Quantifying Application Communication Reliability of Wireless Sensor Networks

AKHILESH SHRESTHA and LIUDONG XING*

Electrical and Computer Engineering Department, University of Massachusetts Dartmouth
285 Old Westport Road, North Dartmouth, MA 02747, USA

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Abstract: In this paper, we consider the problem of modeling and analyzing the application communication reliability of wireless sensor networks (WSN) with different topologies, including star, tree, mesh, and hierarchical clustering. We propose reliability measures that integrate the conventional connectivity-based network reliability with the sensing coverage measure of WSN. We apply a reduced ordered binary decision diagram (ROBDD) based progressive approach for evaluating the proposed coverage-oriented application communication reliability of WSN. Our study will provide useful insights for the WSN designers in choosing the appropriate network topology. We illustrate the basics and advantages of our approach by working through the analysis of an example WSN.

Keywords: application communication, coverage, reduced ordered binary decision diagram, reliability, wireless sensor network

1. Introduction

Due to recent advances in micro-electro-mechanical-systems (MEMS) and wireless technology, wireless sensor networks (WSN) have attracted tremendous research interest in commercial, industrial, and academic areas to build an environmentally aware intelligent network of multimodal sensors. WSN offer a remarkable potential to bridge the gap between the physical world of in-situ sensors and the virtual world of information services, thereby enabling us to assimilate a deep and broad understanding and control of the environment. However, before WSN is truly established as the environment-aware ubiquitous sensing, networking, and computing infrastructure, it is critical that these smart sensors deliver the promised sensing coverage and have reliable and dependable communication. Coverage analysis and reliability evaluation are, therefore, important tasks before the successful deployment of the WSN.

Reference [1] gives different taxonomy to classify sensor networks and proposes that communication within WSN can be conceptually classified into two categories: application and infrastructure. Infrastructure communication relates to the delivery of control, configuration, and maintenance data (e.g. query, path discovery, various policies like degree of coverage) between base station and sensor nodes. Application communication relates to the acquisition of sensed data about the phenomena and reliable delivery of these observed data.

*Corresponding author’s email: ldxing@ieee.org.
*The singular and plural of an acronym are always spelled the same.
from sensor nodes to the sink node. Therefore, in addition to the connectivity requirement, application communication requires that WSN maintain the desired sensing coverage that indicates the quality-of-service (QoS) of WSN. Thus, the measure for describing the application communication reliability (ACR) should integrate the connectivity reliability with the coverage QoS measure of WSN. In our earlier paper [2], we proposed measures and methodologies suitable for infrastructure communication reliability (ICR) of WSN. In this paper, we study the ACR of WSN with different topologies (described in Section 2).

The coverage concept and the connectivity-based network reliability are both widely researched topics. But only little work [3] has studied the two concepts in a unified framework. And no existing work, to the best of our knowledge, has provided quantitative measures integrating the two notions for the reliability analysis of WSN. Therefore, in this paper, we propose new ACR measures that integrate the conventional connectivity reliability with the sensing coverage of WSN. The new measures will provide a more accurate representation of the WSN reliability behavior than the existing measures for application communication paradigm. We present reliability expressions for a few prominent WSN topologies, including star, tree, mesh, and hierarchical clustering. We also propose an efficient approach for evaluating the proposed ACR measures of WSN in an accurate manner.

The remainder of the paper is organized as follows: Section 2 presents a brief background on different topologies of WSN. Section 3 gives the motivation of this research, the problem statement, and assumptions. Section 4 describes an illustrative example WSN. Section 5 presents the general ACR measure and specific reliability expressions for WSN with different topologies. Section 6 presents the proposed progressive reliability analysis approach. Section 7 gives an illustration of the proposed measures and approach through the analysis of the example WSN. The last section presents our conclusions and future work.

2. Background

WSN consist of numerous small and inexpensive sensor nodes with embedded intelligence. The limited power, memory, processing, sensing, and communication capacity of sensor nodes necessitate the dense deployment of sensor nodes in the area of observation. Moreover, due to these resource constraints and operation in unattended and harsh environments, sensor nodes are prone to failures. Therefore, a large number of redundant sensor nodes are usually deployed to achieve fault tolerance and required sensing coverage. Redundancy also facilitates the energy preservation through careful duty-cycle adjustment of sensor nodes [4].

The lack of existing energy and communication infrastructure in WSN demands ad hoc power-aware multi-hop routing operations, collaborative computing, and efficient maintenance of dynamic WSN topology without any centralized administration. The deployed nodes autonomously organize themselves into a communication framework. The communication topology affects the component connectivity and thereby various performance metrics. Presently, star, tree, mesh and hierarchical clustering topologies have emerged as the choice topologies for WSN [2].

