approximate probability of detecting and correctly classifying R before it leaves the domain is more sensitive to the size of the domain the sensor patrols than the number of Ws in the domain; however the number of Ws considered are 25, 50 and 100. Decreasing the size of the domain a single sensor platform patrols also decreases the number of Ws in the domain and hence the false positive rate.

7. Conclusions

We have presented models and results for a maritime domain awareness scenario. In the scenario there are neutral vessels, Whites (W), and one hostile vessel, Red (R) traveling within a domain. A patrolling overhead sensor platform $S$ detects vessels in the domain and classifies them as W or R. $S$ follows each vessel that is classified (perhaps incorrectly) as R until relieved by escorting vessels; during this time it is unavailable to detect additional vessels. The ability to detect, correctly classify, and escort R is influenced by the size of the domain, the number of Ws that are in the domain, and the probability of correctly classifying detected vessels. The introduction of technology such as the Automatic Information System (AIS) to track vessels should decrease the number of Ws in the domain and thus increase the ability to neutralize Rs.
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References


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