Steady-State Availability and MTBF of Systems Subjected to Suspended Animation

SUPRASAD V. AMARI*
Relex Software Corporation
540 Pellis Road, Greensburg, PA 15601, USA

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Abstract - In most practical cases, during a system failure or downtime, all non-failed components are kept idle. This phenomenon is known as suspended animation (SA). In this paper, we provide a simple and efficient method to compute the availability indices of repairable systems subjected to suspended animation. An important aspect of the proposed method is that it is not restricted to exponential failure and repair distributions. Further, the proposed method can be applied to any system configuration with embedded hierarchical k-out-of-n subsystems subjected to suspended animation.

Keywords: suspended animation, steady-state availability, repairable system, MTBF

1. Introduction

Steady-state availability ($A$) and mean time between failures (MTBF) are the two most important measures of repairable systems. The reciprocal of MTBF is known as failure frequency ($\omega$), which is the expected number of failures per unit time. Other measures, such as mean time to failure (MTTF) and mean time to repair (MTTR), can easily be found from $A$ and $\omega$ [1].

In most research papers and textbooks, it is assumed that non-failed components continuously operate, even during a system failure. This assumption simplifies the availability analysis. However, in most practical cases, non-failed components are kept idle to eliminate further damage to the system [2]. This is known as suspended animation (SA) [3]. Suspended animation introduces dependencies among the component states. Schneweiss [4] used conditional probabilities to compute the steady-state availability of series systems with SA. Reference [2] extended this method to analyze $k$-out-of-$n$ systems with SA. Both [2] and [4] assumed exponential failure and repair distributions for the components. To our knowledge, there are no efficient methods for availability analysis of systems subjected to SA. Particularly, there are no efficient methods for the cases with either (a) general system configurations or (b) general failure and repair distributions. In this article, we propose an efficient method to compute the steady-state availability and MTBF of repairable systems subjected to SA. The method is applicable for (a) general configurations with embedded hierarchical $k$-out-of-$n$ subsystems, and (b) general failure and repair distributions.
2. Proposed Method
The proposed method is motivated by two key concepts: (1) conditional probabilities can be used to solve the steady-state availability of series systems with SA [4], and (2) when component failure behaviors are independent, the steady-state availability of a system is independent of the failure and repair distributions of its components [2]. Reference [2] also extended these results for the series systems with SA. By integrating these two concepts, we can guess that steady-state availability of any system with SA can be computing using conditional probabilities. In reference [1], we proved these results for systems consisting of embedded hierarchical \( k \)-out-of-\( n \) subsystems with SA. The results are proved using two key theorems: (1) using infinitely many states (or supplementary variables), any system behavior can be modeled using Markov processes, and (2) elimination of certain states increases the probability of all other states in the same proportion. The actual proofs of these theorems are lengthy and are provided in [1]. As in reference [2], our results are applicable when steady-state availability exists for each component in the system.

Algorithm

1. For each component, compute steady-state availability and failure frequency

\[
A_i = \frac{MTTF_i}{MTTF_i + MTTR_i}; \quad \omega_i = \frac{1}{MTTF_i + MTTR_i}
\]

(1)

2. Starting from the lowest level subsystems to the highest level (system) compute:
   a. Steady-state availability and failure frequency using \( A \) and \( \omega \) of its immediate next level subsystems using Algorithm 1. It should be noted that for the lowest level subsystems, the immediate next level subsystems are the components.

Algorithm 1: \( k \)-out-of-\( n \) subsystems

1. Inputs: \( A_i \) and \( \omega_i \) of each of its next level subsystem \( i, \) \( i = 1, \ldots, n. \)
2. Compute: Availability (\( \text{Avail} \)) and failure frequency (\( \text{Freq} \)):

   \[
P[0] = 1; \quad F[0] = 0;
   \]
   
   For \( i = 1 \) to \( k \) Do
   
   \[
P[i] = 0;
   \]
   
   \[
F[i] = 0;
   \]
   
   Done

   For \( i = 1 \) to \( n \) Do

   For \( j = k-1 \) downto \( 1 \) Do

   \[
F[j] = \omega_i \cdot (P[j-1] \cdot F[j]) + A_i \cdot F[j-1] + (1 - A_i) \cdot F[j];
   \]

   \[
P[j] = A_i \cdot P[j-1] + (1 - A_i) \cdot P[j];
   \]

   Done

   Done

   \[
\text{Avail} = P[k] / P[k-1]; \quad \text{Freq} = F[k] / P[k-1];
   \]

3. Example
We demonstrate the proposed method (or algorithm) through a simple example shown in Figure 1. The system consists of three components: 1, 2, and 3. The system has two subsystems: \( X \) and \( Y \) arranged in series (2-out-of-2 system). The subsystem \( X \) has only one
component. The subsystem $Y$ has two components arranged in parallel (1-out-of-2 subsystem). If both 2 and 3 are failed (subsystem $Y$ is failed), then the component 1 (subsystem $X$) is kept idle. Similarly, if component 1 (subsystem $X$) is failed, both 2 and 3 are kept idle.

![Example System](image)

For simplicity, we assume exponential failure and repair distributions for this example. Hence, $MTTF_i = 1/\lambda_i$ and $MTTF_i = 1/\mu_i$. We first calculate, $A_i$ and $\omega_i$ for each component from (1). Then, using Algorithm 1, we calculate $A$ and $\omega$ for the subsystems $X$ and $Y$.

\[
A_X = A_1 \quad \omega_X = \omega_1 \\
A_Y = A_2 + U_2 \cdot A_3 \quad \omega_Y = \omega_2 + U_2 \cdot \omega_3 - A_3 \cdot \omega_2
\]

(2)

In the final call to Algorithm 1, we compute system level $A$ and $\omega$.

\[
A = \frac{A_X \cdot A_Y}{A_X + U_X \cdot A_Y} \quad \omega = \frac{A_X \cdot \omega_Y + \omega_X \cdot A_Y}{A_X + U_X \cdot A_Y}
\]

(3)

4. Conclusions

We demonstrate a combinatorial method to compute the steady-state availability and MTBF of systems subjected to SA. The method is applicable for a wide range of (a) failure and repair distributions, and (b) system configurations. Our results coincide with the results of special cases provided in [2-4].

References


Suprasad V. Amari is a Senior Reliability Engineer at Relex Software Corporation. Dr. Amari received both his MS and PhD in Reliability Engineering from the IIT, Kharagpur.