**Star Topology** — In star topology, all the peripheral nodes are connected to a central node via direct, point-to-point links, i.e. all the nodes in the WSN are within a single hop from the central node. The central node, being the “hub”, is logically (and/or physically) at the center of the network. The central node can be either the base station itself or a gateway node that is in
direct communication with the base station.

**Tree Topology** — A natural and logical extension of the star topology is the tree topology where the sink node is the root and nodes at different levels in hierarchy are connected via direct links.

**Mesh Topology** — In mesh topology, each sensor node not only sends and receives its own message but also functions as a router to relay messages for its neighbors through the network. Mesh topology facilitates multi-hop communication and multiple communication paths from sensor nodes to the base station.

**Hierarchical Clustering Topology** — In the hierarchical clustering topology [5], all nodes in the WSN are joined at the lowest level. The sensors use their local neighborhood information to form a set of clusters and use some negotiation mechanisms to elect a cluster head (CH) for each cluster. The CH election process can be based on various parameters such as available energy resources, proximity to the base station, and number of neighbors. The CH manages the clusters by assigning duty cycles to sensor nodes and coordinating intra- and inter-cluster transmissions. Usually, it is the CH that performs data aggregation by processing and filtering the possibly redundant data received from its member nodes [6]. In hierarchical clustering topology, the CH at the lowest level are arranged into clusters in a higher level, and a CH is assigned for each cluster at this level. The process is repeated until the highest level in the hierarchy is reached. The number of levels may depend on various criteria including coverage requirement, deployment region, node density, and transceiver and sensing range.

The hierarchical clustering topology maintains a tree rooted at the sink node, with a hierarchy of CH as internal nodes and sensor nodes as leaf nodes of the tree, for network addressing and organization. Whenever a sensor node needs to send a message to the sink or another sensor node, it sends the message to its CH along a multi-hop route. The message is routed progressively to the immediately higher-level CH until it reaches the CH that is the common ancestor of both the source and destination nodes and therefore has the routing information about the destination node. The message is then routed progressively to lower-level CH until it reaches the destination node. Note that unlike the tree topology, the hierarchical clustering topology still maintains the multi-hop mesh routing for actual data communication, *i.e.* each path from a lower level to a higher level and vice versa is actually a multi-hop route.

### 3. Motivation and Problem Statement

It has been shown that the complexity of the network reliability analysis algorithms increases sharply with the increasing number of nodes [7]. Therefore, the traditional analytical network reliability analysis approaches, suitable for networks of moderate size (10~100 nodes), cannot be, at least directly, applied to WSN consisting of hundreds to thousands of nodes.

Besides the inadequacy of the existing analysis approaches, traditional network reliability measures based solely on connectivity is not adequate for describing the application communication behavior of WSN. In WSN, reliable monitoring of the phenomenon depends on both the sensing coverage and the communication of the collected data provided by the subset of sensors in the proximity of the phenomenon to the observer. Therefore the reliability in WSN incorporates the reliability of both data acquisition and data distribution processes. The data distribution behavior can be modeled appropriately by the traditional connectivity-
based reliability measures. Data acquisition aspect of WSN, however, needs to be modeled by the coverage concept [4].

To the best of our knowledge, only little work has been done in modeling and analyzing the reliability of WSN, and the existing approach employs a measure that is similar to the traditional end-to-end terminal-pair reliability measure based solely on the connectivity [8]. In this paper, we consider the problem of evaluating the application communication reliability of WSN by proposing new reliability measures that integrate the conventional connectivity reliability with the sensing coverage of the WSN, and by proposing efficient approaches to computing the proposed coverage-oriented reliability of WSN with various topologies.

Due to a large variety of WSN applications, coverage is subject to a wide range of interpretation. For example, coverage can be defined in terms of spatial or temporal observability. For our study, we consider area (or point) coverage where sensing coverage of a region of interest is defined as the ability to monitor every point in the region by at least one sensor node [3], [4]. In particular, we consider a more general concept of coverage, called $K$-coverage, which requires every point to be covered by at least $K$ sensors [4]. We define $K$-coverage-set as a set of sensor nodes such that all the points in the sensed field are covered by at least $K$ nodes. The degree of sensing coverage required by a WSN depends on the specific application requirements. Therefore, the value of $K$, which is at least one, indicates the fault-tolerant capability and QoS of WSN. In this work, we assume that the nodes providing coverage are stationary. However, as studied in [9], mobile nodes offer better coverage than stationary nodes; albeit the link reliability may deteriorate. Our future work will investigate the effects of node mobility and other coverage concepts on ACR.

We make the following assumptions in our analysis:

- The connectivity aspect of the WSN is modeled by an undirected probabilistic graph $G(V, E)$ [7].
- Both links and nodes fail $s$-independently with known probabilities.
- The failure probability for each link or node is given as a fixed probability for a given mission time or in terms of a lifetime distribution.
- All the sensor nodes are stationary during the mission time.
- In the case of WSN organized using hierarchical clustering topology, each sensor node belongs to a single cluster at any given time.

The accurate reliability analysis of WSN heavily depends on the realistic estimate of the failure parameters or distributions of its components (wireless links and sensor nodes). In particular, estimation of wireless link failure parameters is a challenging task because low-power radio communication is highly variable due to external sources of interference in the spectrum, contention with other nodes, multi-path effects, obstructions, fading, and other changes in the RF conditions, as well as node mobility [10]. Various sophisticated statistical models have been proposed to characterize the failure parameters (e.g., inter-error intervals) of wireless channels, including the simple strategy like the characterization of connectivity between nodes at a point in time using the probability of successful packet transmission [10]. However, most of the existing approaches, primarily based on continuous time Markov chains, for link reliability estimation use only a two-hop scenario due to the complexity of analytical models for multi-hop scenarios [11].

In this paper, we assume component failure parameters are given as input parameters of the problem and we only focus on the system-level reliability analysis. Follow-up research
will include investigation of methods for component failure parameter estimation as well as sensitivity of WSN reliability to changes in the input component parameter values.

4. Illustrative Examples

Figure 1(a) shows the deployment of 28 sensor nodes in a rectangular sensor field and the position of the base station in our example WSN. Figures 1(b), 1(c), and 1(d) show the mesh, tree, and hierarchical clustering configurations respectively for the same deployment in Figure 1(a). Note that each level in the tree configuration in Figure 1(c) has a star topology.

![Deployments and topologies](image)

**Fig. 1: Illustrative WSN**

In the hierarchical clustering WSN (Figure 1(d)), nodes that are connected to nodes in neighboring clusters are referred to as gateway nodes. And, gateway nodes that connect two level-i clusters are referred to as level-i gateway nodes. The cluster head of each cluster i is identified by the node labeled CH_i. These cluster heads represent the lowest level-0 cluster heads in our hierarchy. The single base station represents the sink node. In level-1 of the hierarchy (and highest level for this example), clusters 1, 2, and 3 are organized into a single cluster, and CH_1 is assigned as the level-1 cluster head.

In the following sections, we use this example for illustrating the analysis of ACR for WSN with different topologies.

5. Definitions of Application Communication Reliability (ACR)

Reliability is generally defined as “the probability that the system will perform its intended function under stated conditions for a specified period of time” [12]. In particular, the application communication reliability (ACR) of WSN is defined as the probability that every
point in the sensed field can be observed by at least $K$ sensor nodes (i.e., the field of interest is $K$-covered) and there exists an operational path from the subset of nodes that provides $K$-coverage to the sink node [13]. The value of $K$ should be configurable based on the specific application QoS requirements.

Next we elaborate the above definitions of ACR for different topologies of WSN. The following notations are used: $E_2$ represents the event that there exists an operational communication path between a pair of nodes, and $E_k$ represents the event that there exists an operational communication path between each pair of the $k$ nodes. Also, we assume sensor nodes that provide $K$-coverage for the area of interest constitute the subset $U_i$ here. If there exist $n$ such coverage sets, then each subset $U_i$ $(i = 1, 2, \ldots, n)$, which may contain different number of nodes $|U_i|$, guarantees $K$-coverage. In passing, if there are $n$ $K$-coverage-sets, then the lifetime of the network is increased by a fraction of $n$.

**Star** — The ACR of star topology is the probability that there exists a subset $U_i$ that provides $K$-coverage such that all the nodes in $U_i$ can communicate directly with the sink node.

$$A CR_{star} = \Pr(K\text{-coverage}) = \Pr\left(\bigcup_{\forall U} \left(\cap_{\forall i \in U} E_2 (\text{sink to node } i)\right)\right)$$ (1)

**Tree** — The ACR of tree topology is the probability that there exists a subset $U_i$ that provides $K$-coverage such that all the nodes in $U_i$ can communicate directly with its parent node, which in turn can communicate directly with their parent nodes, and so on, till lastly the highest level ancestors of nodes in $U_i$ can communicate directly with the sink node. Let $PN^{(k)}$ denote the level-$k$ parent node of nodes in $U_i$, $t$ denote the highest hierarchical level in the architecture, and $H_k$ denote the set of parent nodes above the subsets of nodes of interest at parent level $k$, $0 \leq k \leq t$, then:

$$A CR_{tree} = \Pr\left(\bigcup_{\forall U} \left(\cap_{\forall i \in U} E_2 (\text{sink to node } i)\right)\right)$$ (2)

If we assume that each of the coverage sets $U_i$ contains the subsets of nodes that are hierarchically below the same set of $PN^{(0)}$, then each of the terminal-pair events in (2) (except the last event) are identical for each of the $U_i$. Therefore, (2) can be represented as (3) below:

$$A CR_{tree} = \Pr\left(\bigcup_{\forall U} \left(\cap_{\forall i \in U} E_2 (\text{sink to node } i)\right)\right)$$ (3)

Equation (3) can be represented by (4) provided that we account for the failure function of each $PN^{(0)}$ only once.
ACR_{tree} = \prod_{i\in H} \Pr_{T_2} (\text{sink to } \text{PN}^{(0)}_{i}) \cdot \prod_{i\in H, j\in H_{i+1}} \Pr_{T_2} (\text{PN}^{(0)}_{i} \text{ to } \text{PN}^{(i-1)}_{j}) \cdot \ldots \cdot \\
\Pr_{T_2} (\text{PN}^{(0)}_{m} \text{ to } \text{PN}^{(0)}_{n}) \cdot \Pr(\text{K-coverage probability}), \tag{4}

\text{where}

\Pr(\text{K-coverage}) = \Pr \left( \bigcup_{i \in U} \left( \bigcap_{j \in U} E_2 (\text{PN}^{(0)} \text{ of node } i \text{ to node } j) \right) \right) \tag{5}

\text{Mesh} \quad \text{The ACR of mesh topology is the probability that there exists a subset } U_i \text{ that provides K-coverage such that all the nodes in } U_i \text{ can communicate with the sink node.}

ACR_{mesh} = \Pr(\text{K-coverage}) = \Pr \left( \bigcup_{i \in U} E_k (\text{sink and nodes in the subset } U) \right), \tag{6}

k = |U \cup \text{sink node}|

\text{Hierarchical Clustering} \quad \text{In general, the area of interest may cover a subset of clusters. The ACR of a WSN with hierarchical clustering topology is the probability that there exists a subset } U_i \text{ that provides K-coverage such that there exists operational paths from all the nodes in } U_i \text{ to the level-0 CH, from these level-0 CH to the respective parent level-1 CH and so on up to the top level CH, and from these top level CH to the sink node. Let } CH^{(k)} \text{ denote the level-k CH, } t \text{ denote the highest hierarchical level in the architecture, } R \text{ denote the set of clusters that contain the nodes of interest in } U_i, H_0 \text{ denote the set of CH}^{(0)} \text{ for clusters in } R, \text{ and } H_k \text{ denote the set of CH that is hierarchically above } R \text{ at parent level } k, 1 \leq k \leq t, \text{ then:}

ACR_{cluster} = \Pr \left( \bigcup_{i \in U} \left( \bigcap_{j \in U} E_2 (\text{sink to } CH^{(i)}_{j}) \right) \bigcap \ldots \bigcap \left( \bigcap_{w \in R} E_k (\text{CH}^{(0)} \text{ of cluster } \text{w} \cup \text{nodes of } U \text{ in cluster } \text{w}) \right) \right) \tag{7}

If we assume that each of the coverage sets } U_i \text{ contains the subsets of nodes that are contained in the same set of clusters } R, \text{ then each of the terminal-pair events in (7) are identical for each of the } U_i. \text{ Therefore, (7) can be represented as below:}

ACR_{cluster} = \Pr \left( \bigcup_{i \in U} \left( \bigcap_{j \in U} E_2 (\text{sink to } CH^{(i)}_{j}) \right) \bigcap \ldots \bigcap \left( \bigcap_{w \in R} E_k (\text{CH}^{(0)} \text{ of cluster } \text{w} \cup \text{nodes of } U \text{ in cluster } \text{w}) \right) \right) \tag{8}

Furthermore, (8) is tightly lower-bounded by (9), i.e., } ACR_{cluster} \text{ in (8) is at least the value given by (9). This is an important simplification because it is computationally intensive to store and manipulate the huge expressions represented by sub-expressions in (8). Also this is a realistic simplification under the practical assumption that the clusters are non-overlapping, and nodes that participate in communication between } CH^{(k+1)}_c \text{ and } CH^{(k)}_c \text{ do not generally participate in communication between } CH^{(k)}_c \text{ and } CH^{(k-1)}_c; \text{ and when they do participate, their
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contribution is insignificant.

\[ ACR_{\text{cluster}} = \prod_{i \in H} \Pr(\text{sink to } \text{CH}(i)) \times \prod_{i \in H} \Pr_{2}(\text{CH}(i) \text{ to } \text{CH}(i+1)) \times \ldots \times \prod_{m \in H, n \in H} \Pr_{2}(\text{CH}(m) \text{ to } \text{CH}(n)) \times \Pr(K\text{-coverage probability}), \]  

where

\[ \Pr(K\text{-coverage}) = \Pr \left( \bigcup_{\forall U} \left( \bigcap_{\forall w \in R} E_{w}(\text{CH}(0) \text{ of cluster } w \cup \text{nodes of } U \text{ in cluster } w) \right) \right) \]  

6. Proposed Approach

Revisiting the topologies discussed in Section 2 and the reliability definitions in Section 5, it is easy to find that the hierarchical clustering topology is a generic architecture that covers other topologies. Therefore, we use hierarchical clustering topology as a vehicle in describing the generic reliability evaluation algorithm for WSN. The star, mesh, and tree architectures are special cases of our proposed reliability models and evaluation methods.

6.1. The Progressive Approach to Reliability Analysis

Consider the fact that the complexity of network reliability analysis algorithms increases sharply with increasing number of nodes [7]; a progressive reduction scheme [2] is used to reduce the network graph level by level. The number of levels of reduction process is decided by the number of levels in the network hierarchy (refer to Section 2). Each reduced graph will be used for evaluating network reliability related to the corresponding level.

Specifically, consider the example hierarchical clustering WSN in Figure 1(d). The original network graph is referred to as level-0 graph. Therefore, CH1, CH2, and CH3 are referred to as the level-0 cluster heads (CH) of clusters 1, 2, and 3 respectively. The level-0 CH are arranged into a single level-1 cluster, and CH1 is assigned as the CH for this level-1 cluster. The level-0 graph in Figure 1(d) is used to evaluate the last term in (8) (a cluster’s k-terminal reliability) or in (10) (a cluster’s K-coverage probability). The level-0 graph is also analyzed to obtain the occurrence probability of the two-terminal event (i.e., two-terminal reliability) between the level-0 cluster head (denoted by CH(0)) and the level-0 gateway node that connects two level-0 clusters (denoted by g(0)).

![Fig. 2: Reduced Graphs](image)

The level-0 graph in Figure 1(d) is reduced to a level-1 graph containing only the CH(0), g(0), and g(1) as shown in Figure 2(a). The two-terminal reliability between CH(0) and g(0)
computed based on level-0 graph is assigned to the corresponding CH to gateway link at the level-1 graph. The level-1 graph in Figure 2(a) is used to evaluate the second to the last term (two-terminal event/reliability between CH\(^{(1)}\) and CH\(^{(0)}\)) in (8) or (9). The level-1 graph is also analyzed to obtain the two-terminal reliability between CH\(^{(1)}\) and g\(^{(1)}\).

The level-1 graph in Figure 2(a) is further reduced to a level-2 graph containing only the CH\(^{(1)}\) and g\(^{(1)}\) as shown in Figure 2(b). The two-terminal reliability between CH\(^{(1)}\) and g\(^{(1)}\) computed based on level-1 graph is assigned to the corresponding CH to gateway link at the level-2 graph. The level-2 graph in Figure 2(b) is used to evaluate the third to the last term (i.e., the first term, two-terminal reliability between the sink and CH\(^{(1)}\) for this specific example) in (8) or (9).

In general, in the progressive reduction scheme, a level-i graph is reduced to a graph containing only the level-i CH i.e., CH\(^{(i)}\) and level-j inter-cluster gateways, i.e., g\(^{(j)}\), where j >= i. This process is iterated until the graph is reduced to the top level of the hierarchy. Those reduced graphs are used to solve the different sub-problems in (8) and (9). Specifically, the last term is calculated from level-0 graph, the second to the last term is calculated from the level-1 graph, the third last term is calculated from the level-2 graph and so on. Finally, the first term is computed from the top-level graph. The reliability results are integrated using (8) or (9) to obtain the exact or lower-bounded ACR for the entire WSN, respectively.

Note that when using (9) to obtain the tight lower bound on ACR, if a component’s failure probability has been considered at a lower-level graph, it is considered zero in higher-level graphs. For example, the failure probability of each CH and each gateway node g\(^{(0)}\) are already considered in the level-0 two-terminal reliability evaluation, failure probability of zero is assigned to the CH and the gateway nodes in the reduced higher-level graphs. However, the inter-cluster gateway-to-gateway link (g\(^{(0)}\), g\(^{(0)}\)) failure probabilities need to be considered at level-1 graph, as they were not considered at level-0.

For evaluating the two-terminal, all-terminal, and in general k-terminal network reliability, an efficient reduced ordered binary decision diagrams (ROBDD) based method [14], [15] is adopted. The main steps of the method are reviewed as follows:

1. Order the network nodes and links using a good variable ordering heuristic. A heuristic is good in the sense that it yields a compact ROBDD model [16].
2. Generate ROBDD from the probabilistic (reduced) graph of the WSN at the corresponding level, i.e. each sub-expressions in the above reliability definitions are represented by a ROBDD structure.
3. Evaluate network unreliability recursively from the final ROBDD.

6.2. Coverage-Oriented Reliability Measure
The last term of (8), (9) is the cluster K-coverage event/probability. Based on our practical assumption in Section 3 that a sensor node belongs to a single cluster, we analyze each cluster individually to evaluate the K-coverage probability. In general, the K-coverage probabilities for various WSN topologies, viz. star in (1), tree in (5), mesh in (6), and hierarchical clustering topology in (10) can be calculated via the method outlined in this section.

First, we find the K-coverage-sets. This is an extension of the classic art gallery problem that deals with determining the set of observers necessary to cover an art gallery room such that every point is seen by at least one observer (i.e., 1-coverage set). A point p is covered by a node v if their Euclidean distance is less than the sensing range R\(_s\) of v, i.e., \(d(p, v) < R_s\). A K-
coverage-set is defined as a set of sensor nodes in a sensor field such that all the points in the field are covered by at least $K$ nodes [13]. Similar to the traditional minimal cut-set/path-set based reliability evaluation methods [17], we need to identify all the $K$-coverage sets for finding the $K$-coverage probability. As in traditional cut-set/path-set generation, the problem of enumerating the $K$-coverage sets is a NP-complete problem and the most straightforward solution is the brute force way. Specifically, we check each subset in the sample space (with $2^n$ subsets, where $n$ represents number of sensors in the field of interest), identifying whether that subset can $K$-cover the field. Reference [4] proposes a solution to the decision problem of whether every point in the monitored area is covered by at least $K$ sensors via checking the perimeter of every sensor’s sensing range. Reference [18] proposed techniques to formulate and solve the $1$-coverage as integer linear programming (ILP) problems. In this work, we adapted the algorithm in [18] to solve the $K$-coverage problem by redefining sensor intensity matrix as the coverage matrix and redefining the minimum intensity value as $K$. Specifically, we formulate the minimum $K$-coverage problem in WSN as follows:

- **Input:** A sensor set $S = \{s_1, s_2, ..., s_k\}$; a sensor field $A$, where $\{a_1, a_2, ... a_m\}$ is a partitioning of $A$; an area coverage matrix $C_{m \times n}$, where each element $C_{ij}$ is 1 if sensor $s_j \in S$ covers area $a_i$, and it is 0 otherwise.
- **Problem:** Find the minimum $K$-coverage sets of sensors, each set represented by vector $X_{n \times 1}$, where $X_{ij}$ is 1 if sensor $s_j \in S$ is in the set that $K$-covers $a_i$, and 0 otherwise. All the sensors $s_j$ with $X_{ij} = 1$ in the vector $X_{n \times 1}$ constitute a minimal $K$-coverage set. The ILP formulation of the problem is:

$\text{Minimize}: \mathbf{1}_{1 \times n}X_{n \times 1}$

$\text{Where}: C_{m \times n}X_{n \times 1} \geq \mathbf{1}_{1 \times K}$

The objective function being minimized is the number of sensors represented by the sum of elements of vector $X$. The constraints to be satisfied are that each region $a_i$ must be covered by at least $K$ sensors as represented by the inequality. LP_SOLVE, a noncommercial ILP solver can be used to obtain the solution for the above problem [19].

The above procedure can be used to find a single $K$-coverage set. However, note that there can be multiple $K$-coverage sets. Therefore, the above procedure is repeated until all the $K$-coverage sets are obtained. At $i^{th}$ iteration, the ($i$-1) $K$-coverage sets obtained so far, i.e. $\{U_1, U_2, ..., U_{i-1}\}$, is added to the constraint $U_i \in \{U_1, U_2, ..., U_{i-1}\}$ and $U_j \not\subset U_i (j < i)$ of the above optimization problem.

Let $U_i$ be composed of $t_i$ nodes $\{n_{i,1}, n_{i,2}, ..., n_{i,t_i}\}$. Then the probability of obtaining this coverage-set is the probability that there exists a Steiner tree that connects all the nodes $\{n_{i,j} | 1 \leq j \leq t_i\}$ in the coverage set with the cluster head(s), which is a $k$-terminal reliability problem [7], [15] with $k = 1$ $U_i \cup$ cluster head.

After obtaining the $K$-coverage sets, we apply the BDD-based method to find the $K$-coverage probability. Specifically, assume there are $w$ $K$-coverage sets denoted by $\{U_1, U_2, ..., U_w\}$. The $K$-coverage probability is simply the probability that at least one of the $K$-coverage sets is operational, i.e.,

$$\Pr(K\text{-coverage}) = \Pr\left(\bigcup_{U \in \mathcal{U}} \{E_k (CH^{(0)} \cup \text{nodes in the subset } U) \} \right)$$

(11)

To solve this equation, a BDD is generated for each $K$-coverage set $U_i$ by finding the $k$-
terminal reliability between \(\text{CH}^{(0)}\) and the nodes in \(U_i\). The final BDD is obtained by \textit{ORing} the sub-BDD for all the \(K\)-coverage sets. The evaluation of the final BDD using the recursive evaluation algorithm [14] gives the \(K\)-coverage probability.

7. Results
In this section, we illustrate the applications and advantages of our proposed measures and evaluation approaches via the analysis of the tree, mesh and hierarchical clustering topologies of the example WSN in Section 4. We consider the mission time of 10,000 hours. We assume that both links and nodes fail \(s\)-independently and exponentially. Table I shows the constant failure rates of WSN components. Although we assign exponential failure distribution to the nodes and links, our analysis methodology is suitable for any failure distribution. Note that the link failure parameter is a function of distance and transmission power. However, for simplicity of illustration, we assume each link in the mesh and hierarchical clustering topologies has the same failure parameter. Also, we assume that the long range transmission power in tree topology of Figure 1(c) is suitably increased so that the links have the same failure parameters as links in mesh and hierarchical cluster. Intuitively, this will give an upper bound on the reliability of tree WSN with similar transmission energy to that of the mesh and hierarchical clustering WSN. Our future work includes the simulation study of the trade-off in communication energy efficiency and reliability.

| Table I: Failure rates \((hr^{-1})\) for WSN components |
|---|---|---|---|
| Links | Base station | Cluster head | Nodes |
| 2e-6 | 1e-7 | 1e-6 | 1e-6 |

We consider 2-coverage as the QoS requirement for each region (identified by the clusters 1, 2, and 3 in Figure 1(d)) and the entire WSN with tree, mesh and hierarchical clustering topologies. The 2-coverage-sets can be found via the method outlined in Section 6.2. We assume that the 2-coverage-sets are generated as follows:

- **Cluster 1**: \(CS_{11} = \{10, 8, 1, 5\}\), \(CS_{12} = \{10, 8, 6, 4, 5\}\)
- **Cluster 2**: \(CS_{21} = \{17, 15, 11, 12\}\), \(CS_{22} = \{17, 15, 18, 13\}\)
- **Cluster 3**: \(CS_{31} = \{28, 26, 21, 24\}\), \(CS_{32} = \{28, 26, 25, 22, 21\}\)
- **Entire field**:
  - \(CS_{E1} = CS_{11} \cap CS_{21} \cap CS_{31}\)
  - \(CS_{E2} = CS_{11} \cap CS_{21} \cap CS_{32}\)
  - \(CS_{E3} = CS_{12} \cap CS_{22} \cap CS_{31}\)
  - \(CS_{E4} = CS_{12} \cap CS_{22} \cap CS_{32}\)
  - \(CS_{E5} = CS_{12} \cap CS_{21} \cap CS_{31}\)
  - \(CS_{E6} = CS_{12} \cap CS_{21} \cap CS_{32}\)
  - \(CS_{E7} = CS_{12} \cap CS_{22} \cap CS_{31}\)
  - \(CS_{E8} = CS_{12} \cap CS_{22} \cap CS_{32}\)

Note that although the coverage sets for the entire field has been specified as intersect of the coverage sets of clusters 1, 2, and 3; this is not a general rule for obtaining the coverage sets of the entire field (as there may be overlapping areas of coverage in the three clusters).

For illustration, we present the ACR expression of the entire field for the hierarchical clustering topology in Figure 1(d). According to (9), ACR for the entire field is given by,

\[
\text{ACR}_{\text{cluster}} = \prod_{i \in \{3\}} \Pr_2(\text{sink to CH}^{(1)} i) \ast \prod_{i \in \{3\}, j \in \{1, 2\}} \Pr_2(\text{CH}^{(1)} i \text{ to CH}^{(0)} j) \ast \Pr(K\text{-coverage}),
\]

where

\[
\Pr(K\text{-coverage}) = \Pr \left[ \bigcup_{w \in \{1, 2\}} \bigcap_{i \in \{1, 2\}} \bigcap_{j \in \{1, 2\}} E_i \text{ (CH}^{(0)} \text{ of cluster w } \cup \text{ nodes of } U \text{ in cluster w}) \right]
\]
The ACR expressions for each cluster of the hierarchical clustering topology and for other topologies can be similarly established. We tabulate the results of ACR for tree, mesh, and hierarchical clustering topologies for regions within cluster 1, cluster 2, cluster 3, and the entire sensor field of the example WSN from Section 4 in Table II.

<table>
<thead>
<tr>
<th>Cluster #</th>
<th>Tree</th>
<th>Mesh</th>
<th>Hierarchical Clustering</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.88603396</td>
<td>0.98350317</td>
<td>0.94373739</td>
</tr>
<tr>
<td>2</td>
<td>0.88450899</td>
<td>0.95799242</td>
<td>0.92228582</td>
</tr>
<tr>
<td>3</td>
<td>0.85836780</td>
<td>0.95320211</td>
<td>0.89285079</td>
</tr>
<tr>
<td>Entire WSN</td>
<td>0.71573506</td>
<td>0.89809615</td>
<td>0.77886535</td>
</tr>
</tbody>
</table>

Note that in our analysis of ACR for the entire WSN with mesh topology, because there are eight $K$-coverage sets corresponding to eight $k$-terminal reliability sub-problems, we need to generate, store, and manipulate eight ROBDD. Due to memory limitations, we were unable to obtain the exact value of the ACR for this scenario. As we mentioned in Section 3, this is also due to the NP-complete nature of the network reliability problem. The value 0.89809615 given in the Table II for this scenario is the lower bound obtained via multiplication of the reliabilities for cluster 1, cluster 2, and cluster 3. Our future work involves investigating methodologies and heuristics for improving the efficiency of our analysis approach.

For all the scenarios, the mesh topology offers the highest ACR due to multiple paths through the network. The tree topology has the lowest reliability due to the use of only a single direct link between nodes at successive levels in the hierarchy. The hierarchical clustering topology is a compromise between the two extremes. Its ACR is better than the tree’s ACR as it still maintains multi-hop paths, while it has lower reliability than mesh because each communication between nodes at different clusters must route through affiliated cluster heads, which are single-point of failures.

Also, in a very naïve interpretation, cluster 3 with the lowest reliability is the best candidate to improve WSN coverage-oriented application communication reliability. Nevertheless, a better approach for identifying the candidate regions is to perform sensitivity analysis [14], [20] which is our future work.

8. Conclusions

We proposed a novel approach of integrating sensing coverage with conventional network connectivity for the application communication reliability analysis of star, mesh, tree, and hierarchical clustering WSN. The coverage-oriented reliability of WSN provides a better measure of the performance of the WSN than the ones based solely on connectivity or coverage.

As illustrated through the example, our reliability measures can reflect the QoS of WSN through the integration of sensing coverage. The proposed approach for the reliability analysis is practical and easy to implement because it is progressive and is based on the computationally efficient ROBDD approach. However, there is a scope for improvement of the algorithm for exact reliability evaluation.

Our future work also includes the consideration of sensor nodes mobility into the reliability analysis, sensitivity analysis, and integrating other sensing coverage models.
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References


**Akhilesh Shrestha** was awarded the M.S. degree in Computer Engineering from the University of Massachusetts Dartmouth in 2005. He is currently a Ph.D. student in the Electrical and Computer Engineering Department at the University of Massachusetts Dartmouth. His current research interests include fault tolerant and dependable computing, wireless sensor networks, and database systems. He is a student member of IEEE since 2003.

**Liudong Xing** received her M.S. and Ph.D. degrees in Electrical Engineering from the University of Virginia, Charlottesville, VA in 2000 and 2002, respectively. Since 2002, Dr. Xing has been an Assistant Professor with the Electrical and Computer Engineering Department, University of Massachusetts - Dartmouth. Dr. Xing served as a program co-chair for the 2006 IEEE International Symposium on Dependable, Autonomic and Secure Computing, a program vice chair for the 2007 International Conference on Embedded Software and Systems, and an associate guest editor for the Journal of Computer Science on a special issue of “Reliability and Autonomic Management”. She is currently an Editor for Short Communications of the International Journal of Performability Engineering. She is the recipient of the IEEE Region 1 Technological Innovation (Academic) Award in 2007. Her current research interests include dependable computing and networking, reliability engineering, fault-intrusion tolerant computing, and wireless sensor networks. She is a senior member of IEEE and a member of Eta Kappa Nu and ASEE.